Macroeconomic Dynamics

- Accounting System Dynamics Approach-

(On-going Draft Version 2)

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Preface

My Off-Road Journey for A Better World

Futures Studies

In early 1980's, I was told by one of the graduate colleagues at the University of California, Berkeley, that if I continue the research involving Marx and Keynes in addition to neoclassical theory, I would never get a good job offer in the United States. He was right. It was a time for Reganomics which has eventually evolved to the era of globalization in 1990's. Paying little attention to his thoughtful suggestion, I pursued my Ph.D. thesis on the subject "Beyond Walras, Keynes and Marx - Synthesis in Economic Theory Toward a New Social Design", which, alas, became a start of my off-road journey. Main part of the thesis was luckily published with the same title [58], yet it has been left unnoticed among main stream economists.

When I started teaching at the Dept. of Economics, University of Hawaii at Manoa, I almost lost my energy to continue the research on neoclassical mathematical theory for academic survival, because the theory seemed to be totally detached from the economic reality. It was in those lost days when my introduction to the futures studies and Prof. Jim Dator, then secretary general of the World Futures Studies Federation, took place by chance in Hawaii in 1987. Upon arrival to Japan next year, I immediately joined the Federation, and became very active on futures studies for more than ten years since then.

Among the activities of futures studies I have been involved, a major one was the organization of futures seminar series in Awaji Island, Japan, with an objective to establish a future-oriented higher institution dubbed the Network University of the Green World (http://www.muratopia.org/ NUGW). The seminars had been held for seven years from 1993 through 1999, then suspended due to the lack of fund. In the book based on the first seminar in 1993, I have proclaimed that

> Thus, what has been missing in industrial-age scientific research, and hence in the academic curricula of present-day higher institutions, is a study of interrelated wholeness and interdependences [61, p.200].

In order to fill the missing niche, I have tried, with a help by the seminar participants, including Novel laureate Jerome Karle, to establish a new wholistic field of study dubbed FOCAS, meaning Future-Oriented Complexity and Adaptive Studies, in vain. Yet, my conviction on the need for such futures studies for higher education continued to remain as worth being upheld. Faced with the threat of our survival due to climate changes and environmental disasters, future-oriented studies of interrelated wholeness and interdependence is, I believe, more urgently needed for solving these complex problems, since solutions offered by fragmented professionals at the current higher institutions might be the causes of another problems as Asian wisdom connotes. For our survival and sustainability, we need future-oriented higher education which provides wholistic visions and solutions to the present complex problems caused by fragmented science and technology of the present-day higher education. This conviction became a fruit of reward for me at the cost of abandoning neoclassical economic research in a traditional academic stream.

System Dynamics

Throughout the future-oriented activities later on, I was luckily led to the systems view, specifically a method called system dynamics, by chance. It seemed to me a totally new field of study that makes a heavy use of computer simulation for analyzing dynamic behaviors of system structures in physics, chemistry, engineering, environmental studies, business and economics, and public policies, to name a few, in a uniform fashion. In short, its methodology can uniformly cover many fragmented fields of studies, and in this sense it seemed for me to be able to share a similar interdisciplinary vision with future-oriented studies. After many years' frustration on the futures studies, I've jumped in the filed by attending its international conference in Istanbul, Turkey, in 1997. Since then I have been continually attending the system dynamics conferences up to the the present day.

It didn't take much time to realize that, due to its interdisciplinary nature, system dynamics is also facing the similar difficulty in finding an academic position as a discipline in the highly fragmented current higher educational system, as future-oriented studies have bee suffering similarly. In other words, system dynamics and futures studies can have no comfortable places in the current universities. The only difference is the use of computer in the former, and the use of our brain in the latter.

Hence, it seemed to me that future-oriented studies and system dynamics constitute two major fields of future's higher education, using our brain on the one hand and computer on the other hand for a study of interrelated wholeness and interdependence in order to attain human and environmental sustainability. In fact, it has been repeatedly argued at the international conferences whether system dynamics is merely a tool or discipline. For me it seemed to be not to the point and accordingly a fruitless argument,

On the contrary, the following description by Prof. Jay Forrester, a founder of system dynamics, on the nature of system dynamics looked to me to the point.

> Such transfer of insights from one setting to another will help to break down the barriers between disciplines. It means that learning in one field becomes applicable to other fields. There is now a promise of reversing the trend of the last century that has been moving away from the "Renaissance man" toward fragmented specialization. We can now work toward an integrated, systemic, educational process that is more efficient, more appropriate toward a world of increasing complexity, and more compatible with a unity in life [17].

It is a useless effort to search for an appropriate academic citizenship at the current fragmented higher education. In this sense, it seems to be a right choice to introduce the visions and methods of system dynamics to the K-12 education where academic fragmentation does not yet break down into the learning process. The reader may visit a creative learning Web site for its successful introduction at http://clexchange.org.

I felt I have finally been led to a right truck, after more than a decade-long off-road journey, toward a better world. If I had stayed at the economics profession, I would have never encountered system dynamics as most economists are currently still unaware of it. What I have learned from system dynamics is the importance of system design.

My continuing off-road journey got refurbished with this spirit of system design. In the falls of 1999 and 2000, I had a chance to visit MIT where I was introduced systems thinking and system dynamics for the first time as if I was a first-hand learning student by Prof. John Sterman and his doctorate students as well as Prof. Jay Forrester and his undergraduate team of Road Map project (educational self-learning system dynamics program through Web). This became my off-road journey of no return from system dynamics in my profession.

Accounting System Dynamics

Instead of being forced to stay in the economics profession, I was luckily given a chance to teach system dynamics at two management schools in Japan; first at the Osaka Sangyo University in Osaka, then Doshisha Business School in Kyoto. System dynamics obtained its first citizenship in this way as academic subject to be taught in the fragmented higher educational system in Japan.

Eventually, as a faculty member of management and business schools, I strongly felt it necessary to cover accounting system in my system dynamics class, Yet, my search for SD-based accounting system turned out to be unsuccessful, giving me an incentive to develop a SD method of modeling financial statements and accounting system from a scratch. I started working on the SD-based accounting system in the summer of 2001 when I was spending relatively a quiet time on a daily rehabilitation exercise in order to recover from the physical operation on my shoulder in June of the same year. This retreat environment provided me with an opportunity to read books on accounting intensively. My readings mainly consisted of the introductory books such as [27],

[34], [35], [51] and [52], since my knowledge of accounting was limited in those days[1]. Through such readings, I have been convinced that system dynamics approach is very effective not only for understanding the accounting system, but modeling many types of business activities. This conviction fruitfully resulted in my presentation on the principle of accounting system dynamics at the 21st international conference of the System Dynamics Society in New York in 2003 [63], which became a turning point in my off-road journey.

Rekindled in Berkeley, California

In the same summer of 2003, I was luckily offered an 8 months' sabbatical leave, and come back to Berkeley in almost 18 years since I left in 1986, this time as a visiting scholar at the Haas School of Business, not the Economics Department. My old friend, Nobie Yagi, from Berkeley days kindly provided his second house on his site for my family's stay, which gave me a good opportunity to talk with him almost daily. He received Ph.D. in finance and options trading from Berkeley around the same time as I did.

Conversation with him, together with my research environment at the business school rekindled my interest in economics, specifically macroeconomics and finance again. Even so, in those days I have already taken an off-road journey away from main stream economics, and decided to investigate it from my off-road side way. Specifically, I resolved to start reconstructing macroeconomic theories on the basis of the principle of accounting system dynamics which was completed in the same summer.

Since then, being led by the inner logic of accounting system dynamics and macroeconomics, I have spent almost my entire off-road journey on a step-by-step construction of macroeconomic models, which turned into a series of presentation of papers such as [64], [65], [66] and [67]. This series of macroeconomic modeling was completed in 2008 as [68] with a follow-up analytical refinement method of price adjustment mechanism in [69] next year.

An Oasis in Wellington, New Zealand

Second good luck visited me on my off-road journey as two months' short sabbatical leave in 2009 at the Victoria Management School, Victoria University of Wellington, New Zealand. Prof. Bob Cabana, a well-known leading scholar in system dynamics, kindly hosted my visit. This good luck enabled me to review the above paper series uniformly for the publication of this book. Almost daily conversation with him over lunch, as well as a lovely research environment in Wellington, encouraged me to keep working on the draft. Without this stopover in New Zealand as an oasis in my off-road journey, the draft would not have been completed.

[1]In addition to these books, a paper dealing with corporate financial statements [2] was recently in 2002. However, current research for modeling financial statements is independently carried out here with a heuristic objective in mind.

National Model

I was a late comer to the research community of system dynamics. While my step-by-step macroeconomic modeling was advancing, some researchers have kindly suggested at the conferences that I should review the research papers on the National Model project that was led by Prof. Jay Forrester with several Ph.D. students at MIT.

Unfortunately, the national model itself was not available and its related papers were scattered around. Under such situation, my survey managed to cover the following papers [13], [14], [15], [16], [18], [19], [20], [21], [22], [23], [24], [25], [28], [32], [46], [47]. Yet, the review of these papers only gave me an impression as if I were, with my eyes closed, touching various parts of an elephant without knowing what the elephant looks like. During the 23rd international conference of System Dynamics Society in New York, July, 2005, in which I presented a SD-based Keynesian model, I have strongly felt that my research cannot advance without understanding a whole picture of National Model, because my modeling approach, I feared, might have been already taken by the National Model project team.

Without losing time, in September of the same year, I visited Prof. Jay Forrester in his office at MIT. We spent almost two hours on discussing about his National Model. He told me that the national model is still going on, and I may have no chance to take a look at it until it's completed. Even so, the conversation turned out to be a very fruitful to me, out of which I got convinced that my modeling approach on the basis of accounting system dynamics is quite different from his modeling method. This conviction gave me an energy to continue my off-road journey in my own way. At the same time, I truly hoped that the national model would be completed in the near future.

In the spring of 2007, I was invited to review the Ph.D. dissertation by David Wheat at the University of Bergen, Norway, whose title is "The Feedback Method: - A System Dynamics Approach to Teaching Macroeconomics [56]". His model, written by Stella software, seemed to me to be a simple version of the National Model. In this sense it became the first complete macroeconomic model ever presented to the public, and his effort should be congratulated. In the following year, at the 26th international conference of System Dynamics Society, Athens, Greece, I have presented a complete macroeconomic model, written by Vensim, on the basis of accounting system dynamics [68].

At a Vista Point over a Green World

I now feel as if I'm standing at a magnificent vista point in my off-road journey where I can glimpse the peak of a better world I've been searching for. A better world is a green world, which I try to describe in the last chapter 12 of this book. To be specific, this turns out to be a world of MuRatopian economy I pursued in my dissertation in 1980's. It is founded this time on a basis of debt-free money system. I stop my off-road journey at this point. It is my hope that the reader will continue this off-road journey to the summit, so that eventually it becomes a main street for a better and green world.

With many thanks to those who guided me and offered a cordial help during my off-road journey.

August, 2010
Awaji Island, Japan

Kaoru Yamaguchi

Acknowledgments

At the 28th International Conference of the System Dynamics Society, held in Seoul, Korea, July 29, 2010, I have offered the SD Workshop: An Introduction to Macroeconomic Modeling – Accounting System Dynamics Approach – in which the draft of this book was distributed, for the first time, to the participants to obtain their feedbacks. Then, two months later, it is distributed to the participants at the 6th Annual AMI Monetary Reform Conference in Chicago, the United States, Sept. 30 - Oct.3, 2010. Following are those who gave me comments and suggestions.

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I do gratefully acknowledge their cordial feedbacks. All remaining errors, however, are mine.

October, 2010

Kaoru Yamaguchi

Part I

Accounting System Dynamics

Chapter 1

System Dynamics

This chapter[1] introduces system dynamics from a dynamics viewpoint for beginners who have no formal mathematical background. First, dynamics is dealt in terms of a stock-flow relation. Under this analysis, a concept of DT (delta time) and differential equation is introduced together with Runge-Kutta methods. Secondly, in relation with a stock-dependent flow, positive and negative feedbacks are discussed. Then, fundamental behaviors in system dynamics are introduced step by step with one stock and two stocks. Finally, chaotic behavior is explored with three stocks, followed by discrete chaos.

1.1 Language of System Dynamics

What is system dynamics? The method of system dynamics was first created by Prof. Jay Forrester, MIT, in 1950s to analyze complex behaviors in social sciences, specifically, in management, through computer simulations [11]. It literally means a methodology to analyze dynamic behaviors of system. What is system, then? According to Jay Forrester, a founder of this field, "A system means a grouping of parts that operate together for a common purpose [12], page 1-1." For instance, following are examples of system he gave:

- An automobile is a system of components that work together to provide transportation.
- Management is a system of people for allocating resources and regulating the activity of a business.
- A family is a system for living and raising children.

[1] This chapter is based on the paper presented at the 17th International Conference of the System Dynamics Society: *Systems Thinking for the Next Millennium*, New Zealand, July 20 - 23 ,1999; specifically in the session L7: Teaching, Thursday, July 22, 1.30 pm - 3.00 pm, chaired by Peter Galbraith.

According to Edward Deming, a founder of quality control, " A system is a network of interdependent components that work together to try to accomplish the aim of the system [6], p. 50."

Both definitions share similar ideas whose keywords are: interdependent parts or components, and common purpose or aim.

To describe the dynamics of system thus defined, Forrester created a language of system dynamics consisting of four letters: Stock, Flow,Variable and information Arrow. They are illustrated in Figure 1.1. Flow is always connected to Stock. Arrow connects Variable, Flow and Stock.

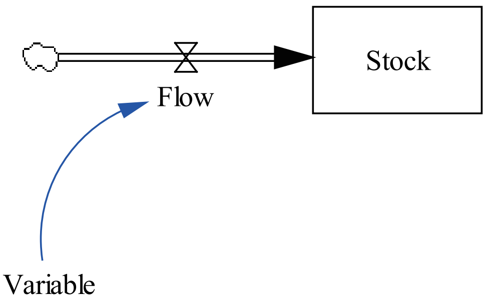

Figure 1.1: Language of System Dynamics

This is a very genius idea. To describe a system, all we need is four letters. If you learn some simple grammar, you could combine these 4 letters to write a sentence, paper, or even a big book. Our body is an excellent example of system that consists of about 30 thousands genes, yet genes are composed of only four types of DNA. Compared with this, English has 26 letters and Japanese has 55 phonetic letters.

Textbooks and Softwares

Several textbooks are now available for learning how to write sentences with the above four letters such as [48] and [53] for business and management modeling, and [8] and [10] for sustainable environment. The reader is strongly recommended to learn system dynamics with these textbooks. I regularly use [48], [53] and [8] in my MBA and policy classes.

Another way to learn quickly is start using SD softwares such as Stella, Vensim and PowerSim with manuals. For the modeling in this book, I have selected Vensim for two reasons; its graphical capability for creating a model and availability of its free version such as Vensim PLE and Vensim Model Reader. It is recommended that the reader runs the model attached to this book simultaneously.

Economics students might need a little bit more rigorous approach to modeling in relation with difference and differential equations, which is, however, not well covered in the above introductory textbooks, This is why I decided to add another introduction to system dynamics method in this chapter.

1.2 Dynamics

1.2.1 Time

For beginners system dynamics seems to be an analysis of systems in terms of feedback mechanisms and interdependent relations. In particular this is true when graphics-oriented softwares of system dynamics become available for PCs and Macs such as Stella, Vensim and PowerSim, enabling even introductory students to build a complicated dynamic model easily without knowing a mechanism of dynamics and differential equations behind the screen.

Accordingly, the analysis of dynamics itself has been de-emphasized in a learning process of system dynamics. Dynamic analysis, however, has to be a foundation of system dynamics, through which systems thinking will be more effectively learned. This is what I have experienced when I encountered a system dynamics as a new research field.

Dynamic analysis needs to be dealt along with a flow of time; an irreversible flow of time. What is time, then? It is not an intention to answer this philosophically deep question. Instead, time is here simply represented as an one dimensional real number, with an origin as its initial starting point, that flows toward a positive direction of the coordinate.

In this representation of time, two different concepts can be considered. The first concept is to represent time as a moment of time or a point in time, denoted here as τ; that is, time is depicted as a real number such that $\tau = 1, 2, 3, \ldots$. The second one is to represent it as a period of time or an interval of time, denoted here as t, such that $t =$ 1st, 2nd, 3rd, $\ldots$, or more loosely $t = 1, 2, 3, \ldots$ (a source of confusion for beginners). Units of the period could be a second, a minute, an hour, a week, a month, a quarter, a year, a decade, a century, a millennium, etc., depending on the nature of the dynamics in question.

In system dynamics, these two concepts of time needs to be correctly distinguished, because stock and flow - the most fundamental concepts in system dynamics - need to be precisely defined in terms of either τ or t as discussed below.

1.2.2 Stock

Let us now consider four letters of system dynamics language in detail. Among those letters, the most important letter is stock. In a sense, system could be described as a collection of stock. What is stock, then? It could be an object to be captured by freezing its movement imaginably by stopping a flow of time, or more symbolically by taking its still picture. The object that can be captured this way is termed as stock in system dynamics. That is, stock is the amount that exists at a specific point in time τ , or the amount that has been piled up or integrated up to that point in time.

Let x be such an amount of stock at a specific point in time τ . Then stock can be defined as $x(\tau)$ where τ can be any real number.

Stocks thus defined may be classified according to their different types of nature as follows.

Physical Stock	Non-Physical Stock
• Natural Stock • Capital Stock • Goods-in-Process and Use	• Information • Psychological Passion • Indexed Figures

Table 1.1: Classification of Stock

- Natural stock consists of those that exist in our natural environment such as the amount of water in a lake, number of trees and birds in a forest and world population.
- Capital stock is a manufactured means of production such as buildings, factories and machines that have been used to produce final goods.
- Goods-in process are those that are in a process of production, which are sometimes called intermediate goods, and goods-in-use are final products that have been used by consumers such as cars and computers.
- Information (and knowledge) is non-physical stock that is stored in various forms of media such as papers, books, videos, tapes, diskettes, CDs and DVDs.
- Psychological passion is emotional stock of human beings such as love, joy, happiness, hatred and anger that have been stored somewhere in our brain tissues.
- Indexed figures are specific forms of information stock that are (scientifically) defined to describe the nature of environment and human activities such as temperature, prices, deposits and sales values.

1.2.3 Stock-Flow Relation

Since Newton, it has been a challenge in classical mechanics to describe a change in stock. One of the methods widely employed is to capture the amount of stock at various discrete points in time, $\tau = 0, 1, 2, 3, \cdots$ and consider a change in stock at the next point as the amount of the stock at the present point and its increment between the present and next points; that is, τ and $\tau + 1$. Let us call such an interval of time between these two points a unit interval. The length of unit could be, as already mentioned above, a second, a minute, an hour, a day, a week, a month, a year, or whichever unit to be suitable for capturing the movement of the stock in question. Hence, a period of time t could be defined as a t-th unit interval or period, counting from the origin; that is, $\tau = 0$.

Flow is defined as an increment (or decrement) of stock during a unit interval, and denoted here by $f(t)$. Flow that can only be defined at each discrete period of time is called discrete flow.

It is important to note that flow defined in this way is the amount between two points in time or a unit interval, while stock is the amount at a specific point in time. In other words, τ which is used for defining stock implies a point in time and t which is used for defining flow means a t-th unit interval between a point in time τ and its next period $\tau + 1$.

In this way, any dynamic movement can be operatively understood in terms of stock and flow. This stock-flow relation becomes fundamental to a dynamic analysis. It is conceptually illustrated in Figure 1.2.

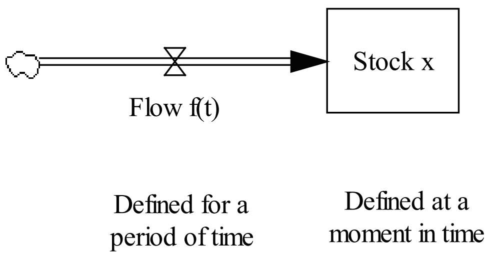

Figure 1.2: Stock-Flow Relation

It is essential to learn from the figure that flow is a part of stock, and in this sense physical or quantitative unit of flow and stock has to coincide. For instance, flow of oil cannot be added to the stock of water. As a system dynamics model becomes complicated, we tend to forget this essential fact.

Stock-flow relation can be formally written as

$$x(\tau + 1) = x(\tau) + f(t) \qquad \tau \text{ and } t = 0, 1, 2, 3, \cdots \tag{1.1}$$

To avoid a confusion derived from dual notations of time, τ and t, we need to describe stock-flow relation uniformly in terms of either one of these two concepts of time. Which one should, then, be adopted? A point in time τ could be interpreted as a limit point of an interval of time t. Hence, t can portray both concepts adequately, and can be chosen.

Since t represents a unit interval between τ and $\tau + 1$, the amount of stock at the t-th interval $x(t)$ could be defined as a balance at a beginning point τ of the period or an ending point $\tau + 1$ of the period; that is,

$$x(t) = x(\tau) : \text{Beginning balance of stock} \tag{1.2}$$

or

$$x(t) = x(\tau + 1) : \text{Ending balance of stock} \tag{1.3}$$

When the beginning balance of the stock equation (1.2) is applied, the stock-flow equation (1.1) becomes as follows:

$$x(t + 1) = x(t) + f(t) \qquad t = 0, 1, 2, 3, \cdots \tag{1.4}$$

In this formula, stock $x(t+1)$ is valued at the beginning of the period $t+1$; that is, flow $f(t)$ is added to the present stock value to give a stock value of the next period.

When the ending balance of the stock equation (1.3) is applied, the stock-flow equation (1.1) can be rewritten as

$$x(t) = x(t-1) + f(t) \qquad t = 1, 2, 3, \cdots \tag{1.5}$$

In this way, two different concepts of time - a point in time and a period of time - have been unified. It is very important for the beginners to understand that time in system dynamics always implies a period of time which has a unit interval. Of course, periods need not be discrete and can be continuous as well.

1.2.4 Integration of Flow

Discrete Sum

Without losing generality, let us assume from now on that $x(t)$ is an amount of stock at its beginning balance. If $f(t)$ is defined at a discrete time $t = 1, 2, 3, \ldots$, then the equation (1.4) is called a difference equation. In this case, the amount of stock at time t from the initial time 0 can be summed up or integrated in terms of discrete flow as follows:

$$x(t) = x(0) + \sum_{i=0}^{t-1} f(i) \tag{1.6}$$

This is a solution of the difference equation (1.4).

Continuous Sum

When flow is continuous and its measure at discrete periods does not precisely sum up the total amount of stock, a convention of approximation has been employed such that the amount of $f(t)$ is divided into n sub-periods (which is here defined as $\frac{1}{n} = \Delta t$) and n is extended to an infinity; that is, $\Delta t \to 0$. Then, the equation (1.4) can be rewritten as follows:

$$\begin{aligned} x(t) &= x\left(t - \frac{1}{n}\right) + \frac{f\left(t - \frac{1}{n}\right)}{n} \\ &= x(t - \Delta t) + f(t - \Delta t)\Delta t \end{aligned} \tag{1.7}$$

Let us further define

$$\lim_{\Delta t \to 0} \frac{x(t) - x(t - \Delta t)}{\Delta t} \equiv \frac{dx}{dt} \tag{1.8}$$

Then, for $\Delta t \to 0$ we have

$$\frac{dx}{dt} = f(t) \tag{1.9}$$

This formulation is nothing but a definition of differential equation. Continuous flow and stock are in this way transformed to differential equation, and the amount of stock at t is obtained by solving the differential equation. In other words, whenever a stock-flow diagram is drawn as in Figure 1.2, differential equation is constructed behind the screen in system dynamics.

The infinitesimal amount of flow that is added to stock at an instantaneously small period in time can be written as

$$dx = f(t)dt \tag{1.10}$$

Here, dt is technically called *delta time* or simply DT. Then an infinitesimal (or continuous) flow becomes a flow during a unit period t times DT.

Continuous sum is now written, in a similar fashion to a discrete sum in equation (1.6), as

$$x(t) = x(0) + \int_0^t f(u)du \tag{1.11}$$

This gives a general formula of a solution to the differential equation (1.9). The notational difference between continuous and discrete flow is that in a continuous case an integral sign is used instead of a summation sign. A continuous stock is, thus, alternatively called an integral function in mathematics.

In this way, stock can be described as a discrete or a continuous sum of flow[2]. This stock-flow relation becomes a foundation of dynamics (and, hence, system dynamics). It cannot be separable at all. Accordingly, among 4 letters of system dynamics language, stock-flow relation becomes an inseparable new letter. Whenever stock is drawn, flows have to be connected to change the amount of stock. This is one of the most essential grammars in system dynamics.

1.3 Dynamics in Action

We are now in a position to analyze a dynamics of stock in terms of stock-flow relation. What we have to tackle here is how to find an efficient summation and integration method for different types of flow. Let us consider the most fundamental type of flow in the sense that it is not influenced (increased or decreased) by outside forces. In other words, this type of flow becomes autonomous and dependent only on time. Though this is the simplest type of flow, it is indeed worth being fully analyzed intensively by the beginners of system dynamics. Examples of this type to be considered here are the following:

- constant flow
- linear flow of time
- non-linear flow of time squared

[2]In Stella the amount of stock is described, similar to the equation (1.7), as

$$x(t) = x(t - dt) + f(t - dt) * dt,$$

while in Vensim it is denoted, similar to the equation (1.11), as

$$x(t) = INTEG(f(t),\ x(0)).$$

- random walk

Another examples would be trigonometric flow, present value, and time-series data, which are left uncovered in this book.

1.3.1 Constant Flow

The simplest example of this type of flow is a constant amount of flow through time. Let a be such a constant amount. Then the flow is written as

$$\boxed{f(t) = a} \tag{1.12}$$

This constant flow can be interpreted as discrete or continuous. A discrete interpretation of the stock-flow relation is described as

$$x(t+1) = x(t) + a \tag{1.13}$$

and a discrete sum of the stock at t is easily calculated as

$$x(t) = x(0) + at \tag{1.14}$$

On the other hand, a continuous interpretation of stock-flow relation is represented by the following differential equation:

$$\frac{dx}{dt} = a \tag{1.15}$$

and a continuous sum of the stock at t is obtained by solving the differential equation as

$$\begin{aligned} x(t) &= x(0) + \int_0^t a \, du \\ &= x(0) + at. \end{aligned} \tag{1.16}$$

From these results we can easily see that, if a flow is a constant amount through time, the amount of stock obtained either by discrete or continuous flow becomes the same.

1.3.2 Linear Flow of Time

We now consider autonomous flow that is linearly dependent on time. The simplest example of this type of flow is the following[3]:

[3] When time unit is a week, $f(t))$ has a unit of (Stock unit/ week) . Accordingly, in order to have the same unit, the right hand side has to be multiplied by a unitary variable of unit converter which has a unit of (Stock unit / week / week).

$$f(t) = t * \text{unit converter}$$

This process is called "unit check", and system dynamics requires this unit check rigorously to obtain equation consistency. In what follows in this introductory chapter, however, this unit check is not applied.

$$\boxed{f(t) = t} \tag{1.17}$$

Let the initial value of the stock be $x(0) = 0$. Then the analytical solution becomes as follows:

$$x(t) = \int_0^t u \, du = \frac{t^2}{2} \tag{1.18}$$

At the period $t = 10$, we have $x(10) = 50$. This is a true value of the stock. Stock and flow relation of the solution is shown in Figure ??. The amount of stock at a time t is depicted as a height in the figure, which is equal to the area surrounded by the flow curve and time-coordinate up to the period t.

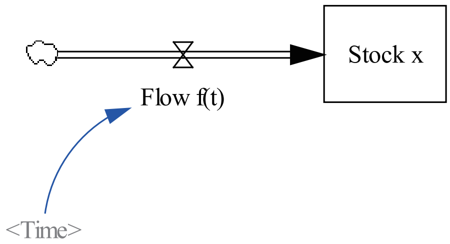

Figure 1.3: Linear Flow of Time

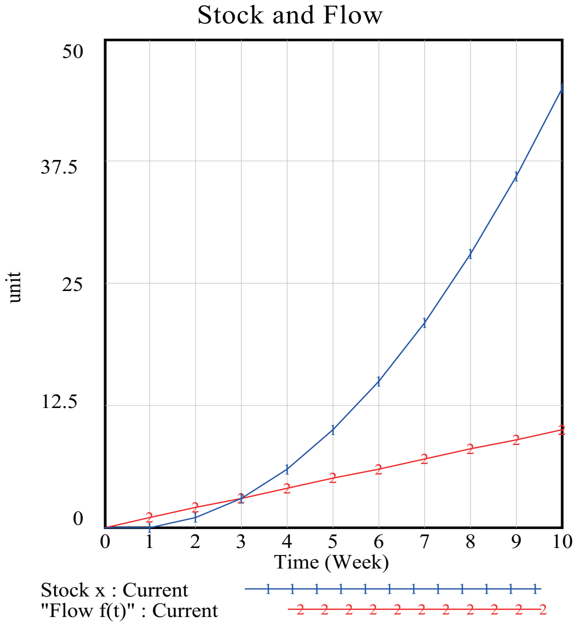

Figure 1.4: Linear Flow and Stock

Discrete Approximation

In general, the analytical (integral) solution of differential equation is very hard, or impossible, to obtain. The above example is a lucky exception. In such a general case, a numerical approximation is the only way to obtain a solution. This is done by dividing a continuous flow into discrete series of flow. Let us try to solve the above equation in this way, assuming that no analytical solution is possible in this case. Then, a discrete solution is obtained as

$$x(t) = \sum_{i=0}^{t-1} i \tag{1.19}$$

and we have $x(10) = 45$ at $t = 10$ with a shortage of 5 being incurred compared with a true value of 50. Surely, the analytical solution is a true solution. Only when it is hard to obtain it, a discrete approximation has to be resorted as an alternative method for acquiring a solution. This approximation is, however, far from a true value as our calculation shows.

a) Continuous Flow $dt \to 0$

Two algorithms have been posed to overcome this discrepancy. First algorithm is to make a discrete period of flow smaller; that is, $dt \to 0$, so that discrete flow appears to be as close as to continuous flow. This is a method employed in equation (1.7), which is known as the Euler's method. Table 1.2 shows calculations by the method for $dt = 1$ and 0.5. In Figure 1.4 a true value at $t = 10$ is shown to be equal to a triangle area surrounded by a linear flow and time-coordinate lines; that is, $10 * 10 * 1/2 = 50$. The Euler's method is, graphically speaking, to sum the areas of all rectangles created at each discrete period of time. Surely, the finer the rectangles, the closer we get to a true area.

Table 1.3 shows that as $dt \to 0$, the amount of stock gets closer to a true value of $x(10) = 50$, but it never gets to the true value. Meanwhile, the number of calculations and, hence, the calculation time increase as dt gets finer.

b) 2nd-Order Runge-Kutta Method

Second algorithm to approximate a true value is to obtain a better formula for calculating the amount of $f(t)$ over a period dt so that a rectangular area over the period dt becomes closer to a true area. The 2nd-order Runge-Kutta method is one such method. According to it, a value of $f(t)$ at the mid-point of dt is used.

$$x(t + dt) = x(t) + f(t + \frac{dt}{2})dt \tag{1.20}$$

Table 1.2: Linear Flow Calculation of $x(10)$ for dt=1 and 0.5

t	$x(t)$	dx
0	0	0
1	0	1
2	1	2
3	3	3
4	6	4
5	10	5
6	15	6
7	21	7
8	28	8
9	36	9
10	45	

where $dt = 1$ and $dx = f(t)dt = t$.

t	$x(t)$	dx	t	$x(t)$	dx
0.0	0.00	0.00	5.0	11.25	2.50
0.5	0.00	0.25	5.5	13.75	2.75
1.0	0.25	0.50	6.0	16.50	3.00
1.5	0.75	0.75	6.5	19.50	3.25
2.0	1.50	1.00	7.0	22.75	3.50
2.5	2.50	1.25	7.5	26.25	3.75
3.0	3.75	1.50	8.0	30.00	4.00
3.5	5.25	1.75	8.5	34.00	4.25
4.0	7.00	2.00	9.0	38.25	4.50
4.5	9.00	2.25	9.5	42.75	4.75
			10.0	47.50	

where $dt = 0.5$ and $dx = f(t)dt = 0.5t$.

In our simple linear example here, it is calculated as

$$f(t + \frac{dt}{2}) = t + \frac{dt}{2} \tag{1.21}$$

Applying the 2nd-order Runge-Kutta method, we can obtain a true value even for $dt = 1$ as shown in Table 1.3.

1.3.3 Nonlinear Flow of Time Squared

We now consider non-linear continuous flow that is dependent only on time. The simplest example is the following:

$$\boxed{f(t) = t^2} \tag{1.22}$$

Let the initial value of stock be $x(0) = 0$. Then the analytical solution is obtained as follows:

$$x(t) = \int_0^t u^2 du = \frac{t^3}{3} \tag{1.23}$$

At the period $t = 6$, a true value of the stock becomes $x(6) = 72$. Figure 1.5 illustrates a stock and flow relation for this solution. Stock is shown as a height at a time t, which is equal to an area surrounded by a nonlinear flow curve and a time-coordinate up to the period t.

Table 1.3: Discrete Approximation for $x(10)$

$f(t)\backslash dt$	1	1/2	1/4	1/8	1/16
Euler	45	47.5	48.75	49.375	49.6875
Runge-Kutta 2	50				

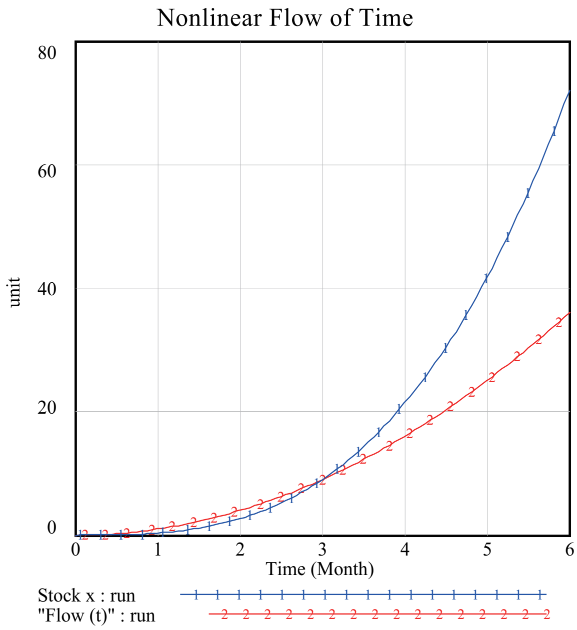

Figure 1.5: Nonlinear Flow and Stock

Discrete Approximation

A discrete approximation of this equation is obtained in terms of a stock-flow relation as follows:

$$x(t) = \sum_{i=0}^{t-1} i^2 \tag{1.24}$$

At the period $t = 6$, this approximation yields a value of 55, resulting in a large discrepancy of 17. In general, there exists no analytical solution for calculating a true value of the area surrounded by a nonlinear flow curve and a time-coordinate. Accordingly, two methods of approximation have been introduced; that is, the Euler's and 2nd-order Runge-Kutta methods. Table 1.4 shows such approximations by these methods for various values of dt. As shown in the table, both the Euler's and 2nd-order Runge-Kutta methods are not efficient to attain a true value for nonlinear flow even for smaller values of dt.

$$f(t + \frac{dt}{2}) = t^2 + tdt + \frac{dt^2}{4} \tag{1.25}$$

Clearly, in a case of nonlinear flow, the 2nd-order Runge-Kutta method, whether it be Stella or Madonna formula , fails to attain a true value at $t = 6$; that is, $x(6) = 72$.

Table 1.4: Discrete Approximation for $x(6)$

$f(t)\backslash dt$	1	1/2	1/4	1/8	1/16
Euler	55	63.25	67.5625	69.7656	70.8789
Runge-Kutta 2	71.5	71.875	71.9688	71.9922	71.998
Runge-Kutta 4	72.0				

4th-Order Runge-Kutta Method

The 4th-order Runge-Kutta method is a further revision to overcome the inefficiency of the 2nd-order Runge-Kutta method in a nonlinear case of flow as observed above. Its formula is given as[4]

$$x(t+dt) = x(t) + \frac{f(t) + 4f(t+\frac{dt}{2}) + f(t+dt)}{6} dt \tag{1.26}$$

Table 1.4 and 1.5 show that the 4th-order Runge-Kutta method are able to attain a true value even for $dt = 1$. The reader, however, should be reminded that this is not always the case as shown below.

Table 1.5: 2nd- and 4th-Order Runge-Kutta Method ($dt = 1$)

t	x(t)	Runge-Kutta 2	$x(t)$	Runge-Kutta 4
0	0	0.25	0	0.33
1	0.25	2.25	0.33	2.33
2	2.5	6.25	2.66	6.33
3	8.75	12.25	9	12.33
4	21	20.25	21.33	20.33
5	41.25	30.25	41.66	30.33
6	71.5		72	

1.3.4 Random Walk

Stochastic flow is created by probability distribution function. The simplest one is uniform random distribution in which random numbers are created between minimum and maximum. Let us consider the stock price whose initial value is \$ 10, and its price goes up and down randomly between the range of maximum \$1.00 and minimum - \$1.00.

$$\boxed{f(t) = \text{RANDOM UNIFORM (Minimum, Maximum)}} \tag{1.27}$$

Figure 1.6 is produced for a specific random walk. It is a surprise to see how a random price change daily produces a trend of stock price.

[4]Fore detaied explanation, see [9], section 2.8, pp.103 - 107 and 388 - 391.

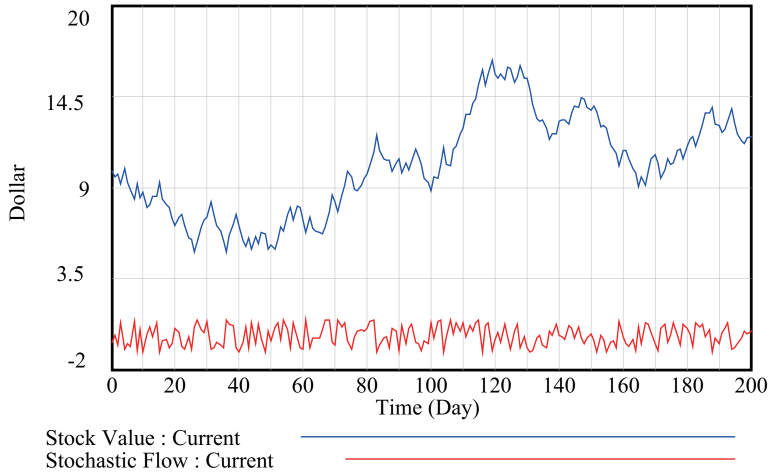

Figure 1.6: Random Walk

1.4 System Dynamics

1.4.1 Exponential Growth

So far the amount of flow is assumed to be created by autonomous outside forces at each period t. Next type of flow we now consider is the one caused by the amount of stock within the system. In other words, flow itself, being caused by the amount of stock, is causing a next amount of flow through a feedback process of stock: that is to say, flow becomes a function of stock. Whenever flow is affected by stock, dynamics becomes system dynamics.

When flow is discrete, a stock-flow relation of this feedback type is described as follows:

$$x(t+1) = x(t) + f(x(t)), \qquad t = 0, 1, 2, \cdots \tag{1.28}$$

In the case of a continuous flow, it is presented as a differential equation as follows.

$$\frac{dx}{dt} = f(x) \tag{1.29}$$

The simplest example of stock-dependent feedback flow is the following:

$$\boxed{f(x) = ax} \tag{1.30}$$

Figure 1.7 illustrates this stock-dependent feedback relation.

Its continuous flow is depicted as an autonomous differential equation:

$$\frac{dx}{dt} = ax \tag{1.31}$$

From calculus, an analytical solution of this equation is known as the following exponential equation:

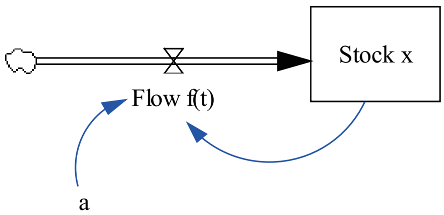

Figure 1.7: Stock-Dependent Feedback

$$x(t) = x(0)e^{at} \quad \text{where } e = 2.71828182845904523 54\cdots \tag{1.32}$$

It should be noted that the initial value of the stock $x(0)$ cannot be zero, since non-zero amount of stock is always needed as an initial capital to launch a growth of flow.

What happens if such an analytical solution cannot be obtained? Assuming that flow is only discretely defined, we can approximate the equation as a discrete difference equation:

$$x(t+1) = x(t) + ax(t), \ \ t = 0, 1, 2, \cdots \tag{1.33}$$

Then, a discrete solution for this equation is easily obtained as

$$x(t) = x(0)(1+a)^t \tag{1.34}$$

A true continuous solution of the equation could be obtained as an approximation from this discrete solution (1.34), first by dividing a constant amount of flow a into n sub-periods, and secondly by making n sub-periods into infinitely many finer periods so that each sub-period converges to a moment in time.

$$\begin{aligned} x(t) &= x(0)\left[\lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n\right]^t \\ &= x(0)\left[\lim_{n\to\infty}\left(1+\frac{1}{\frac{n}{a}}\right)^{\frac{n}{a}}\right]^{at} \\ &= x(0)e^{at} \end{aligned} \tag{1.35}$$

Let an initial value of the stock be $x(0) = 100$ and $a = 0.1$. Then, a true value for the period $t = 10$ is $x(10) = 100e^{0.1*10} = 100e^1 = 271.8281828459\cdots$. This is to obtain a compounding increase in 10% for 10 periods. The amount of initial stock is shown to be increased by a factor of 2.7 when a growth rate is 10%. Table 1.6 shows numerical approximations by the Euler's, 2nd-order and 4th-order Runge-Kutta methods. It is observed from the table that even the 4th-order Runge-Kutta method cannot obtain a true value of the exponential e.

Table 1.6: Discrete Approximation

$f(t)\backslash dt$	1	1/2	1/4	1/8	1/16
Euler	259.374	265.33	268.506	270.148	270.984
Runge-Kuttta 2	271.408	271.7191	271.8004	271.8212	271.8264
Runge-Kutta 4	271.8279	271.828169	271.828182	271.828182	271.828182

In Figure 1.7, the amount of flow is shown to be determined by its previous amount through the amount of stock. Structurally this relation is sketched as a following flow chart: an increase in flow$\uparrow$ $\longrightarrow$ an increase in stock$\uparrow$ $\longrightarrow$ an increase in flow$\uparrow$. At an annual growth rate of 10%, for instance, it takes only seven years for the initial amount of stock to double, and 11 years to triple, and 23 years to become 10 folds. In fifty years, it becomes about 150 times as large. This self-increasing relation is called a *reinforcing or positive feedback* (see Figure 1.7.) Left-hand diagram in Figure 1.14 illustrates such a positive feedback growth.

Constant Doubling Times

One of the astonishing features of exponential growth is that a doubling time of stock is always constant. Let $x(0) = 1$ and $x(t) = 2$ in the equation (1.32), then it is obtained as follows.

$$ln2 = at \implies t = \frac{0.693147}{a} \tag{1.36}$$

For instance, when $a = 0.02$, that is, an annual growth rate is 2%, then doubling time of the stock becomes about 35 years. That is to say, every 35 years the stock becomes twice as big. When $a = 0.07$, or an annual growth rate is 7%, stock becokes dobled about every 10 years. Consider an economy growthing at 7% annually. its GDP becomes 8 folds in 30 years. This enormous power of exponential is usually overlooked or under estimated. See the section of "Misperception of Exponential Growth" on pages 269 - 272 in [48].

Examples of Exponential Growth (Reinforcing Feedback)

In system dynamics, this exponential growth is called *positive* or *reinforcing* feedback. Figure 1.8 illustrates some examples of these reinforcing stock-dependent feedback relation. Left-hand diagram illustrates our financial system in which our bank deposits keeps increasing as long as positive interest rate is guaranteed by our banking system. This financial system creates the environment that "the rich becomes richer exponentially".

1.4.2 Balancing Feedback

If system consists of only exponential growth or reinforcing feedback behaviors, it will sooner or later explode. System has to have "a common purpose" or "the aim of the system" as already quoted in the beginning of this chapter. In other words, it has to be stabilized to accomplish its aim.

To attain a self-regulating stability of the system, another type of feedback is needed such that whenever a state of the system $x(t)$ is off the equilibrium x^*, it tries to come back to the equilibrium, as if it's being attracted to the equilibrium. If system has this feature, it will be stabilized at the equilibrium. In economics, it is called global stability. Free market economy has to have this price stability as a system to avoid unstable price fluctuations.

Structurally the stability is attained if stock-flow has a relation such that an increase in flow↑ ⟶ a decrease in stock↓ ⟶ a decrease in flow↓. This stabilizing relation is called a *balancing or negative feedback* in system dynamics. Figure 1.9 illustrates this balancing feedback stock-dependent relation.

Mathematically, this is to guarantee the stability of equilibrium.

Let x^* be such an equilibrium point, or target or objective of the stock $x(t)$). Then stabilizing behavior is realized by the following flow

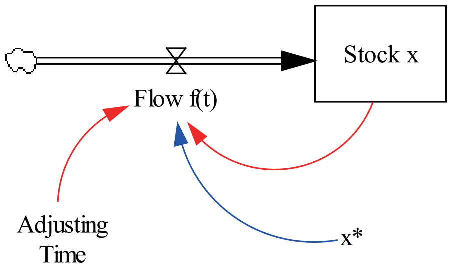

Figure 1.9: Balancing Feedback

$$\boxed{f(x) = \frac{x^* - x(t)}{AT}} \tag{1.37}$$

where AT is the adjusting time of the gap between x^* and x.

Fortunately, this differential equation can be analytically solved as follows. First rewrite the equation (1.37) as

$$\frac{d(x(t) - x^*)}{x(t) - x^*} = -\frac{dt}{AT} \tag{1.38}$$

Then integrate both side to obtain

$$ln(x(t) - x^*) = -\frac{t}{AT} + C, \tag{1.39}$$

where ln denotes natural logarithm. This is further rewritten as

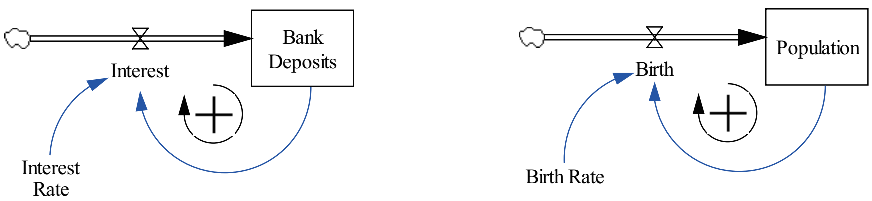

Figure 1.8: Examples of Exponential Feedback

$$x(t) - x^* = e^{-\frac{t}{AT}} e^C \tag{1.40}$$

At the initial point in time, we have

$$e^C = x(0) - x^* \text{ at } t = 0 \tag{1.41}$$

Thus, the amount of stock at t is analytically obtained as

$$x(t) = x^* - (x^* - x(0))e^{-\frac{t}{AT}} \tag{1.42}$$

Examples of Balancing Feedback

Figure 1.10 illustrates some examples of these balancing stock-dependent feedback relation.

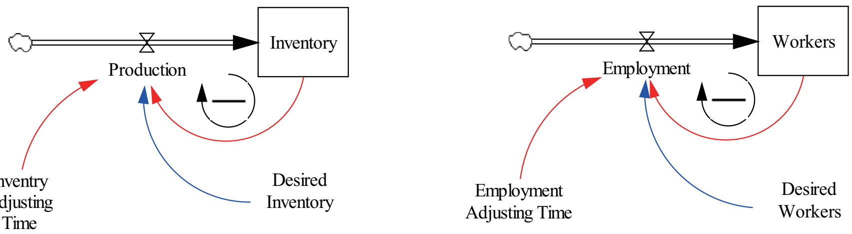

Figure 1.10: Examples of Balancing Feedback

Exponential Decay

When $x^* = 0$, system continues to decay or disappear. In other words, stock begins to decrease by the amount of its own divided by the adjusting time.

$$\boxed{f(x) = -\frac{x(t)}{AT}} \tag{1.43}$$

This decay process is called exponential decay. Whenever exponential decay appears, flow has only negative amount. In this case in system dynamics we draw flow out of stock so that it becomes more intuitive to understand the outflow of stock. Figure 1.11 illustrates such stock-outflow relation.

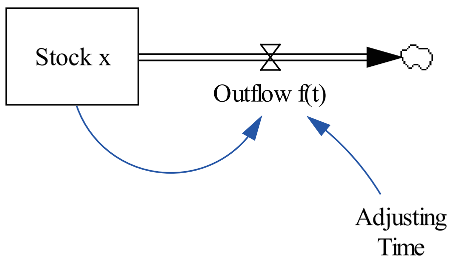

Figure 1.11: Exponential Decay

At an annual declining rate of 10%, for instance, the initial amount of stock decreases by half in seven years, by one third in 11 years, and by one tenth in 23 years, balancing to a zero level eventually. Right-hand diagram in Figure 1.14 illustrates such a negative feedback decay.

Examples of Exponential Decay

Figure 1.12 illustrates this stock-dependent feedback relation.

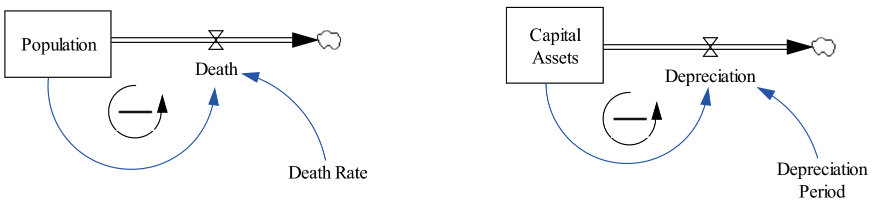

Figure 1.12: Examples of Exponential Decay

1.5 System Dynamics with One Stock

1.5.1 First-Order Linear Growth

We have now learned two fundamental feedbacks in system dynamics; reinforcing (exponential or positive) feedback and balancing (negative) feedback. Let us now consider the simplest system dynamics which have these two feedbacks simultaneously. It is called first-order linear growth system. “First-order” implies that the system has only one stock, while “linear” means that its inflow and outflow are linearly dependent on stock. Figure 1.13 illustrates our first system dynamics model which has both reinforcing and balancing feedback relations.

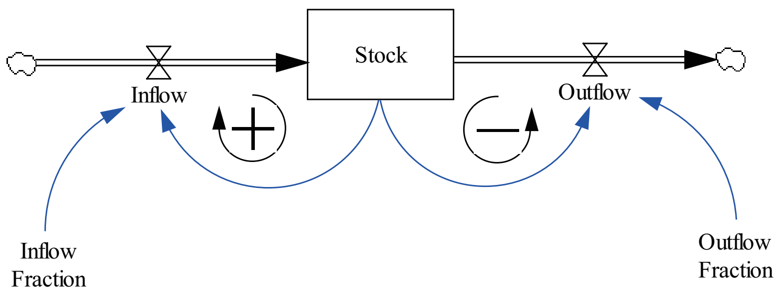

Figure 1.13: First-Order Linear Growth Model

Table 1.7 describes its equation.

Left-hand diagram of Figure 1.14 is produced for the inflow fraction value of 0.1 and outflow fraction value of zero, and has a feature of exponential growth. Right-hand diagram is produced by the opposite fractional values, and has a feature of exponential decay. It is easily confirmed that whenever inflow fraction is greater than outflow fraction, the system produces exponential growth behavior. When outflow fraction is greater than inflow fraction, it causes an

Table 1.7: Equations of the First-Order Growth Model

Inflow= Stock*Inflow Fraction
Units: unit/Year
Inflow Fraction= 0.1
Units: 1/Year [0,1,0.01]
Outflow= Stock*Outflow Fraction
Units: unit/Year
Outflow Fraction= 0.04
Units: 1/Year [0,1,0.01]
Stock= INTEG (Inflow-Outflow, 100)
Units: unit

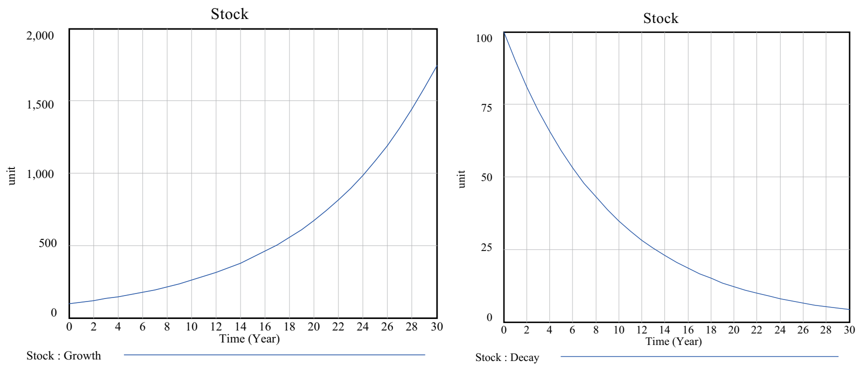

Figure 1.14: Exponential Growth and Decay

exponential decay behavior. In this way, the first-order linear system can only produces two types of behaviors: exponential growth or decay.

This model can best describe population dynamics. Suppose the world birth rate (inflow fraction) is 3.5%, while its death rate (outflow fraction) is 1.5%. This implies that world population grows exponentially at the net growth rate of 2%.

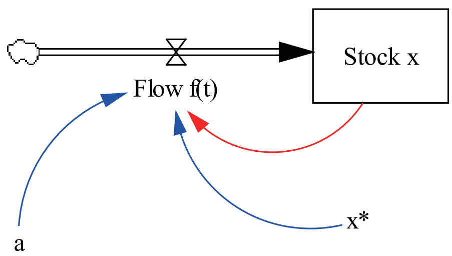

Figure 1.15: S-shaped Growth Model

1.5.2 S-Shaped Limit to Growth

In the first-order linear model, the system may explode if inflow fraction is greater than outflow fraction. Population explosion is a good example. To stabilize the system, the exponential growth $ax(t)$ has to be curbed by bringing another balancing feedback which plays a role of a break in a car. Specifically, whenever $x(t)$ grows to a limit x^*, it begins to be regulated as if population is control ed and speed of the car is reduced. This is a feedback mechanism to stabilize the system. It could be done by the following flow:

$$\boxed{f(x) = ax(t)b(t), \text{ where } b(t) = \frac{x^* - x(t)}{x^*}} \tag{1.44}$$

Apparently $b(t)$ is bounded by $0 \leq b(t) \leq 1$, and reduces to zero as $x(t)$ approached to its limit x^*. Figure 1.15 is such model in which both reinforcing (exponential) and balancing feedback are brought together. It is called S-shaped growth in system dynamics,

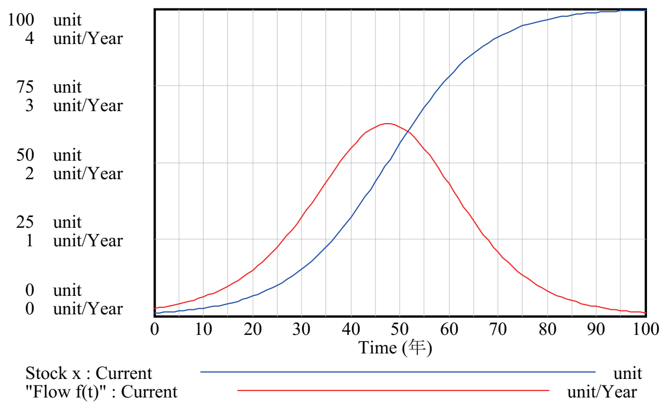

Figure 1.16: S-Shaped Limit to Growth 1

1.5.3 S-Shaped Limit to Growth with Table Function

Another way to regulate the growth of $x(t)$ is to increase $b(t)$ to the value of a as $x(t)$ grows to its limit x^* as shown below.

$$\boxed{f(x) = (a - b(t))x(t)} \tag{1.45}$$

Specifically, any functional relation that has a property such that $b(t)$ approaches a whenever $x(t)/x^*$ approaches 1 works for this purpose.

One of the simplest function is

$$b(t) = a\frac{x(t)}{x^*} \tag{1.46}$$

In this case the above function becomes

$$\boxed{f(x) = (a - b(t))x(t) = a\left(\frac{x^* - x(t)}{x^*}\right)x(t)} \tag{1.47}$$

which becomes the same as the above S-shaped limit to growth.

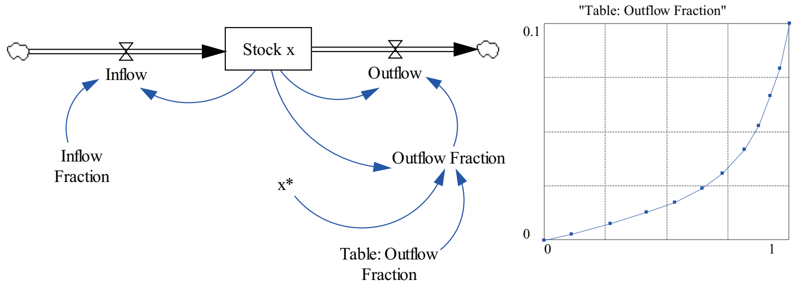

Figure 1.17: S-Shaped Limit to Growth Model with Table Function

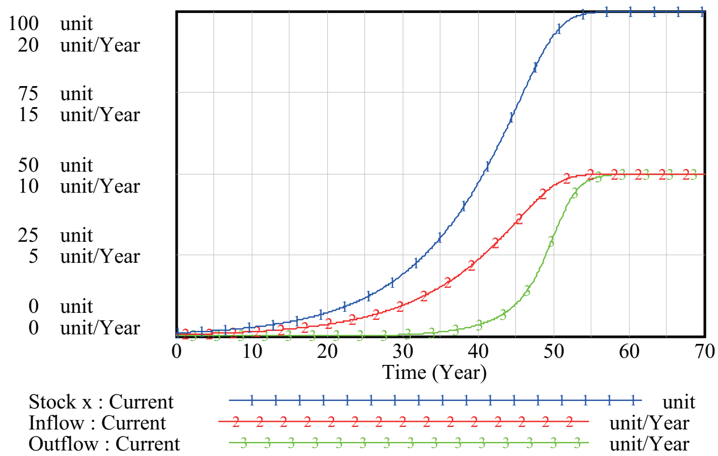

Figure 1.18: S-Shaped Limit to Growth 2 with Table Function

If mathematical function is not available, still we can produce S-shaped behavior by plotting the relation, which is called table function. One of such table function is shown in the right-hand diagram of Figure 1.17. Left-hand diagram illustrates S-shaped limit to growth model.

1.6 System Dynamics with Two Stocks

1.6.1 Feedback Loops in General

When there is only one stock, two feedback loops are at maximum produced as in first-order linear growth model. When the number of stocks becomes two, at maximum three feedback loops can be generated as illus rated in Figure 1.19

Mathematically, general feedback loop relation with two stocks can be represented by a following dynamical system in which each flow is a function of stocks x and y.

$$\frac{dx}{dt} = f(x,y) \quad (1.48)$$

$$\frac{dy}{dt} = g(x,y) \quad (1.49)$$

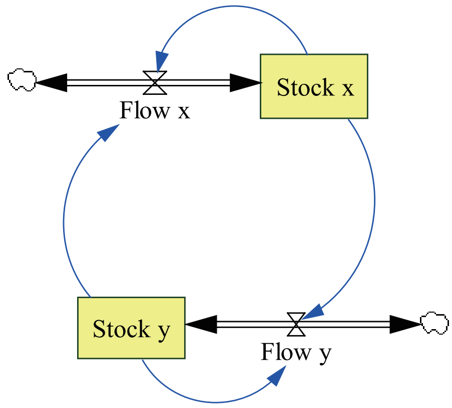

Figure 1.19: Feedback Loops in General

1.6.2 S-Shaped Limit to Growth with Two Stocks

Behaviors in system dynamics with one stock are limited to exponential growth and decay generated by the first-order linear growth model, and S-shaped limit to growth. To produce another fundamental behaviors such as overshoot and collapse, and oscillation, at least two stocks are needed. System dynamics with two stocks are called second-order system dynamics.

Let us begin with another type of S-shaped limit to growth behavior that can be generated with two stocks x and y. When the total amount of stock x and stock y is limited by the constant available resources such that $x+y=b$, and the amount of stock x flows into stock y as shown in Figure 1.20, stock y begins to create a S-shaped limit to growth behavior. A typical system causing this behavior is described as follows.

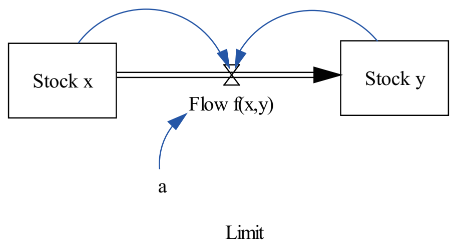

Figure 1.20: S-shaped Growth Model

$$\frac{dx}{dt} = -f(x,y) \quad (1.50)$$

$$\begin{aligned} \frac{dy}{dt} &= f(x,y) \\ &= ax(t)y(t) \\ &= a(b-y(t))y(t) \quad (1.51) \end{aligned}$$

where $a = 0.001$ and $b = 100$. This relation is also reduced to

$$\frac{dx}{dt} + \frac{dy}{dt} = 0 \tag{1.52}$$

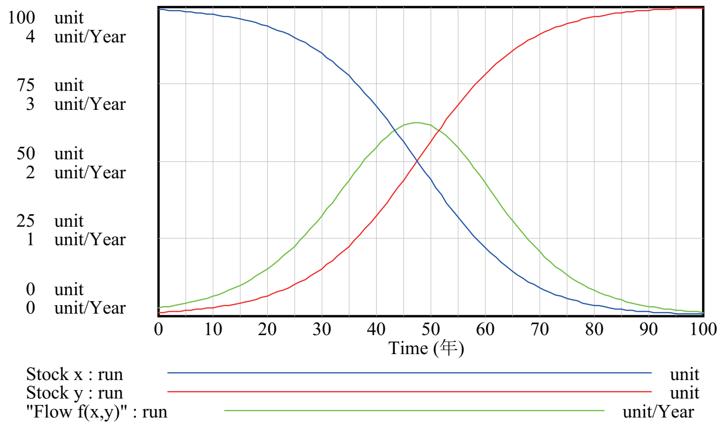

Figure 1.21: S-Shaped Limit to Growth 3

Mathematically, the above equation (1.51) is similar to the S-shaped limit to growth equation (1.44). In other words, $x(t) = b - y(t)$ begins to diminish as $y(t)$ continues to grow. This is a requirement to generate S-shaped limit to growth.

Examples of this type of S-shaped limit to growth are abundant such as logistic model of innovation diffusion in marketing.

So far we have presented three different figures to illustrate S-shaped limit to growth. Mathematically, all of the S-shaped limits to growth turn out to have the same structure. The same structures can be built in three different models, depending on the issues we want to analyze. This indicates the richness of system dynamics approach.

1.6.3 Overshoot and Collapse

Next behavior to be generated with two stocks is a so-called overshoot and collapse. It is basically caused by the S-shaped limit to growth model with table function. However, coefficient $b(t)$ is this time affected by the stock y. Increasing stock x causes stock y to decrease, which in turn makes the availability of stock y smaller, which then increases outflow fraction. This relation is described by the table function in the right-hand diagram of Figure 1.22. Increasing fraction collapses stock x. The model is shown in the left-hand diagram. Behaviors of overshoot & collapse is shown in Figure 1.23.

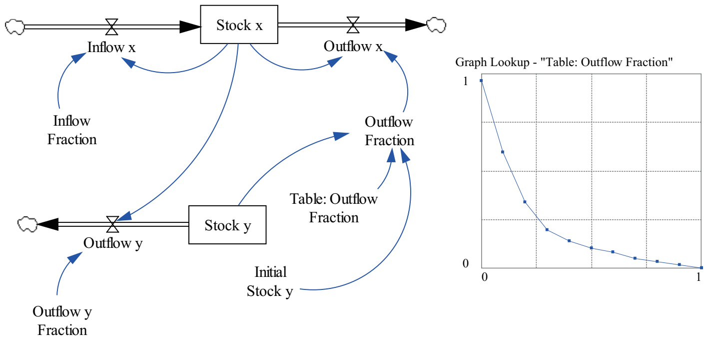

Figure 1.22: Overshoot & Collapse Model

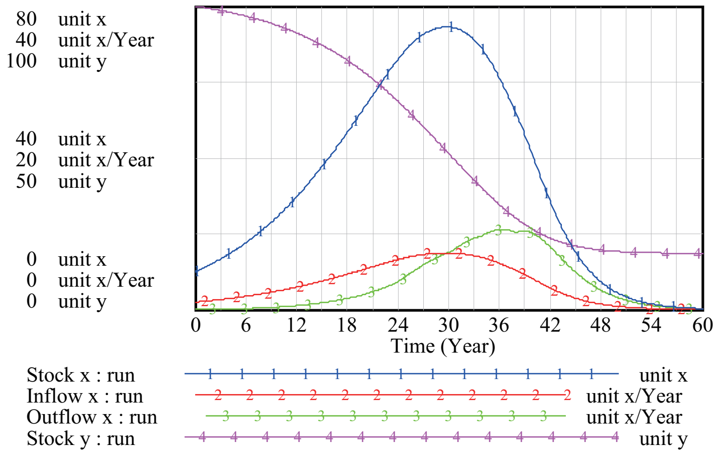

Figure 1.23: Overshoot & Collapse Behavior

Examples of Overshoot and Collapse

My favorite example of overshoot and collapse model is the decline of the Mayan empire in [3].

1.6.4 Oscillation

Another behavior that can be created with two stocks is oscillation A simple example of system dynamics with two stocks which is illustrated in Figure 1.24

It can be formally represented as follows:

$$\frac{dx}{dt} = ay, \;\; x(0) = 1, \;\; a = 1 \quad (1.53)$$

$$\frac{dy}{dt} = -bx, \;\; y(0) = 1, \;\; b = 1 \quad (1.54)$$

This is nothing but a system of differential equations, which is also called a dynamical system in mathematics. Its solution by Euler method is illustrated in Figure 1.25 in which DT is set to be $dt = 0.125$.

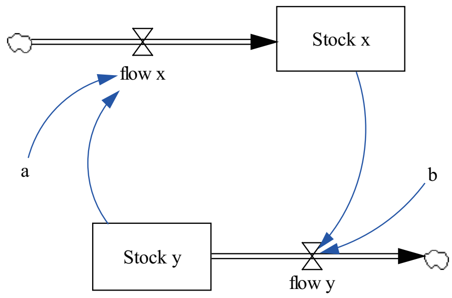

Figure 1.24: An Oscillation Model

Movement of the stock y is illustrated on the y-axis against the x-axis of time (left figure) and against the stock of x (right figure). In this Euler's solution, the amount of stock keeps expanding even a unit period is divided into 8 sub-periods for better computations. In this continuous case of flow, errors at each stage of calculation continue to accumulate, causing a large deviation from a true value.

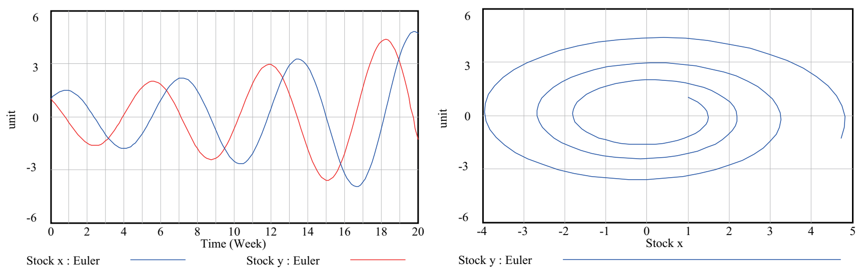

Figure 1.25: Oscillation under Euler Method

On the other hand, the 2nd-order Runge-Kutta solution eliminates this deviation and yields a periodic or cyclical movement as illustrated in Figure 1.26. This gives us a caveat that setting a small number of dt in the Euler's method is not enough to approximate a true value in the case of continuous flow. It is expedient, therefore, to examine the computational results by both methods and see whether they are differentiated or not.

Examples of Oscillation

Pendulum movement is a typical example of oscillation. It is shown in [4] that employment instability behavior is produced by the same system structure which generates pendulum oscillation.

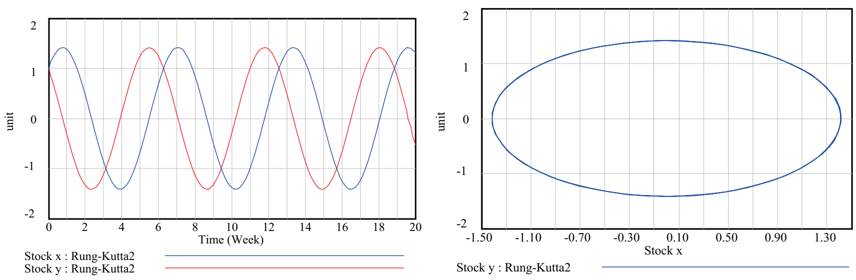

Figure 1.26: Oscillation under Runge-Kutta2 Method

1.7 Delays in System Dynamics

1.7.1 Material Delays

First-Order Material Delays

System dynamics consists of four letters: stock, flow, variable and arrow, as already discussed in the beginning of this chapter. To generate fundamental behaviors, these letters have to be combined according to its grammatical rules: reinforcing (positive) and balancing (negative) feedback loops, and delays. So far reinforcing and balancing feedback loops have been explored. Yet delays have been already applied in our models above without focusing on them. Delays play an important role in model building. Accordingly, it is appropriate to examines the meaning of delays in system dynamics in this section.

Delays in system dynamics has a structure illustrated in Figure 1.27. That is, output always gets delayed when input goes through stocks. This is an inevitable feature in system dynamics. It is essential in system dynamics to distinguish two types of delays: material delays and information delays. Let us start with material delays first.

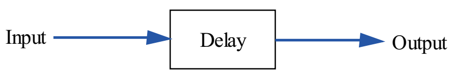

Figure 1.27: Structure of Delays

When there is only one stock, delays becomes similar to exponential decay for the one-time input, which is called *pulse*. In Figure 1.28, 100 units of material are input at time zero. This corresponds to the situation, for instance, in which 100 units of goods are purchased and stored in inventory, or 100 letters are dropped in the post office. Delay time is assumed to be 6 days in this example. In other words, one-sixth of goods are to be delivered daily as output.

Second-Order Material Delays

In the second-order material delays, materials are processed twice as illustrated in the top diagram of Figure 1.29. Total delay time is the same as 6 days.

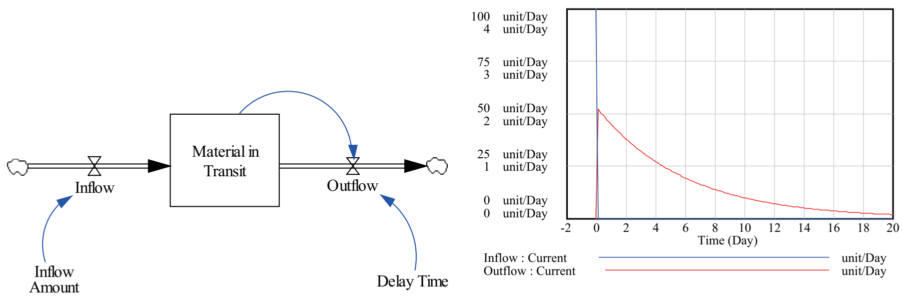

Figure 1.28: First-Order Material Delays

Accordingly, delay time for each process becomes 3 days. In this case, output distribution becomes bell-shaped. The reader can expand the delays to the n-th order and see what will happen to output.

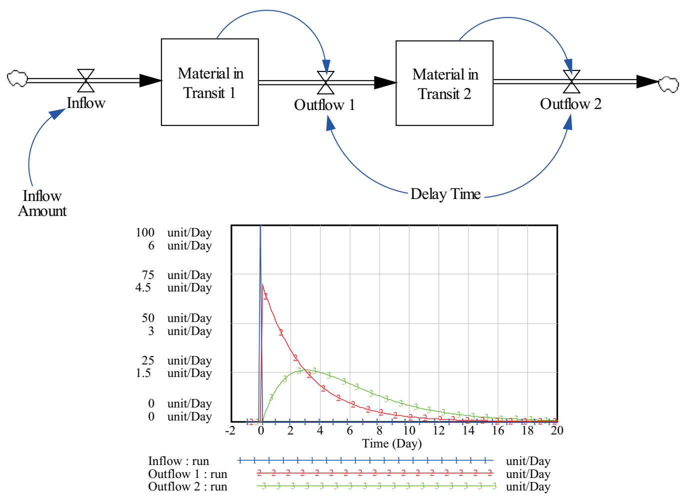

Figure 1.29: Second-Order Material Delays

1.7.2 Information Delay

First-Order Information Delays

Information delays occur because information as input has to be processed by human brains and implemented as action output. Information literally means in-form; that is, being input to brain which forms it for action. This is a process to adjust our perceived understanding in the brain to the actual situation outside the brain. Structurally this is the same as balancing feedback explained above to fill the gap between x^* and x, as illustrated in the left-hand diagram of Figure 1.30.

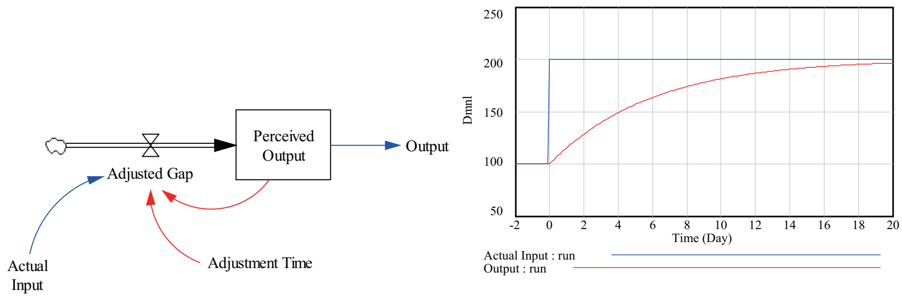

Figure 1.30: First-Order Information Delays

Our perceived understanding, say, on daily sales order, is assumed here to be 100 units, yet actual sales jumps to 200 at the time zero. Our suspicious brain hesitates to adjust to this new reality instantaneously. Instead, it slowly adapts to a new reality with the adjustment time of 6 days. This type of adjustment is explained as adaptive expectations and exponential smoothing in [48].

Second-Order Information Delays

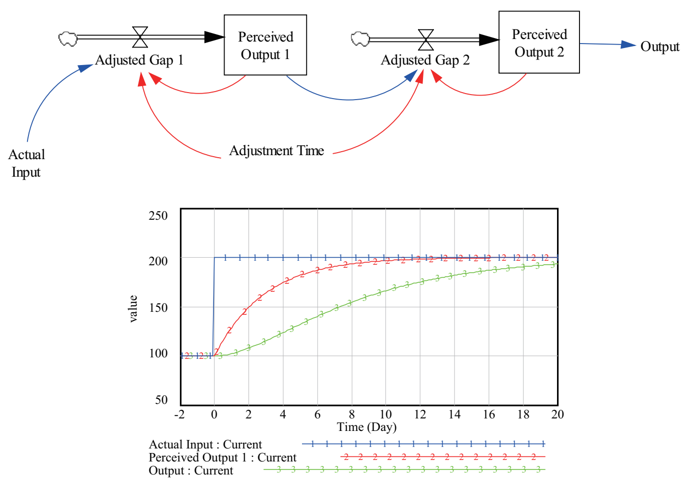

Figure 1.31: Second-Order Information Delays

Second-order information delays imply that information processing occurs through two brains. This is the same as re-thinking process for one person or a process in which information is being sent to another person. This structure is modeled in the top diagram of Figure 1.31.

In either case, second-order adaptation process becomes slower than the first-order information delays as illustrated in the bottom diagram.

Adaptive Expectations for Random Walk

First-order information delays are called adaptive expectations or exponential smoothing because perceived output tries to adjust gradually to the actual input. When random walk becomes actual input, output becomes exponentially smoothing as illustrated in Figure 1.32.

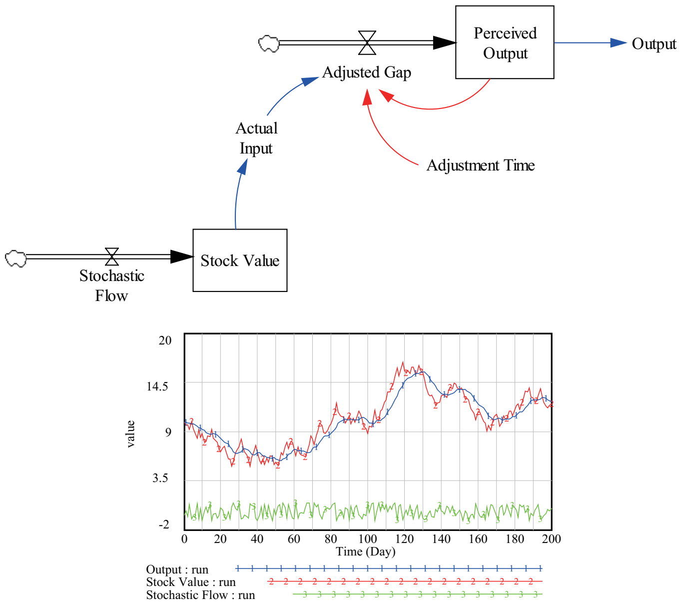

Figure 1.32: Adaptive Expectations for Random Walk

1.8 System Dynamics with Three Stocks

1.8.1 Feedback Loops in General

It has been shown that system dynamics with two stocks can mostly produce all fundamental behavior patters such as exponential growth, exponential decay, S-shaped limit to growth, overshoot and collapse, and oscillation. Actual behaviors observed in complex system are combinations of these fundamental behaviors. We are now in a position to build system dynamics model based on these fundamental building blocks. And this introductory chapter on system dynamics seems appropriate to end at this point, and we should go to next chapter in which how system dynamics method can be applied to economics.

Yet, the exists another behaviors which can not be produced with two stocks; that is a chaotic behavior! Accordingly, we stay here for a while, and consider a general feedback relation for the case of three stock-flow relations. Figure 1.33 illustrates a general feedback loops.

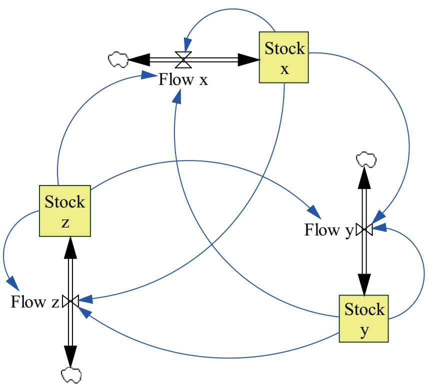

Figure 1.33: Feedback Loop in General

Each stock-flow relation has its own feedback loop and two mutual feedback loops. In total, there are 6 feedback loops, excluding overlapping ones. As long as we observe *the parts* of mutual loop relations, that's all loops. However, if we observe *the whole*, we can find two more feedback loops: that is, a whole feedback loop of $x \to y \to z \to x$, and $x \to z \to y \to x$. Therefore, there are 8 feedback loops as a whole. The existence of these two whole feedback loops seems to me to symbolize a complex system in terms of loops; that is, *the whole is more than the sum of its parts.*

> A complex system is one whose component parts interact with sufficient intricacy that they cannot be predicted by standard linear equations; so many variables are at work in the system that its overall behavior can only be understood as an emergent consequence of the holistic sum of all the myriad behaviors embedded within. Reductionism does not work with complex systems, and it is now clear that a purely reductionist approach cannot be applied when studying life: in living systems, *the whole is more than the sum of its parts* (emphasis is made by the author) [37], pp. 7-8.

Mathematically, this general feedback loop relation can be represented by a following dynamical system in which each flow is a function of all stocks x, y and z.

$$\frac{dx}{dt} = f(x, y, z) \tag{1.55}$$

$$\frac{dy}{dt} = g(x, y, z) \tag{1.56}$$

$$\frac{dz}{dt} = h(x, y, z) \tag{1.57}$$

1.8.2 Lorenz Chaos

As a special example of the general feedback loops by three stock-flow relations, let us consider well-known Lorenz equations which yield a chaotic movement.

Mathematical equations of the Lorenz chaos are written as

$$\frac{dx}{dt} = -a(x-y) \tag{1.58}$$

$$\frac{dy}{dt} = -xz + bx - y \tag{1.59}$$

$$\frac{dz}{dt} = xy - cz \tag{1.60}$$

where initial values are assigned as $x(0) = 0$, $y(0) = 2$ and $z(0) = 0$ and parameters are as originally set at $a = 10$, $b = 28$ and $c = 8/3$ by Lorenz. (See Chapter 14 The Lorenz System in [33]).

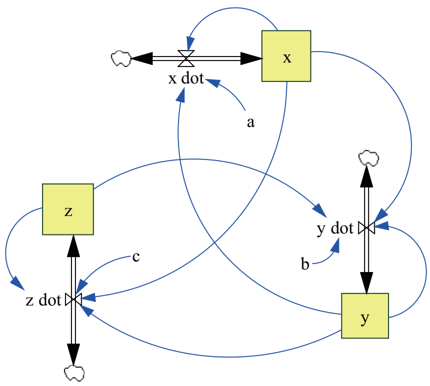

Figure 1.34: Lorenz Feedback Loop

Figure 1.34 illustrates feedback loops of the Lorenz equations. Compared with a general case of the left-hand diagram, a link from the Stock z to the Flow x is missing. Accordingly, we have in total 6 feedback loops - a loss of two loops ! (We don't know whether this loss of loops is related with a chaos to be discussed below.)

Figure 1.35 illustrates a phase diagram of Lorenz chaos in which movements of the stock y and z are illustrated on the y-axis against the stock x on the x-axis. Calculations are done by the 4th-order Runge-Kutta method at $dt = 0.0078125$; that is, at each of 128 sub-periods in a unit period. With such a small sub-period, computational errors may arise less likely as explained above.

Sensitive Dependence on Initial Conditions

In the above Lorenz phase diagram, movement of stocks does not converge to a fixed point or a limit cycle, or diverge to infinity. Instead, wherever it starts, it seems to be eventually attracted to a certain region and continue fluctuating in it, with the information of its start being lost eventually. That region is called *a strange attractor* or *chaos*. One of the main features of chaos is a sensitive dependence on initial conditions. This is numerically explained as

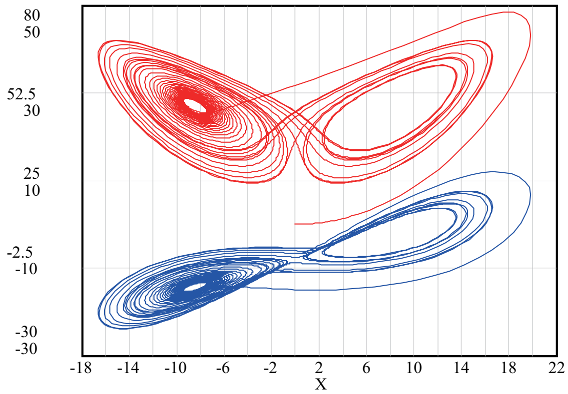

Figure 1.35: Lorenz Chaos

follows. Suppose a true initial value of the stock y in the Lorenz equations is $y^*(0) = 1.0001$ instead of $y(0) = 1.0$, and denote its true value by y^*. At the period $t = 10.25$ those two values of the stock are calculated as $y(t) = 5.517$, and $y^*(t) = 5.518$. The difference is only 0.001 and they stay very close each other. This makes sense, because both started at the very close distance of 0.0001. To our surprise, however, at the period $t = 27$, they are calculated as $y(t) = -2.546$, and $y^*(t) = 13.77$; a large difference of 16.316 is made. Small amount of differences at an initial time eventually turns out to cause a big difference later. In other words, stock values sensitively depend on their initial conditions. Figure 1.36 illustrates how values of the stock y begin to diverge from a true value y^* around the period $t = 18$.

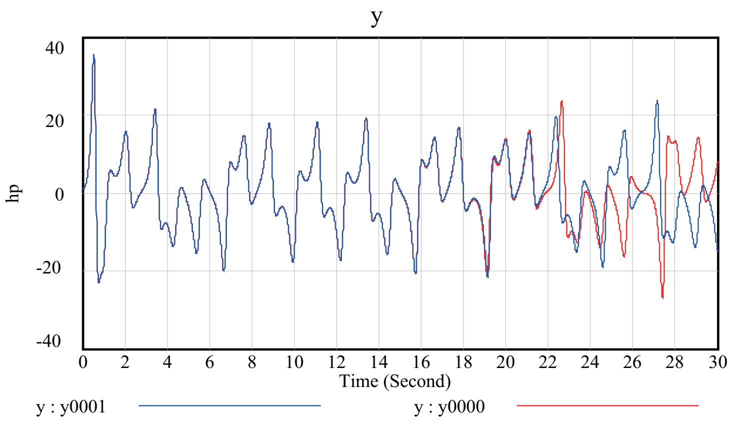

Figure 1.36: Sensitive Dependence on Initial Conditions

Why could it be possible? It is caused by the power of exponential magnification empowered by feedback loops. As illustrated in Figure 1.14, for instance, a simple calculation yields that an initial difference of 0.001 is exponentially magnified to 22.02 by the time $t = 100$, more than twenty-two thousand factors larger, because of a positive feedback loop. Chaos is a region called strange attractor to which infinitely many iterated and exponentially magnified values are confined. Hence, it is intuitively understood that exponentially magnified values in a chaos region sensitively depend on initial conditions; in other words, values whose initial conditions differ only very slightly cannot stay close and begin to diverge eventually.

This chaotic feature creates annoying problem in system dynamics: unpredictability in the future. It is almost impossible in reality to obtain true initial values due to some observation errors and round-off errors of measurement and computations. These errors are magnified in a chaotic system dynamics to a point where predictions of the future and forecasting become almost meaningless and misleading. If analytical solutions of differential equations could be found, this would never happen, because solutions are continuous function of time and we could easily predict or approximate the future behavior of the system even if initial conditions are missed slightly. Without the analytical solutions, the future has to be iterated step by step, causing an exponential magnification by feedback loop. Unfortunately as discussed above, it is almost impossible to find analytical solutions in a nonlinear dynamics and system dynamics. In such cases, if a true initial value fails to be specified, then we cannot predict the future at all, even if we try to make calculations as precise as possible by employing Runge-Kutta methods and making sub-periods smaller as discussed in the previous sections. Hence, system dynamics becomes inefficacious as a forecasting simulation method.

What's a good use of system dynamics, then? If a dynamic system is chaotic, all values of stocks are attracted to a region of strange attractor; in other words, information of initial conditions will be lost eventually and only patterns or structures of the system begin to reveal themselves. In system dynamics, these patterns and structures help us learn the behavior of the system we want to explore. System dynamics is a very effective learning method in that direction, not in the direction of futures prediction.

1.9 Chaos in Discrete Time

1.9.1 Logistic Chaos

Famous logistic function which produces chaos is the following:

$$x_{t+1} = ax_t(1 - x_t) \qquad t = 0, 1, 2, 3, \cdots \tag{1.61}$$

(See Chapter 15 Discrete Dynamical Systems in [33])

To fit into our system dynamics presentation, its flow can be rewritten as follows:

$$\boxed{f(t) = ax_t(1 - x_t) - x_t} \tag{1.62}$$

As coefficient a increases stock x produces n-period oscillations $n = 2, 4, 8, \cdots$, and eventually produces chaos. Left-hand diagram of Figure 1.37 shows an chaotic movement for $a = 3.95$ and right-hand diagram is its phase diagram. Chaotic movements do not fill in all space but fit into parabola shape as strange attractor.

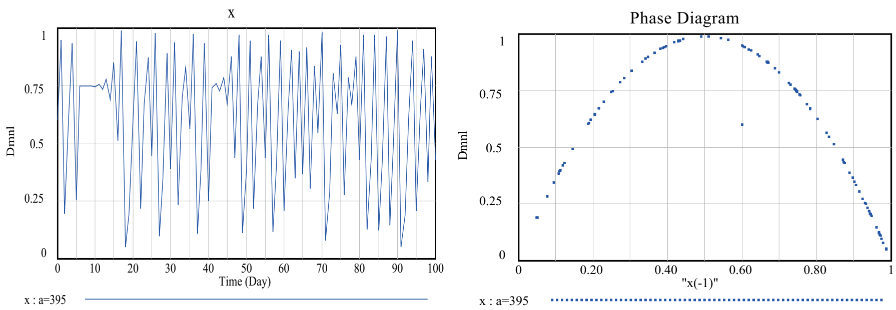

Figure 1.37: Chaos in Logistic Function

This type of chaos in discrete time gradually disappears when DT(Delta Time) in simulation becomes smaller.

1.9.2 Discrete Chaos in S-shaped Limit to Growth

The equation (1.51) in the S-shaped limit to growth can be rewritten in a discrete format as

$$y_{t+1} = y_t + ay_t(b - y_t) \qquad t = 0, 1, 2, 3, \cdots \tag{1.63}$$

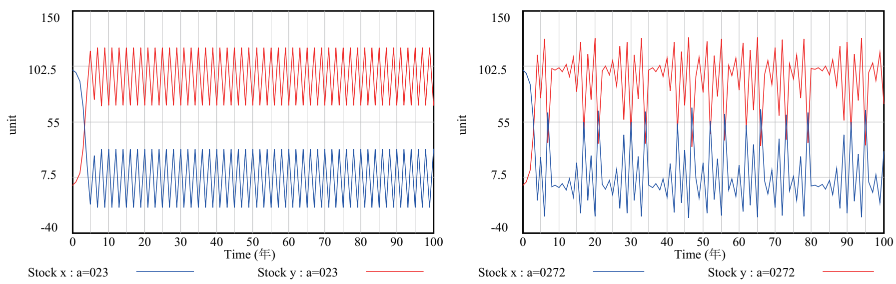

Figure 1.38: Chaos in S-shaped Limit to Growth

In other words, flow becomes

$$\boxed{f(t) = ay_t(b - y_t)}, \tag{1.64}$$

which also becomes the same as the right-hand side of the equation (1.61) for $b = 1$.

To our surprise, it turns out that the flow (1.64) can also produce chaos if $f(t)$ is allowed to take negative values. In other words, S-shaped limit to growth behavior turns out to be chaotic if flows move forward and backward between stocks x and y.

Left-hand diagram of Figure 1.38 is a two-period cycle of stocks x and y for $a = 0.023$ and right-hand diagram is chaotic movements for $a = 0.0272$.

Appendix: Runge-Kutta Methods in General

Flow in equation (1.29) can be more generally described as a function of time and stock; that is,

$$\frac{dx}{dt} = f(t, x) \tag{1.65}$$

Accordingly, the Runge-Kutta methods need to be more generally formulated as follows[5]

2nd-Order Runge-Kutta Method

$$\begin{aligned}
dx_1 &= f(t,x)dt && (1.66)\\
dx_2 &= f(t+dt, x+dx_1)dt && (1.67)\\
dx &= \frac{dx_1+dx_2}{2}\\
&= \frac{f(t,x)+f(t+dt,x+dx_1)}{2}dt && (1.68)
\end{aligned}$$

4th-Order Runge-Kutta Method

$$\begin{aligned}
dx_1 &= f(t,x)dt && (1.69)\\
dx_2 &= f(t+\frac{dt}{2}, x+\frac{dx_1}{2})dt && (1.70)\\
dx_3 &= f(t+\frac{dt}{2}, x+\frac{dx_2}{2})dt && (1.71)\\
dx_4 &= f(t+dt, x+dx_3)dt && (1.72)\\
dx &= \frac{dx_1+2dx_2+2dx_3+dx_4}{6} && (1.73)
\end{aligned}$$

Table 1.8: Same Number of Calculation

	dt	dt	dt
Euler	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$
Runge-Kuttta 2	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$
Runge-Kutta 4	1	$\frac{1}{2}$	$\frac{1}{4}$

Compared with the Euler's methods, the 2nd-order Runge-Kutta method requires twice as many calculations and the 4th-order Runge-Kutta method requires 4 times as many calculations. In other words, the number of calculation of the Euler's method for $dt = \frac{1}{4}$ is the same as the 2nd-order Runge-Kutta method for $dt = \frac{1}{2}$, which is also the same as the 4th-order Runge-Kutta method for $dt = 1$. Table 1.8 shows a combination of dt and three methods that induces the same

[5]Technical Documentation, STELLA Software, High Performance Systems, Inc. pp. 13-6, 13-7, 1997.

number of calculations. Even so, from the results in Table 1.6, it can be easily verified that the Runge-Kutta methods produce better approximations for the same number of calculations.

Chapter 2

Demand and Supply

This chapter[1] first examines the neoclassical foundation of price adjustment mechanism built on logical time, using system dynamics modeling. Then it is argued that similar workings could be done in a real market economy running on historical time by the interplay of price, inventory and their interdependent feedback relations. This implies that off-equilibrium analysis built on historical time without neoclassical concept of auctioneer is a better way of representing market activities. This approach can be one of the foundations of our macroeconomic modeling.

2.1 Adam Smith!

"There's a person who has influenced upon us more than Jesus Christ! Who's he?" An instructor of Economics 1, an introductory course for undergraduate students at the Univ. of California, Berkeley, challenged his students cheerfully. I was sitting in the classroom as a Teaching Assistant for the course. This was in early 80's when I was desperately struggling to unify three schools of economics in my dissertation; that is, neoclassical, Keynesian and Marxian schools of economics.

"He's the author of the Wealth of Nations written in 1776; his name is Adam Smith!", claimed the instructor. Adam Smith's idea of free market economy has been a core doctrine throughout the so-called Industrial Age which started in the middle of the eighteenth century. It has kept influencing our economic life even today with a simple diagram such as Figure 2.1.

Those who have studied economics are very familiar with this diagram of demand and supply, which intuitively illustrates a market mechanism of price adjustment processes. Price is taken on vertical axis and quantity is taken on

[1] This chapter is based on the paper presented at the 27th International Conference of the System Dynamics Society, Albuquerque, New Mexico, USA, July 26-30, 2009. It is written during my short-term sabbatical leave at the Victoria Management School, Victoria University of Wellington, New Zealand in March 2009.

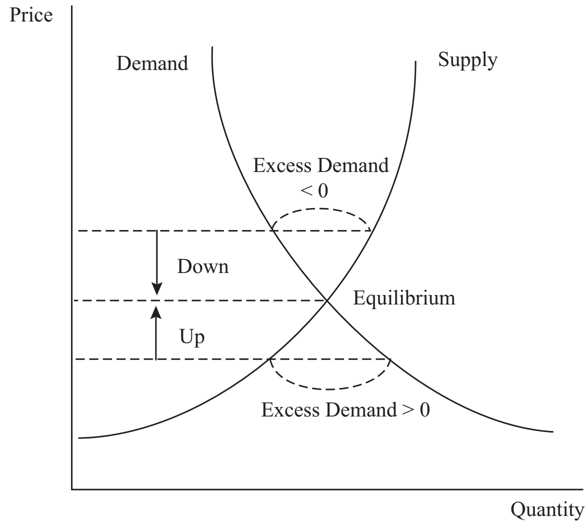

Figure 2.1: Price Mechanism of Demand and Supply

horizontal axis. Demand is illustrated as a downward sloping curve, indicating the attitude of consumers that their demand decreases for higher prices and increases for lower prices. This relation is theoretically derived from a utility maximization principle of consumers. Supply is illustrated as an upward sloping curve which exhibits the behavior of producers that their supply increases for higher prices and decreases for lower prices. This relation results from a principle of profit maximizing behavior by producers. Market equilibrium, in which the amount of demand is equal to the amount of supply and market clears, is shown to exist at a point where demand and supply curves intersect in the diagram.

When price is higher than the equilibrium, there exists an excess supply or unsold and increased amount of inventory (which is also called a negative excess demand), and price is eventually forced to go down to attract more consumers to buy the product. On the other hand, if price is lower, there exists an excess demand or the shortage of product which eventually pushes up the price. In either case, price tends to converge to an equilibrium price. This adjusting market force is provided by an *invisible hand*, Adam Smith believed. It is called a price adjustment mechanism, or tâtonnement process, in modern microeconomics.

This price adjustment mechanism works not only in commodity markets but also in labor markets as well as financial capital markets. For instance, let us consider a labor market by taking a wage rate on the vertical axis and the quantity of labor on the horizontal axis. Then, demand curve is interpreted as the demand for labor by producers and supply curve represents the attitude of workers to work. Producers do not employ as many workers as before if wage rate increases, while more workers want to work or they want to work longer hours if their wage rate is higher, and vice versa. Market equilibrium in the labor market denotes full employment. If wage rate is higher than the equilibrium, unemployment comes off and eventually workers are forced to accept a wage cut. In the case of lower wage rate, labor shortage develops and eventually wage rate is pushed up. In this way, price adjustment mechanism works similarly in the labor market.

In a financial capital market, price on the vertical axis becomes an interest rate, and it become a foreign exchange rate in a case of a foreign exchange market. Price mechanism works in a similar fashion in those markets.

In this way, workings of a price adjustment mechanism could be explicated uniformly in all markets by the same framework. Our daily economic activities are mostly related with these market mechanisms governed by the *invisible hand*. This is why the instructor at the UC Berkeley amused his students, saying that Adam Smith has been more influential than Jesus Christ!

Unfortunately, however, this doctrine of *invisible hand*, or neoclassical school of economic thought has failed to obtain unanimous acceptance among economists, and two opposing schools of economics eventually have been struggling to fight against the workings of market price mechanism depicted by Figure 2.1 They are Keynesian and Marxian schools. Mutually-antagonistic dissents of these school created the East-West conflicts, Cold War since the World War II, and domestic right-left wing battle till late 80's when these battles of ideas finally seemed to have ended with a victory of neoclassical school. Since then, the age of the so-called privatization (of public sectors), and globalization with the help of IT technologies have started as if the doctrine of the *invisible hand* has been the robust foundation of free market fundamentalism similar to religious fundamentalisms.

Accordingly most of us believed there would be no longer conflicts in economic thoughts as well as in our real economic life until recently when we were suddenly hit by severe financial crises in 2008; the worst recession ever since the Great Depression in 1929. The battle of ideas seems to be re-kindled against the doctrine of the *invisible hand*. Indeed, the instructor at the UC Berkeley was right. Today Adam Smith seems to be getting more influential globally, not because his doctrine is comprehensive enough to accomplish a consensus on the workings of a market economy, but because it caused many serious socio-economic conflicts and wars instead.

2.2 Unifying Three Schools in Economics

As a graduate student in economics in late 70's and early 80's, I was struggling to answer the question: Why did three schools disagree? As a proponent of Adam Smith's doctrine, neoclassical school believes in a price adjusting mechanism in the market. As shown above, however, this price mechanism only works so long as prices and wages move up and down flexibly in order to attain an equilibrium. Therefore, if disequilibria such as recession, economic crisis and unemployment happen to occur, they believe, it's because economic agents such as monopoly, government and trade unions refuse to accept price and wage flexibility and distort the workings of market mechanism.

Keynesian school considers that market has no self-restoring forces to establish an equilibrium once economic recessions and unemployment occur, because prices and wages are no longer flexible in a modern capitalist market economy. To attain an equilibrium, therefore, government has to stimulate the economy through fiscal and monetary policies. In Figure 2.1 these policies imply to shift the demand curve to the right so that excess supply (and negative excess demand) will be eliminated.

Marxian school believed that market disequilibria such as economic crisis and unemployment are inevitable in a capitalist market economy, and proposed a planned economy as an alternative system. After the collapse of the Soviet Union in 1989, Marxian school ceased to exercise its influence because the experiments of a planned economy in the former socialist countries turned out to be a failure. Even so, they manage to survive under the names of post-Keynesian, environmental economics and institutional economics, etc.

Accordingly, only neoclassical and Keynesian schools remain to continue influencing today's economic policies. In the United States, Republican policies are deeply affected by the doctrine of neoclassical school such as free market economy and small government through deregulation. Meanwhile, Democrats favor for Keynesian viewpoint of public policies such as regulations by wise (not small) government. Current financial crises may reinforce the trend of regulation against hand-free financial and off-balance transactions.

Why do we need three different glasses to look at the same economic reality? Why do we need three opposing tools to analyze the same economic phenomena? These were naive questions I posed when I started studying economics as my profession. In those days I strongly believed that a synthesis of three schools in economics is the only way to overcome Cold War, East-West conflicts and domestic right-left wing battles. By synthesis it was meant to build a unified general equilibrium framework from which neoclassical, Keynesian and Marxian theories can be derived respectively as a special case. My intention was to show that different world views were nothing but a special case of a unified economic paradigm.

While continuing my research toward the synthesis, I was suddenly encountered by a futuristic viewpoint of *The Third Wave* by Alvin Toffler [50]. It was on December 23, 1982, when I happened to pick up the book which was piled up in a sociology section at the Berkeley campus bookstore. The most unimaginable idea to me in the book was the one that both capitalism and socialism were the two sides of the same coin in the industrial age against the leftist doctrine that socialism is an advanced stage of economic development following capitalism. What's an economic system of the Third Wave, then? Can a new economic system in the information age comply with either neoclassical or Keynesian school of economics developed in the industrial age? I kept asking these questions many times in vain, because Tollfer failed to present his economic system of the information age in a formal and theoretical fashion.

Being convinced by Toffler's basic idea, however, I immediately decided to develop a simple economic model which could be a foundation of a new economic framework for the information age. In this way, the Third Wave became a turning point of my academic research in economics, and since then my work has been focused on a new economic system of the information age. My effort of synthesizing three schools in economics and creating a future vision of a new economic system fortunately resulted in a publication of the book [58]. Its main message was that three schools in economics are effete in a coming information age, and a new economic paradigm suitable for the new age has to be established[2].

My idea of economic synthesis was to distinguish logical time on which neoclassical school's way of thinking is based, from historical time on which Keynesian and Marxist schools of economic thought are based. Yet, the working tools available in those days are paper and pencil. Under such circumstances I was fortunate to encounter by chance system dynamics in middle 90's through the activities of futures studies. Since then, system dynamics modeling gradually started to re-kindle my interest in economics. This chapter examines a true mechanism of the working of market economy, which is made possible by the application of system dynamics modeling.

2.3 Tâtonnement Adjustment by Auctioneer

Let us now construct a simple SD model to examine how a market economy of demand and supply works. In this simple economy buyers and sellers have demand and supply schedules of shirts per week as shown in Table 2.1. These figures are taken from a paper in [57] under the supervision of Professor Jay W. Forrester[3]. The reader can easily replace them with his or her own demand and supply schedules.

Price	Quantity Demanded $D = D(p)$	Quantity Supplied $S = S(p)$
\$ 5	100	0
\$ 10	73	40
\$ 15	57	57
\$ 20	45	68
\$ 25	35	77
\$ 30	28	84
\$ 35	22	89
\$ 40	18	94
\$ 45	14	97
\$ 50	10	100

Table 2.1: Demand and Supply Schedules in [57]

[2]This theme will be further explored in part IV: New Macroeconomic System.

[3]MIT System Dynamics in Education Project (http://sysdyn.clexchange.org/sdep.htm) offers a collection of SD models and papers called Road Maps for self-taught learning of system dynamics. The reader is encouraged to explore these profound resources of SD modeling.

In microeconomics these schedules are called demand and supply functions of market prices and derived rigorously from the axiomatic assumptions of consumers and producers. Demand and supply schedules (or functions $D = D(p)$ and $S = S(p)$) are illustrated in Figure 2.2 in which price is taken on horizontal axis while demand and supply are plotted on vertical axis. This is a standard presentation of functions in mathematics. On the other hand, in standard textbooks of economics price has been traditionally taken on vertical axis as illustrated in Figure 2.1.

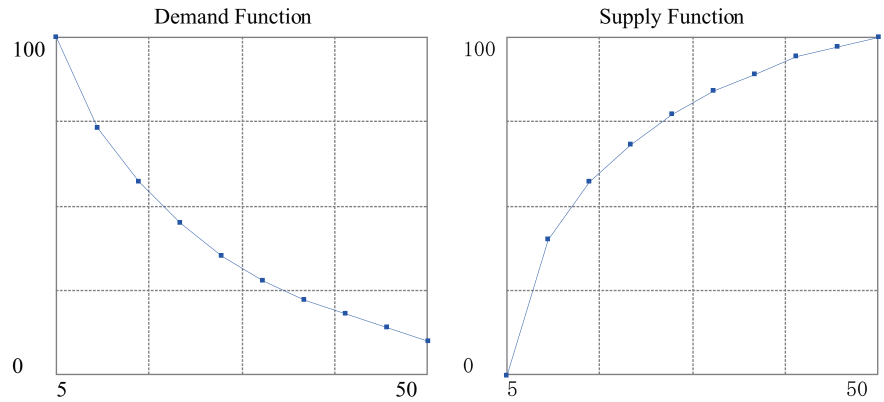

Figure 2.2: Demand and Supply Functions)

Now buyers and sellers meet in the market to buy and sell their products according to their schedules of demand and supply. In order to make this market economy work, we need the third player called *auctioneer* who quotes a price. His role is to raise a price if demand is greater than supply, and lower it if demand is less than supply. His bids continue until the equilibrium is attained where demand is simply equal to supply. This process is called Walrasian or neoclassical price adjustment mechanism or tâtonnement.

The important rule of this market game is that no deal is made until market equilibrium is attained and buyers and sellers can make contracts of transactions. In this sense, time for adjustment is not a real time in which economic activities such as production and transactions take place, but the one needed for calculation. The time of having this nature is called *logical time* in [58]. In reality, there are very few markets that could be represented by this market except such as stock and auction markets. Even so, neoclassical school seems to cling to this framework as if it represents many real market transactions.

Equilibrium

Does this market economy work? This question includes two different inquiries: an existence of equilibrium and its stability. If equilibrium does not exist, the auctioneer cannot finish his work. If the equilibrium is not stable, it's impossible to attain it. Let us consider the existence problem first.

The auctioneer's job is to find an equilibrium price at which demand is equal to supply through a process of the above-mentioned tâtonnement or groping process. Mathematically this is to find the price p^* such that

$$D(p^*) = S(p^*) \tag{2.1}$$

In our simple demand and supply schedules in Table 2.1, the equilibrium price is easily found at \$ 15. The existence proof of general equilibrium in a market economy has annoyed economists over a century since Walras. It was finally proved by the so-called Arrow-Debreu model in 1950's. For detailed references, see Yamaguchi [58]. Arrow received Nobel prize in economics in 1972 for his contribution to "general economic equilibrium and welfare theory". He was a regular participant from Stanford University to the Debreu's seminar on mathematical economics when I was in Berkley. Debreu received Nobel prize in economics in 1983 for his contribution to "new analytical methods into economic theory and for his rigorous reformulation of the theory of general equilibrium". I used to attend his seminar on mathematical economics in early 80's, and still vividly remember the day of his winning the prize, followed by a wine party spontaneously organized by faculty members and graduate students.

Stability

The second question is how to find or attain the equilibrium. From the demand and supply schedules given above, there seems to be no difficulty of finding the equilibrium. In realty, however, the auctioneer has no way of obtaining these schedules. Accordingly, he has to grope them by quoting different prices. To describing this groping process, a simple SD model is built as in Figure 2.3 [Companion model: 1 Auctioneer.vpm].

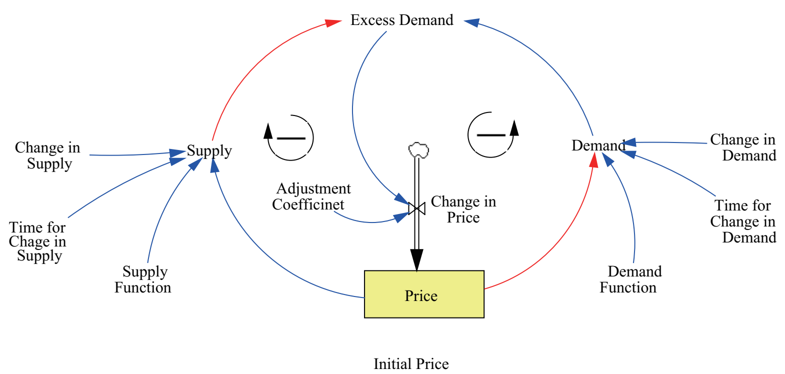

Figure 2.3: Auctioneer's Tâtonnement Model

Mathematically, the model is formulated as follows:

$$\frac{dp(t)}{dt} = f(D(p) - S(p), \lambda) \tag{2.2}$$

where f is excess demand function and λ is a price adjustment coefficient. In the model f is further specified as

$$f = \lambda \ \frac{D(p) - S(p)}{D(p) + S(p)} \ p \tag{2.3}$$

From the simulations in our simple model the idea of tâtonnement seems to be working well as illustrated in Figure 2.4. The left-hand diagram shows that the initial price of \$10 tends to converge to an equilibrium price of \$15. Whatever values of initial price are taken, the convergence can be similarly shown to be attained. In this sense, the market economy can be said to be globally stable. With this global stability, the auctioneer can start with any quotation of initial price to arrive at the equilibrium successfully.

In the right-hand diagram, demand schedule is suddenly increased by capricious buyers by 20 units at the week of 15, followed by the reactive increase of the sellers in the same amount of supply at the week of 30, restoring the original equilibrium. In this way, the auctioneer can easily respond to any changes or outside shocks and attain new equilibrium states. These shifts of demand and supply curves are well known in microeconomics as comparative static analysis.

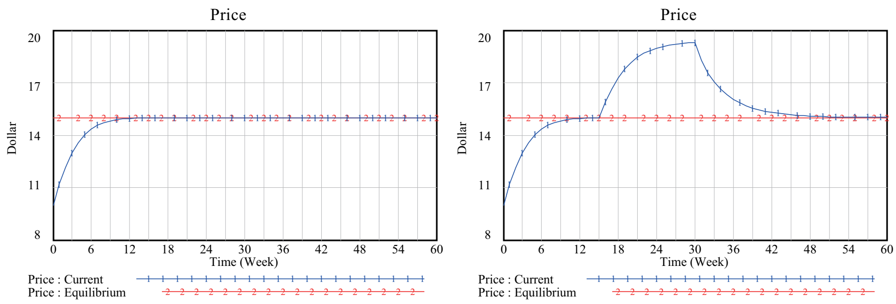

Figure 2.4: Stability of Equilibrium

Chaos

So far, neoclassical price mechanism seems to be working well. To attain the equilibrium in our model, a price adjustment coefficient is set to be 0.4. What will happen if the auctioneer happens to increase the adjustment coefficient from 0.4 to 3 in order to speed up his tâtonnement process? Surprisingly this has caused a period 2 cycle of price movement with alternating prices between 10.14 and 18.77, as illustrated in the left-hand diagram of Figure 2.5. When the coefficient is increased a little bit further to 3.16, price behavior suddenly becomes very chaotic as the right-hand diagram illustrates. I encountered this chaotic price behavior unexpectedly when I was constructing a pure exchange economic model using S language under UNIX environment in [59].

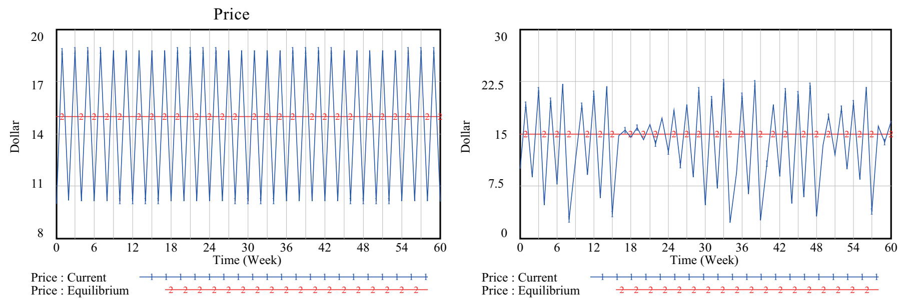

Figure 2.5: Chaotic Price Behavior

Under such a chaotic price behavior, it is obvious that the auctioneer fails to attain an equilibrium price. Accordingly, under the failure of finding the equilibrium, market transactions can never take place according to the neoclassical rule of the market game. This indicates a fundamental defect in neoclassical framework of market economy based on the idea of *logical* time.

Short-side Transactions

Tired with an endless struggle by the auctioneer to attain an equilibrium in a chaotic price behavior, buyers and sellers may force their actual transactions to resume at a short-side of demand and supply. In other words, if demand is greater than supply, the amount supplied at that price is traded, while the amount demanded is purchased if supply is greater than demand.

To allow this off-equilibrium transactions, the auctioneer has to have enough amount of inventory at hand before the market starts. To calculate the enough amount of inventory, a slightly revised model is built as shown in the left-hand diagram of Figure 2.6 [Companion model: 2 Auctioneer(Inventory).vpm].

When the auctioneer quotes an initial price below equilibrium at $5, allowing the short-side trade, unrealized excess demand keeps piling up as backlog due to an inventory shortage and the amount accumulates up to 325.30 shirts. When market price is initially quoted above equilibrium at $25, excess supply causes inventory of unsold shirts to piles up to 137.86 shirts, as illustrated in the right-hand diagram of Figure 2.6. If the auctioneer is allowed to have these amount of inventories from the beginning, he could find an equilibrium price even by allowing these inter-auction transactions. Since no shirts are made available until the equilibrium contract is made and production activities start under the neoclassical rule of market game, this short-side off-equilibrium deal is logically impossible. In other words, no feedback loop is made available without inventory from the viewpoint of system dynamics. In conclusion, the existence of chaotic price behavior and neoclassical assumption of market economy are inconsistent.

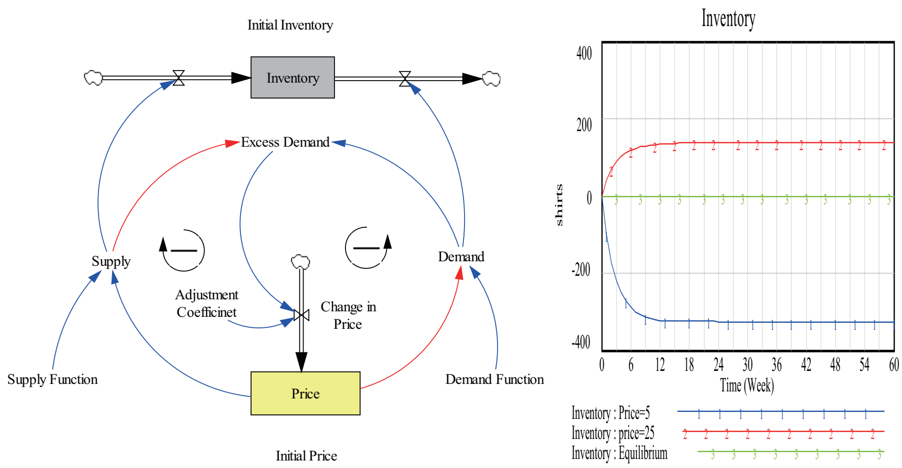

Figure 2.6: Short-side Transaction Model and Inventory

2.4 Price Adjustment with Inventory

The above analysis indicates it's time to abandon the neoclassical framework based on logical time. In reality, production and transaction activities take place week by week, and month by month at short-side of product availability, accompanied by piled-up inventory or backlog. Time flow on which these activities keep going is called *historical time* in [58]. In system dynamics, demand and supply are regarded as the amount of flow per week, and flow eventually requires its stock as inventory to store products. Thanks to the inventory stock, transactions now need not be waited until the auctioneer finishes his endless search for an equilibrium. This is a common sense, and even kids understand this logic. In other words, a price adjustment process turns out to require inventory from the beginning of its analysis, which in turn makes off-equilibrium transactions possible on a flow of historical time.

This disequilibrium approach is the only realistic method of analyzing market economy, and system dynamics modeling make it possible. The model running on historical time for simulations, which is based on [57], is drawn in Figure 2.7 [Companion model: 3 Inventory.vpm].

Price no longer need to respond to the excess demand, instead it tries to adjust to the gap between inventory and desired inventory. To avoid a shortage under off-equilibrium transactions, producers usually try to keep several weeks of the demanded amount as inventory. This amount is called desired inventory. An inventory ratio is thus calculated as the inventory divided by the desired inventory. And market prices are assumed to respond to this ratio. Table 2.2 specifies the effect of the ratio on price. For instance, if the actual inventory is 20% larger than the desired inventory, price is assumed to be lowered by 25%. Vice versa, if it's 20% smaller, then price is assumed to be raised by 35%.

Mathematically, the model is formulated as follows:

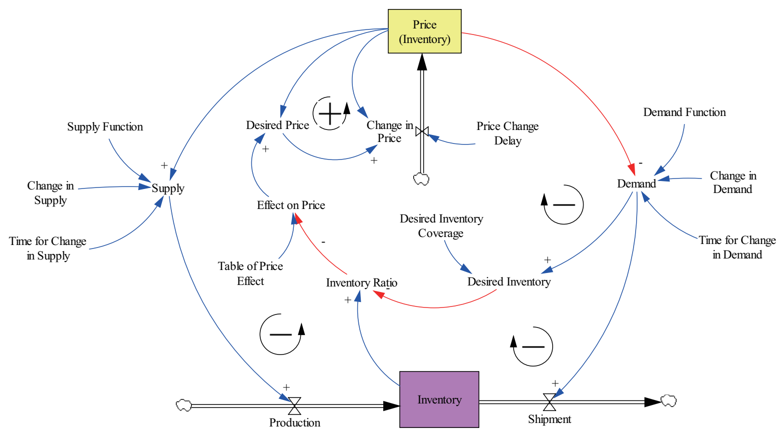

Figure 2.7: Price Adjustment Model with Inventory

Inventory Ratio	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4
Effect on Price	1.8	1.55	1.35	1.15	1	0.875	0.75	0.65	0.55

Table 2.2: Effect of Inventory Ratio on Price

$$\frac{dp(t)}{dt} = \frac{p^* - p(t)}{PCD} \tag{2.4}$$

where PCD is a parameter of price change delay, and p^* is a desired price such that

$$p^* = p(t)g\left(\frac{x(t)}{x^*}\right) \tag{2.5}$$

Function g is a formal presentation of the numerical relation in Table 2.2, and $x(t)$ and x^* denote inventory and desired inventory, respectively, such that

$$\frac{dx(t)}{dt} = S(p) - D(p) \tag{2.6}$$

$$x^* = \alpha D(p) \tag{2.7}$$

where α is a parameter of desired inventory coverage[4].

[4]Without using a table function, function g could be mathematically represented as

$$g = \frac{1}{x^e}, \text{ where } x = \frac{x(t)}{x^*} \tag{2.8}$$

Elasticity of the function g can be calculated as

$$\text{Elasticity} \equiv -\frac{dg}{g}\Big/\frac{dx}{x} = -\frac{dg}{dx}\frac{x}{g} = -\left(-\frac{e}{x^{e+1}}\right)\frac{x}{g} = e \tag{2.9}$$

Hence, the function g has a uniform elasticity e over its entire range.

Under such circumstances, the initial price is set here at \$10 as in the case of the auctioneer's tâtonnement. Price (line 5) now fluctuates around the equilibrium price of \$15 by overshooting and undershooting alternatively, then tends to converge to the equilibrium as illustrated in Figure 2.8. Inventory gap (= desired inventory - inventory) is the gap between line 4 and 3, and price responds to this gap rather than the excess demand (the gap between line 1 and 2). The reader can easily confirm that price tends to rise as long as the inventory gap is positive, or inventory ratio is lower than one, and vice versa.

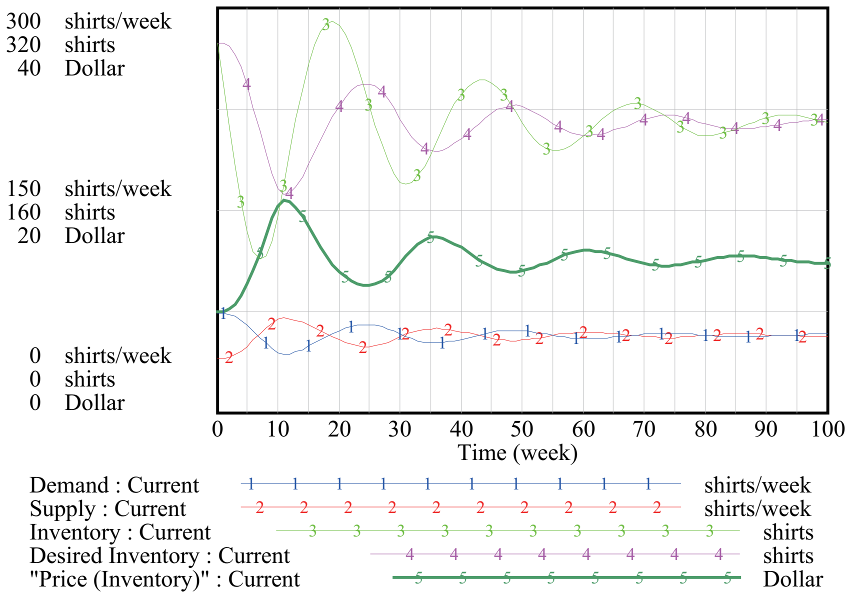

Figure 2.8: Price Adjustment with Inventory

In the left-hand diagram of Figure 2.9, demand is increased by 20 units at the week of 15, followed by the increase in the same amount of supply at the week of 30, restoring the original equilibrium as in the case of the auctioneer's tâtonnement, though overshooting this time. These shifts of demand and supply curves, however, may no longer be appropriate to be called comparative static analysis method in microeconomics, because we are no longer comparing two different states of equilibrium points. Right-hand diagram illustrates how price cycle is triggered by reducing the original inventory coverage of 4 weeks to 2.3

Desired price p^* can be now rewritten as

$$p^* = p(t)\frac{1}{x^e} \tag{2.10}$$

Price adjustment processes are constructed in this fashion in our macroeconomic models below.

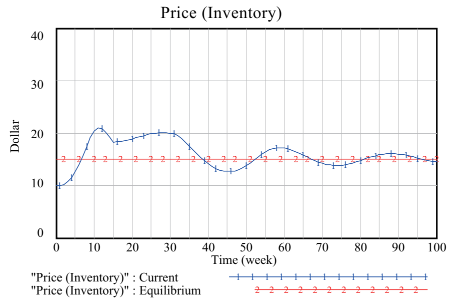

Figure 2.9: Effects of the Changes in Demand, Supply and Inventory Coverage

weeks. In conclusion, system dynamics modeling makes it possible to describe the actual off-equilibrium transactions and price behaviors along the historical time.

2.5 Logical vs Historical Time

A combined model is created in Figure 2.10 to compare how the above two price adjustment processes behave differently; one is running on logical time and the other on historical time [Companion model: 4 Comparison.vpm].

Left-hand diagram of Figure 2.11 is produced to show similar patterns by setting the auctioneer's adjustment coefficient to be 2.7. In both cases it takes about 100 weeks to attain the equilibrium. The difference is that under logical time production and transactions never take place until the equilibrium is attained around the logical time of 100 weeks, while a real economy running on the historical time is suffering from the fluctuation of inventory business cycles for 100 weeks until a real equilibrium price is attained.

What will happen if the demand suddenly increases by 20 at week 50. Right-hand diagram illustrates the real economy can no longer attain the equilibrium in 100 weeks. In this way the market economy is forced to be fluctuating around off-equilibrium points forever in face of continued outside shocks, compared with a quick realization of the equilibrium by the auctioneer around the logical time of week 70.

The meaning of logical and historical times is now clear. Microeconomic textbooks are full of logical time analyses when dynamics of price movements are discussed. The reader now has the right to ask if the time in textbooks is logical or historical. If historical, price has to be always accompanied by the inventory on historical time.

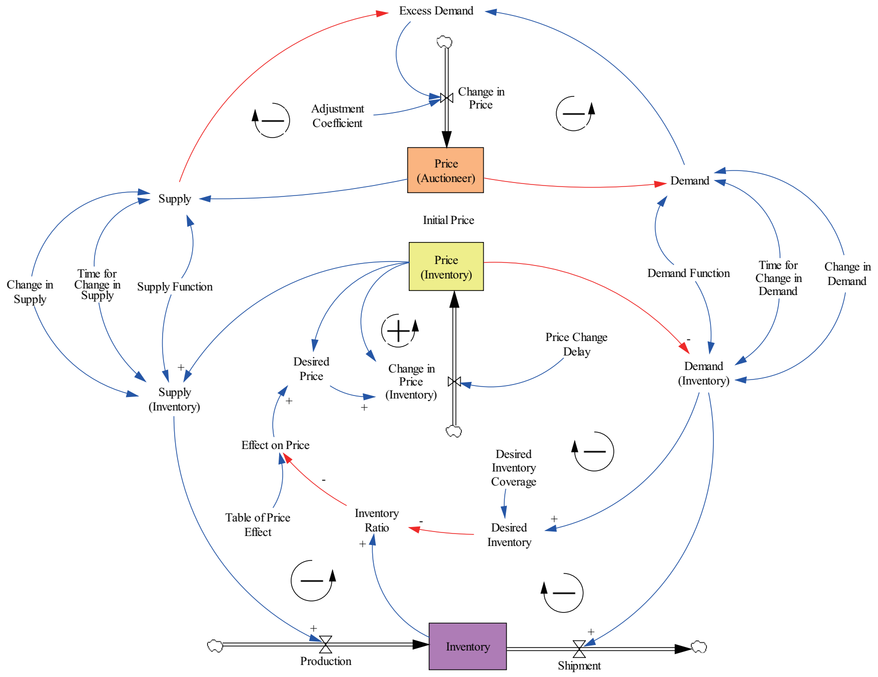

Figure 2.10: Auctioneer vs Inventory Price Mechanism Compared

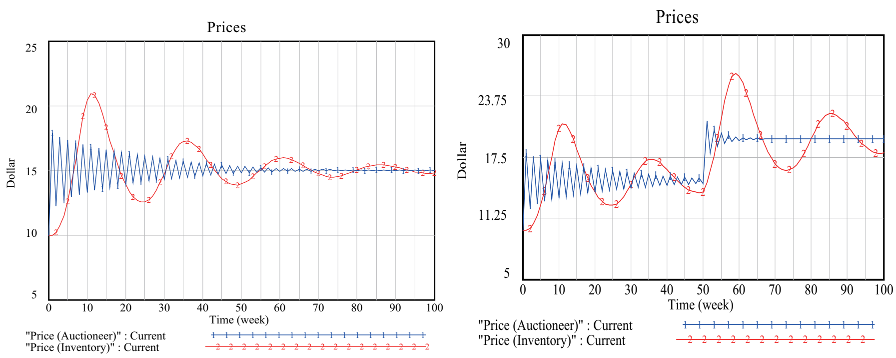

Figure 2.11: Auctioneer vs Inventory Price Behaviors

2.6 Stability on A Historical Time

Which path, then, should we follow to analyze free market economic activities? Neoclassical analysis of logical time is mathematically rigorous, yet free price behavior is no longer stable, as preached by market fundamentalists, due to the appearance of chaos as shown above. In other words, market economy could be chaotic even on the basis of neoclassical doctrine.

On the other hand, analysis running on historical time is off-equilibrium and looks unstable, full of business cycles; that is, chaotic as well. Yet, there's a way to make the historical time analysis stable and free from business cycles. To do so, let us now change the seller's supply (production) schedule so that it can reflect the inventory gap as follows:

$$\begin{aligned} \text{Supply (Inventory)} &= \text{Supply Function (Price (Inventory))} \\ &\quad + \text{Inventory Gap / Inventory Adjustment Time} \qquad (2.11) \end{aligned}$$

Mathematically, equation (2.6) is replaced with the following:

$$\frac{dx(t)}{dt} = S^*(p) - D(p) \qquad (2.12)$$

$$S^*(p) = S(p) + \frac{x^* - x(t)}{IAT} \qquad (2.13)$$

where IAT is inventory adjustment Time.

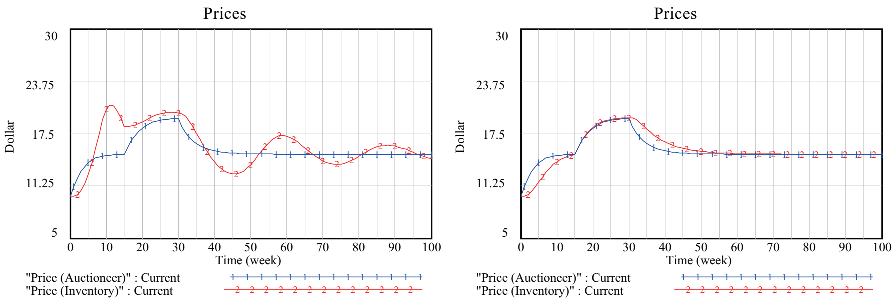

Figure 2.12: Historical Price Stability with Adjusted Supply Schedule (1)

Left-hand diagram of Figure 2.12 illustrates how price behaviors are different between logical time (line 1) and historical time (line 2) when demand is increased by 20 units at the week of 15, followed by the increase in the same amount of supply at the week of 30 [Companion model: 5 Comparison(Supply).vpm)]. In both cases prices try to restore the original equilibria, though their speed and meaning are different. In the right-hand diagram, newly adjusted supply schedule is now applied with the inventory adjustment time of 3 weeks. To our surprise, almost the same price behavior is obtained as the one on logical time.

In the left-hand diagram of Figure 2.13, price behavior on the logical time is illustrated as line 1 for the initial price at \$10, while the same price behavior on the historical time is illustrated as line 2 for the inventory coverage of 2.3 weeks, similar to the right-hand diagram of Figure 2.9. Now the new supply schedule is applied to the same situation, which results in line 3. Again, the line 3 becomes very similar to the price behavior (line 1) on the logical time.

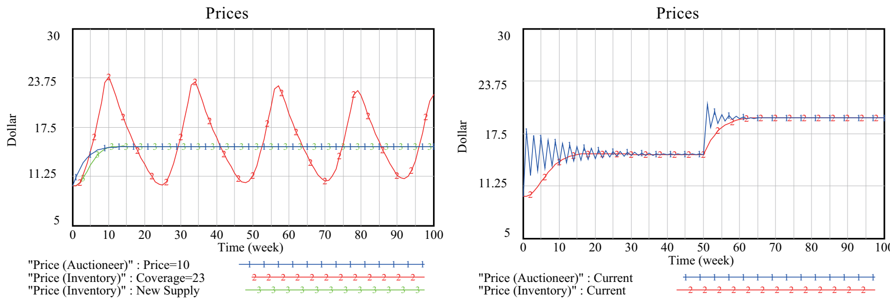

Figure 2.13: Historical Price Stability with Adjusted Supply Schedule (2)

Finally let us apply the new supply schedule to the right-hand diagram of Figure 2.11, that is previously explained as the case in which "the real economy can no longer attain the equilibrium in 100 weeks." Right-hand diagram of Figure 2.13 is the result obtained by newly adjusted supply with the inventory adjustment time of 3 weeks. Again almost similar price behavior is restored as the one on the logical time.

These simulation results may indicate that our market economy could behave as close as the one predicted by neoclassical equilibrium analysis on logical time so long as economic agents behave appropriately on the historical off-equilibrium time. In other words, we no longer need a help from auctioneer running on logical time to attain an equilibrium in a market economy. Price, inventory and their interdependent feedback relations can do the same job in a real market economy.

2.7 A Pure Exchange Economy

2.7.1 A Simple Model

Chaotic price behavior observed in tâtonnement adjustment is not specific to a partial or single market. To show a chaos in a general equilibrium framework, let us consider a pure exchange economy: the most favored economy used by neoclassical economists in textbooks. A pure exchange economy is a kind of game without production in which initially endowed goods are exchanged on the basis of traders' own preferences such that their utilities are maximized. Such an exchange economy is profoundly criticized by Joan Robinson [43] as an irrelevant game in a prison camp in which prisoners are given fairly equal amounts of commodities irrespective of their personal tastes so that an exchange game based on their tastes can easily proceed. I have also criticized its appropriateness as a capitalist economic model [58, Chapter 7], and posed a more comprehensive model comprising the analysis of both logical and historical time for a better understanding of the functioning of a capitalist market economy[58, Chap.3-6].

Yet, the exchange model is still used in most textbooks on microeconomics as a first approximation to a market mechanism. If there still exists something that we can learn from a pure exchange model, it is the functioning of a tâtonnement price adjustment mechanism. The structure of the price mechanism is basically the same for a more general economy with production. Thus, Hildenbrand and Kirman justify the analysis of a pure exchange economy as saying "if we cannot solve, in a reasonably satisfactory way, the exchange problem, then there is not much hope for the solution of the more general one [31, pp.51-51]." I have indicated [58] that this justification is only applicable to the analysis of *logical* time, but not to that of *historical* time. A pure exchange model should, therefore, be confined to a heuristic use for understanding a price mechanism of logical time.

Understanding the exchange economy this way, do we still have unanswered problems? The answer seemed to me to be negative at first, since the economy has been comprehensively studied in the literature, for instance, [31] and [45]. However, there still exist some interesting questions in the area of numerical computations and simulations of price adjustment mechanisms using system dynamics modeling.

The economy is explained as follows. It consists of at least two traders (and consumers simultaneously) who bring their products to the market for exchange. Their products are called *initial endowment* in economics, which becomes the source of supply in the market. We assume following endowment for consumer 1 and 2.

$$\begin{cases} \text{Consumer 1} & = (10,\ 6) \\ \text{Consumer 2} & = (\ 6, 15) \end{cases} \tag{2.14}$$

The economy can thus evade the analysis of production. That's why it is called a pure exchange economy.

In the pure exchange economy only relative prices matter due to the Walras law[5]. Let us assume that commodity 1 becomes a numeráire, that is, its price is unitary: $p_1 = 1$, $p_2 = p$ be a relative price of commodity 2.

When the price is quoted in the economy, traders evaluate a market value of their products as a source of their income for further exchange or purchase of the products in the market. Then as consumers, they try to maximize their utility (which is derived from the consumption of the products purchased in the market) according to their own preferences subject to their income constraint. In this way their demand for products are calculated as a function of prices, income (which in turn is a function of prices) and preferences. Total demand is obtained as a sum of these individual consumer's demand, which is then compared with the total supply. Excess demand is defined as the difference between total demand and total supply, and becomes a function of prices and preferences. Figure 2.14 illustrates a causal loop diagram of the pure exchange economy.

[5] See the appendix for detail.

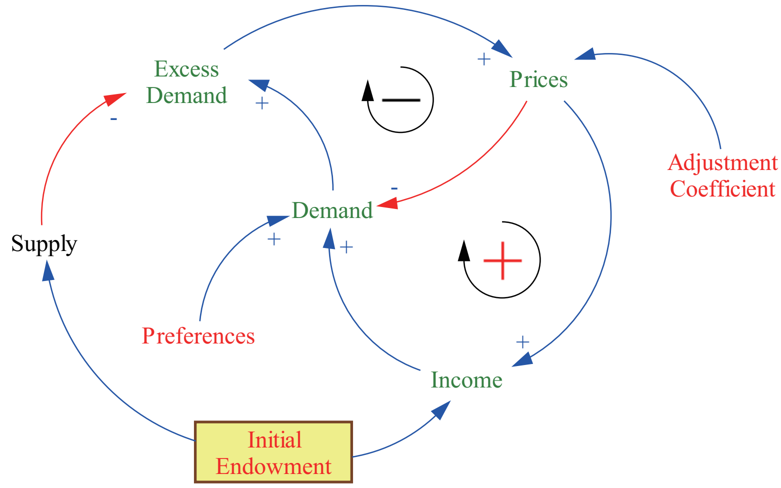

Figure 2.14: A Causal Loop of A Pure Exchange Economy

Market prices have two causal loops; one positive and one negative feedback loops. In the figure they are indicated by plus and minus signs. Positive loop in general tries to reinforce the original move stronger, while negative loop tries to counterbalance it. Thus, a moving direction of market prices depends on which loop is dominating: positive or negative? When a positive feedback loop dominates, prices tend to diverge, while a negative feedback loop reverses the direction of the price movement. These opposite and complicated movements are caused by the values of two parameters: adjustment coefficient and preferences. Pure exchange model is illustrated in Figure 2.15 [Companion model: 6 PureExchange.vpm].

2.7.2 Tâtonnement Processes on Logical Time

A step-by-step calculation process of price adjustment is depicted in Figure 2.16, where P_t denotes prices at the period t, a function f denotes the amount of excess demand, and α, λ denotes preferences and price adjustment coefficient, respectively. Preferences and adjustment coefficient are the only parameters in the economy which have to be exogenously determined. Once these are given and present prices are quoted, excess demand can be calculated. If it is positive, prices at the next period are increased by the amount of the excess demand multiplied by the adjustment coefficient. Hence, adjustment coefficient determines the degree of a price increase in the next period. When excess demand is negative, prices at the next period are decreased by the the amount of the excess demand multiplied by the adjustment coefficient.

As illustrated in Figure 2.1, a convergence of prices to the equilibrium is expected where demand and supply curves intersect. Indeed, they did for a very small value of adjustment coefficient; that is, prices are shown to be globally

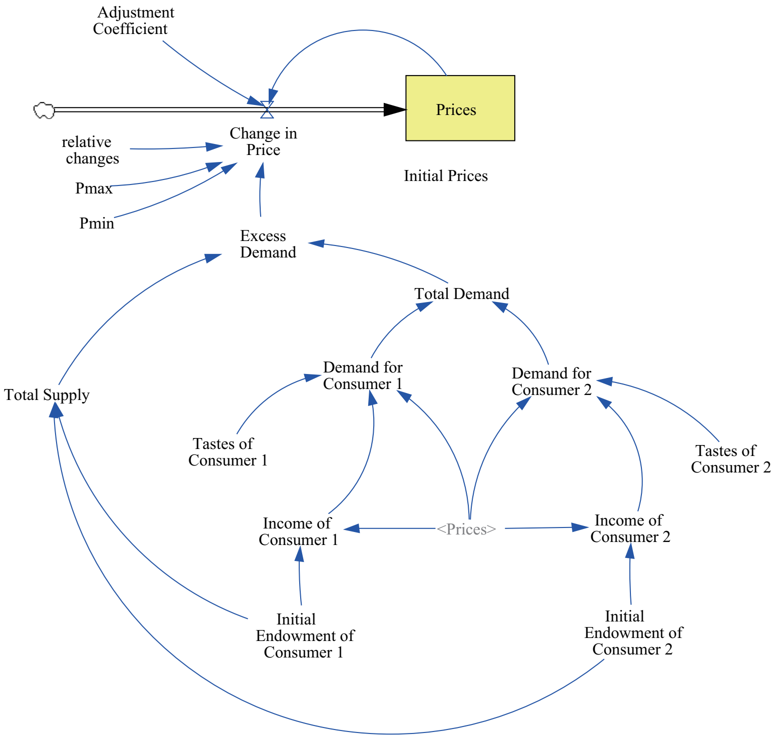

Figure 2.15: A Pure Exchange Economy Model

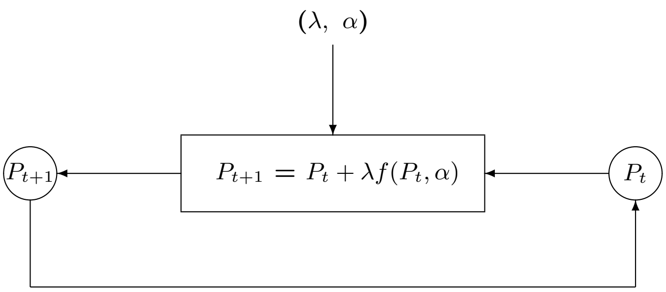

Figure 2.16: Tâtonnement Adjustment Process

stable.

To our surprise, however, something strange happened as the value of the coefficient increased. As Figure 2.17 indicates, the adjustment process begins to produce a clear bifurcation, or an oscillation of prices in period 2 when price adjustment coefficient is $\lambda = 0.148$. Furthermore, an increasing adjustment coefficient continues to create new bifurcations or price oscillations of period

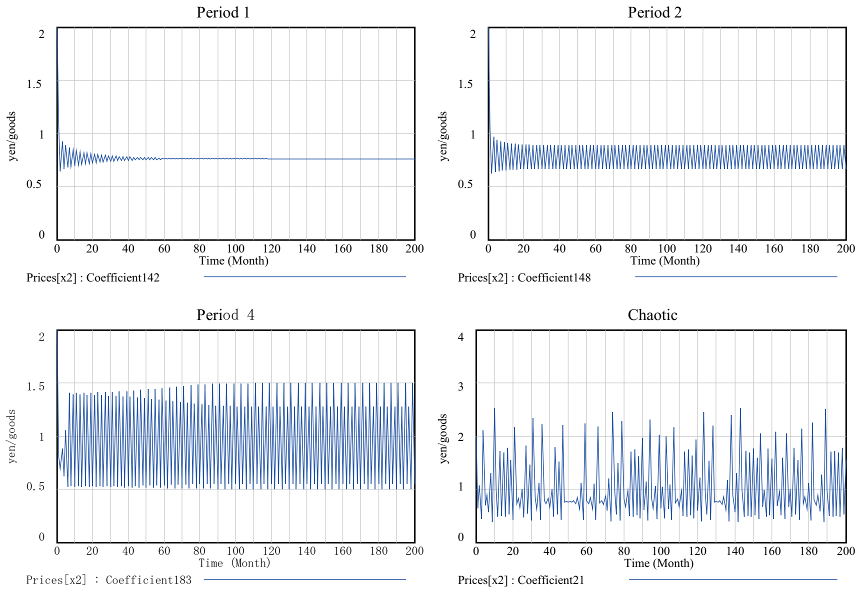

Figure 2.17: Price Movement of Period 1, 2, 4 and Chaos

$2n$, $n = 1, 2, \ldots$ until it became totally chaotic. In other words, instead of converging to an equilibrium or diverging to infinity, market prices seemed to be eventually attracted to a certain region and continue fluctuating in it, with the information of initial values being lost shortly.

Figure 2.18 illustrates the bifurcation of prices as the adjustment coefficient increases. The region is called *a strange attractor* or *chaos*. Hence, a price mechanism in a market economy turned out to be chaotic!. In such a chaotic region, market economy becomes far from the equilibrium and globally unstable, and economic disequilibria such as recession and unemployment become dominant.

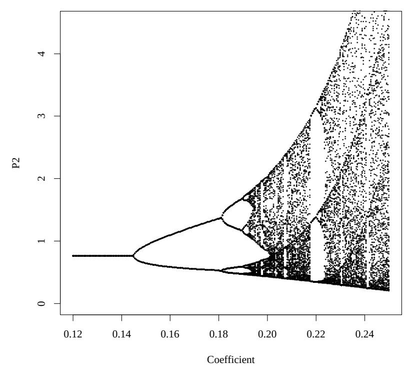

Figure 2.18: Chaotic Price for λ

One of the main features of chaos is a sensitive dependence on initial conditions. This means that a very small difference of initial values will create a big difference later on and a long run prediction of the movement will become eventually impossible. This is confirmed by plotting prices as time-series data. In Figure 2.19 dotted and real lines represent a time-series of two prices whose initial values only differ by 0.0001. Line 1 is obtained at $\lambda = 0.21001$, while line 2 is obtained at $\lambda = 0.21002$. Evidently two lines begin to diverge as time passes around month 20, which proves that prices are indeed chaotic (See [59] for details).

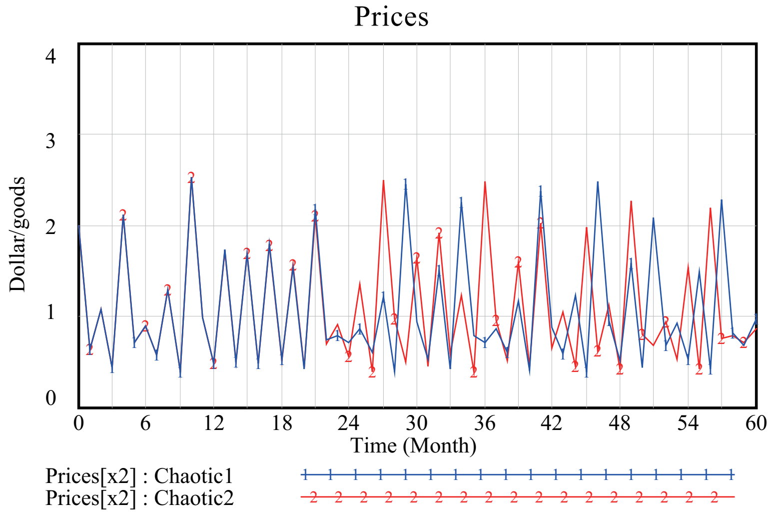

Figure 2.19: Sensitive Dependence on Initial Conditions

It is almost impossible in reality to obtain a true initial value due to some observation errors and round-off errors of measurement and computations. These errors are exponentially magnified in a chaotic market to a point where predictions of future prices and forecasting are almost meaningless and even misleading.

This is a wholly unexpected feature for a neoclassical doctrine of market stability originated by Adam Smith's idea of *invisible hand*. Even so, this chaotic situation could be harnessed so long as the value of adjustment coefficient is small enough; in other words, prices are regulated to fluctuate only within a small range so that no violent jumps of prices are allowed - a relief to the neoclassical school.

2.7.3 Chaos Triggered by Preferences

What will happen, then, if preferences, an another parameter in the economy, vary? Can whimsical preferences of consumers are also powerful enough to drive a stable economy into chaos? To examine this, I started with a globally stable situation in which a price, wherever its initial position is, converges to an equilibrium price; specifically at the adjustment coefficient of $\lambda = 0.138$. Then tastes of goods 1 for consumer 1 is increased from 0.5 slightly up. Figure

2.20 illustrates periodic behavior of price caused by the changes in consumer 1's tastes.

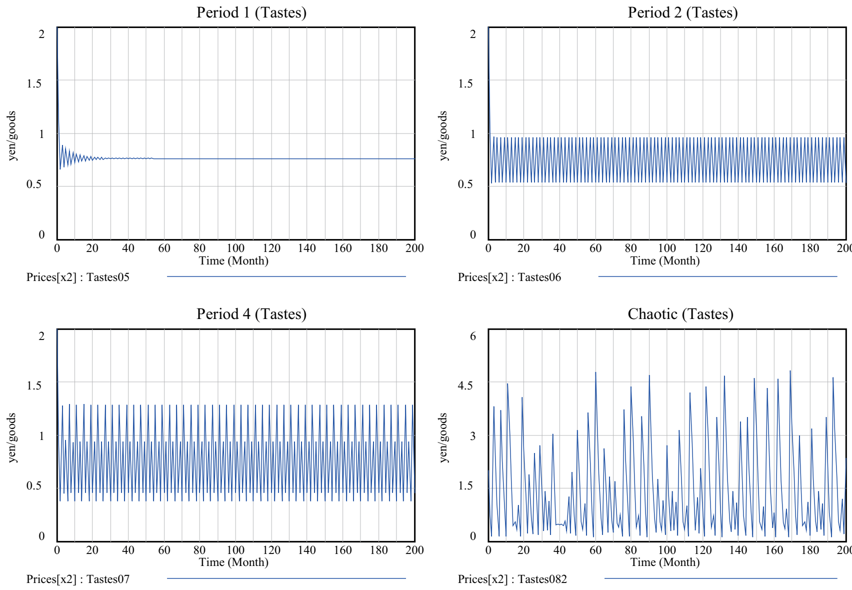

Figure 2.20: Price Movement of Period 1, 2, 4 and Chaos Caused by Tastes

Figure 2.21 is produced by changing the values of preferences α. It indicates that as the values of α increase an equilibrium price tends to be going down up to a bifurcation point. Except this decreasing equilibrium price, to our surprise, both diagrams in Figures 2.18 and 2.21 turned out to be structurally similar; that is, chaos is similarly caused by the changes in preferences (Fore details see [59] and [60]).

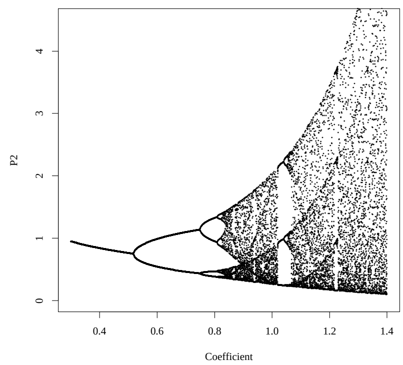

Figure 2.21: Chaotic Price for α

This seems to be a serious challenge against a neoclassical doctrine of price stability. Market equilibrium can no longer be restored even by a small value of adjustment coefficient. That is to say, price regulations suggested above are no longer effective to harness a chaos in the market. The price stability attained by a small value of adjustment coefficient can be easily driven into a chaos by whimsical preferences of consumers. Capricious behaviors of consumers themselves are the cause of chaos and, to be worse, no regulations are possible to control consumers' preferences. It is concluded, therefore, that chaos is inherent in the market, to be precise, of logical time.

2.7.4 Off-Equilibrium Transactions on Historical Time

What is an economic implication of this chaotic price adjustment, then? Pure exchange economy works only when its auctioneer can find equilibrium prices at which traders and consumers make their transactions. If the auctioneer cannot find the equilibrium, market failure arises according to neoclassical framework of market economy. The auctioneer could become totally helpless in the face of an unpredictability of market prices and the existence of chaos itself in the market economy.

Chaos is caused by the values of two parameters; adjustment coefficient and preferences. The auctioneer could find the coefficient value which attains price stability and eventually equilibrium. This could be done by harnessing a chaotic movement of prices, as mentioned above, by imposing a price regulation directly or setting a market rule for price changes. These policies of the auctioneer inevitably begin to justify a Keynesian school's idea of utilizing public policies by wise government.

Yet, chaos is triggered by another parameter of consumers' preferences. This time the auctioneer has no direct or indirect control over preferences and tastes of consumers. This means consumers' whimsical preferences have a chance to nullify price adjustment stabilization and drive a stable economy into chaos again.

Accordingly, it has to be concluded that in a pure exchange market economy there is no way to avoid a chaotic price movement and a global instability. We will be all of a sudden thrown into a chaotic world against a neoclassical world of a stable price mechanism. From the simulation results above, it could be even concluded that disequilibrium states are normal in a market economy!. In other words, a stable price adjustment mechanism propounded by neoclassical school is rather exceptional in a market economy that is prevalently chaotic.

No one could deny this conclusion, because it is drawn from a most fundamental exchange model of a market economy. This conclusion forces us to drastically change our vision on a classical doctrine of *invisible hand* that has been believed for more than 200 years since Adam Smith.

Traditional classical and neoclassical doctrine of economics has been constructed on a linear framework of a classical Newtonian mechanics. Modern neoclassical theory of price adjustment mechanism is nothing but an application of such a classical mechanics to economics. Keynes once warned that our economic thoughts are easily enslaved by those of professional economists. It turned out that economists themselves were enslaved by classical physicists.

Modern economic theory has not only failed to provide remedies for overcoming these disequilibria caused by a chaotic market, but also has stubbornly clung, to be worse, to a traditional belief in a globally stable market economy.

Market economic analysis now has to be based on off-equilibrium transactions on historical time. Once economic analysis is freed from the control of *invisible hand*, market disequilibria such as recession and unemployment can be better handled on historical time with system dynamics method.

MuRatopian Economy

After the collapse of the former Soviet Union in 1989, a capitalist market economy has become the only remaining alternative, no matter how violent and chaotic it is. Accordingly, free market principles are enforced globally such as market and financial deregulations, restructuring and re-engineering by business corporations, resulting in recessions and higher unemployment rates. And government tries repeatedly to exercise traditional fiscal and financial policies in vain.

In the book [58], information age is shown to be incompatible with a capitalist market economy and a mixed economy of welfare state. It then poses a necessity of new economic paradigm suitable for the information age. As one such new paradigm, I have proposed an economic system called MuRatopian economy. Interested readers are referred to "Sustainability and MuRatopian Economy" [61, Chapter 5] and "Toward A New Social Design" [58, Chapter 8].

Now that disequilibria on historical time are shown to be normal states in a capitalist market economy, the doctrine of Adam Smith should not be influential anymore in the information age of the 21st century. We need to change the way we think about a market economy. We have to create a new economic system that is beyond a chaotic capitalist market economy and is preferable in the new information age. This will be challenged in the last part of this book.

Before going so far, we have to explore how market economies and macroeconomies running on historical time work.

2.8 Co-Flows of Goods with Money

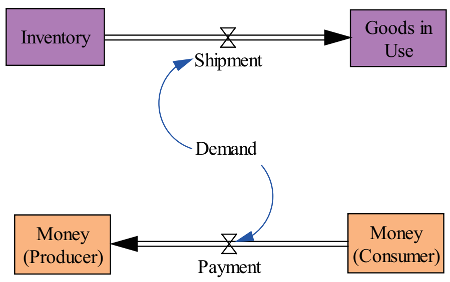

Figure 2.22: Co-flows of Goods and Money

So far, we have focused on the attainment of the equilibrium in a market economy through price adjustment. In a market economy, however, attainment of equilibrium is necessary, but not sufficient to make transactions possible if the economy is not a so-called pure exchange economy, and it is running on historical time.

Whenever transactions are allowed at off-equilibrium prices, money as a medium of exchange has to be introduced. This is what human history tells us, as explored in [70]. In other words, goods and money flows simultaneously as illustrated in Figure 2.22.

Accordingly, we have to explore how to model such co-flow transactions. It will be done in the next chapter by examining accounting system.

Appendix: A Pure Exchange Economy

A Model

A pure exchange model can be represented as constituting n commodities and H consumers who own the initial endowments:

$$\bar{x}^h = (\bar{x}_1^h, \bar{x}_2^h, \dots, \bar{x}_n^h), \quad h \in H. \tag{2.15}$$

Total supply of commodities in the economy is obtained as

$$\bar{x}_i = \sum_{h \in H} \bar{x}_i^h, \quad i = 1, 2, \dots, n. \tag{2.16}$$

Moreover, for a given price vector $p = (p_1, p_2, \dots, p_n)$, a consumer h's notional income is calculated as

$$I_h(p) = p\bar{x}^h = \sum_{i=1}^{n} p_i \bar{x}_i^h, \quad h \in H. \tag{2.17}$$

As a consumer h's preferences, let me assume a following Cobb-Douglas utility function in a logarithmic form where $\alpha^h > 0$:

$$u^h(x^h, \alpha^h) = \sum_{i=1}^{n} \alpha_i^h \log x_i^h. \tag{2.18}$$

It is well known that a utility function thus defined is strongly quasi-concave.

The consumer h is now assumed to seek to maximize $u^h(x^h, \alpha^h)$ subject to his budget constraint $px^h \leq I_h(p)$. Then, by a simple calculation his demand functions are obtained as

$$x_i^h(p) = \hat{\alpha}_i^h \frac{I_h(p)}{p_i}, \quad i = 1, 2, \dots, n, \tag{2.19}$$

$$\text{where } \hat{\alpha}_i^h = \frac{\alpha_i^h}{\sum_{i=1}^n \alpha_i^h} \text{ and } \sum_{i=1}^{n} \hat{\alpha}_i^h = 1. \tag{2.20}$$

These non-linear demand functions are shown to be homogeneous of degree zero in price p.

Total demand for commodities is defined as

$$x_i(p) = \sum_{h \in H} x_i^h(p), \quad i = 1, 2, \dots, n. \tag{2.21}$$

Then, excess demand functions are calculated as

$$\zeta_i(p) = \frac{1}{p_i} \sum_{h \in H} \hat{\alpha}_i^h I_h(p) - \bar{x}_i, \quad i = 1, 2, \dots, n. \tag{2.22}$$

An equilibrium of the economy is defined to be a situation in which all markets clear for some price p^*, that is,

$$\zeta(p^*) = \{\zeta_1(p^*), \zeta_2(p^*), \ldots, \zeta_n(p^*)\} = 0. \tag{2.23}$$

The existence of such an equilibrium price is reduced to find a solution in the following linear system:

$$\begin{bmatrix} a_{11} - \bar{x}_1 & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} - \bar{x}_2 & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} - \bar{x}_n \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{2.24}$$

where $a_{ij} = \sum_{h \in H} \hat{\alpha}_i^h \bar{x}_j^h$.

It is shown in [31, p.100] that there exists a non-trivial positive equilibrium price $p^* \gg 0$. The existence of equilibria in an exchange economy is more generally shown by Smale[45]. Such an equilibrium price is known to be unique up to n-1 prices. This can be easily confirmed by the fact that the column sums of the above matrix are zero, or from the Walras' law: $p\zeta(p) \equiv 0$. That is to say, only relative prices are determined in the equilibrium.

Two commodities and two consumers

Let us simplify the exchange economy as consisting of two commodities and two consumers. In this simplified economy, excess demand functions are calculated as

$$\zeta_1(p) = \hat{\alpha}_1^1 \frac{I_1(p)}{p_1} + \hat{\alpha}_1^2 \frac{I_2(p)}{p_1} - \bar{x}_1, \tag{2.25}$$

$$\zeta_2(p) = \hat{\alpha}_2^1 \frac{I_1(p)}{p_2} + \hat{\alpha}_2^2 \frac{I_2(p)}{p_2} - \bar{x}_2. \tag{2.26}$$

These excess demand functions are obviously homogeneous of degree zero, and Walras' law in this economy is shown to be

$$p_1\zeta_1(p) + p_2\zeta_2(p) \equiv 0. \tag{2.27}$$

Therefore, a relative equilibrium price $p^* = (p_1^*, p_2^*)$ satisfying $\zeta_i(p^*) = 0,\ i = 1, 2$, is calculated as follows.

$$\frac{p_1^*}{p_2^*}_{|\zeta_1=0} = \frac{\hat{\alpha}_1^1\bar{x}_2^1 + \hat{\alpha}_1^2\bar{x}_2^2}{(1-\hat{\alpha}_1^1)\bar{x}_1^1 + (1-\hat{\alpha}_1^2)\bar{x}_1^2}. \tag{2.28}$$

$$\frac{p_1^*}{p_2^*}_{|\zeta_2=0} = \frac{(1-\hat{\alpha}_2^1)\bar{x}_2^1 + (1-\hat{\alpha}_2^2)\bar{x}_2^2}{\hat{\alpha}_2^1\bar{x}_1^1 + \hat{\alpha}_2^2\bar{x}_1^2}. \tag{2.29}$$

From a relation: $1 - \hat{\alpha}_1^i = \hat{\alpha}_2^i,\ i = 1, 2$, it can be shown that these two relative equilibrium prices are equal, that is,

$$\frac{p_1^*}{p_2^*}_{|\zeta_1=0} = \frac{p_1^*}{p_2^*}_{|\zeta_2=0} \tag{2.30}$$

In sum, it is demonstrated that in this simplified economy an equilibrium price exists, and only a relative price is determined, as expected from the analysis of the general model above.

Constructing Tâtonnement Processes

How can we attain an equilibrium price when it cannot be directly computed? In such a case, a tâtonnement price adjustment process is the only available method to determine an equilibrium price or even estimate it. A standard adjustment process that is often used in the literature is the following in which an adjustment coefficient λ is given exogenously.

$$p_i(t+1) = \text{Max}\ \{p_i(t) + \lambda\zeta_i(p(t)), 0\}, \quad i = 1, 2. \tag{2.31}$$

As an alternative tâtonnement adjustment (a), the following process is also employed:

$$p_i(t+1) = p_i(t) + \lambda \text{Max}\ \{\zeta_i(p(t)), 0\}, \quad i = 1, 2. \tag{2.32}$$

When prices are bounded by some minimum and maximum values, the following minmax tâtonnement adjustment (m) is occasionally applied:

$$p_i(t+1) = \text{Min}\ \{\bar{p}_i, \text{Max}\ \{p_i(t) + \lambda\zeta_i(p(t)), \underline{p}_i\}\}, \quad i = 1, 2. \tag{2.33}$$

This process is a generalization of the above standard tâtonnement adjustment process whose maximum price is assumed to be infinite.

In these processes the adjustment coefficient λ has to be arbitrarily chosen by an auctioneer. To avoid this arbitrariness, let us construct another processes in which an adjustment coefficient λ is determined by a relative weight of prices at the iteration period t such that

$$\lambda_i(t) = \frac{p_i(t)}{\sum_{i=1}^{2} p_i(t)}, \quad i = 1, 2, \tag{2.34}$$

and call these revised coefficients composite coefficients. Thus, these composite coefficients are applied to the above three adjustment processes respectively as follows.

Standard composite tâtonnement adjustment (c)

$$p_i(t+1) = \text{Max}\ \{p_i(t) + \lambda_i(t)\zeta_i(p(t)), 0\}, \quad i = 1, 2. \tag{2.35}$$

Alternative composite tâtonnement adjustment (ac)

$$p_i(t+1) = p_i(t) + \lambda_i(t)\text{Max}\ \{\zeta_i(p(t)), 0\}, \quad i = 1, 2. \tag{2.36}$$

Minmax composite tâtonnement adjustment (mc)

$$p_i(t+1) = \text{Min}\ \{\bar{p}_i, \text{Max}\ \{p_i(t) + \lambda_i(t)\zeta_i(p(t)), \underline{p}_i\}\}, \quad i = 1, 2. \tag{2.37}$$

In this way six different tâtonnement price adjustment processes can be constructed.

Global Stability

Can any arbitrarily-chosen initial price attain an equilibrium in an exchange economy? If it can, the economy is called globally stable. Arrow, Block and Hurwicz [1] proved such a global stability under the assumptions of Walras' law, homogeneity of excess demand function and gross substitutability. Since then it has been generally adopted in the literature on microeconomics and mathematical economics, for instance [49, pp.321-329]. Walras' law and homogeneity are already shown to hold in the exchange economy. It is also shown here that for $(p_1,\ p_2) > 0$ a gross substitutability holds in the simplified economy as follows.

$$\frac{\partial \zeta_1(p)}{\partial p_2} = \frac{1}{p_1}(\hat{\alpha}_1^1 \bar{x}_2^1 + \hat{\alpha}_1^2 \bar{x}_2^2) > 0. \tag{2.38}$$

$$\frac{\partial \zeta_2(p)}{\partial p_1} = \frac{1}{p_2}(\hat{\alpha}_2^1 \bar{x}_1^1 + \hat{\alpha}_2^2 \bar{x}_1^2) > 0. \tag{2.39}$$

Equilibrium prices are attained under a condition that *an adjustment coefficient λ is fixed at its original default value.* Hence, the simplified exchange economy turned out to be globally stable.

Chapter 3

Accounting System Dynamics

Understanding financial statements is imperative for better management of corporations, while system dynamics (SD) offers dynamic modeling and simulation skills for better strategies of management. This chapter[1] tries to present a consolidated principle of accounting system dynamics on the basis of simple principles from SD and accounting system. It is, then, specifically applied to model corporate financial statements (income statement, balance sheet and cash flow statement) described in the book [34]. It is shown that cash flow statement is indispensable for modeling financial statements. At the same time, a limitation of the current accounting system as a dynamic guidance for management strategies is pointed out. This demonstrates the importance of SD modeling in the field of accounting system.

3.1 Introduction

Business accounting system consists of three financial statements such as income statement, balance sheet and cash flow statement. Success or failure of corporations has been measured by these financial statements. In this sense, accounting system has been and will be a foundation for our business activities, on which macroeconomic activities are further built.

Accounting system is recently undergoing radical reforms in Japan in order to catch up with its global defacto standard of the American accounting system. The so-called Japanese version of financial Big Bang began to be implemented in March 2000. One of its major reforms is a legal requirement of cash flow statement which had been neglected in the Japanese accounting system until recently. Since then many introductory accounting books focusing on cash flows

[1] This chapter is based on the paper: Principle of Accounting System Dynamics – Modeling Corporate Financial Statements – in "Proceedings of the 21st International Conference of the System Dynamics Society", New York City, USA, July 20-24, 2003, ISBN 0-9672914-9-6.

have been lined up in many bookstores, attracting attention to many business people in Japan.

Under such circumstances, recent financial scandals such as Enron and WorldCom were a surprise to most Japanese who have been trying to introduce the American accounting system as the most trustworthy system. What went wrong with them? One of the reflecting arguments was that the practice of the current accounting system is heavily dependent on professional accountants and specialized accounting software. If current accounting system were more friendly to managers and employees, then abnormal behaviors of financial practices such as mentioned above would have been avoided at its earlier stage, I thought.

It occurred to me then that SD approach to the accounting system could make it more friendly. Furthermore, it would be more practical, I thought, if corporate SD models could incorporate financial statements directly or indirectly, since model performances are better evaluated in terms of financial statements as done in the real world of business.

With these beliefs in mind, I began to search for references on a system dynamics method of modeling corporate financial statements. My search has been unsuccessful except the book [38] which was by chance suggested in the discussions among SD mailing community. It took more than a year to obtain the book through the Amazon on-line search for used books. It turned out, however, that the book was written with DYNAMO, and accordingly has been left unnoticed in my bookshelf.

Failure of the search gave me an incentive to develop a SD method of modeling financial statements from a scratch. I started working in the summer of 2001 when I was spending relatively a quiet time on a daily rehabilitation exercise in order to recover from the physical operation on my shoulder in June of the same year. This environment gave me a good chance for reading books on accounting. My readings mainly consisted of the introductory books such as [27], [34], [35], [51], [52], since my knowledge of accounting was limited[2]. Through such readings, I have been convinced that system dynamics approach is very effective for understanding the accounting system.

The purpose of this chapter is, therefore, to understand the accounting system in terms of system dynamics. A consolidated principle of accounting system dynamics will be constructed for this purpose. It is then applied to model corporate financial statements exemplified in [34]. In the due course, it will be shown how cash flow statement plays an indispensable role in modeling corporate financial systems, contrary to the practice that it has not been required in the Japanese financial statements. I wondered why such an essential cash flow statement has been neglected until recently in Japan. System dynamics approach indeed sheds light on the wholeness of the current accounting system.

On the other hand, SD business models seem to have also neglected the importance of incorporating financial statements for better evaluation of model performances. Business models without such financial statements, whether they

[2] In addition to these books, a paper dealing with corporate financial statements [2] is recently published. However, current research for modeling financial statements is independently carried out here with a heuristic objective in mind.

are explicitly or implicitly built in them, would be indeed incomplete, because they fail to reflect the wholeness of dynamic business activities. In this sense, a corporate financial model that will be suggested at the end of this chapter would provide a kind of pecuniary archetype for corporate financial modeling.

3.2 Principles of System Dynamics

System is a self-functioning whole consisting of interdependent parts that are interacting with one another with some influence from its outside world. Examples of systems are abundant such as our bodies, communities, corporations, and public organizations as well as subsystems within these systems. System dynamics is a discipline that tries to describe dynamic movements of these systems. For the understanding of financial accounting system, which is a main purpose of this chapter, it would be enough to consider the following three principles of system dynamics.

Principle 1 (System as a collection of stocks) System can be described by a collection of state variables, called *stocks* in system dynamics, whose levels or volumes are measured at a *moment in time*.

In other words, state variables (stocks) of the system are the entity that can be pictured or recorded for its description.

Principle 2 (Stock-flow relation) Levels of a stock can only be changed by the amount of *flows* measured for a *period of time*. The amount of flow that increases the stock is called inflow, while the one that decreases it is called outflow.

In this way, stock and flow constitute an inseparable relational unit in system dynamics [62]. Stock-flow relation is illustrated in Figure 3.1.

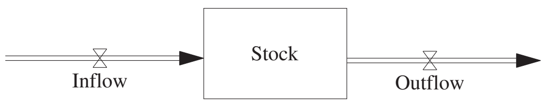

Figure 3.1: Stock-Flow Relation

Principle 3 (Information feedback) The amount of inflows and outflows is directly or indirectly determined either by the information obtained from the stocks through their feedback loops, or parameters obtained outside the system.

As will be clarified below, modeling dynamic accounting system mostly depends on the parameters of transaction data obtained outside the system.

3.3 Principles of Accounting System

Accounting system of modern corporations consists of three financial statements such as balance sheet, income statement and cash flow statement. Examples of these statements used in this chapter are replicated from the book [34].

Balance Sheet

A	Cash
B	Accounts Receivable
C	Inventories
D	Prepaid Expenses
A + B + C + D = E	Current Assets
F	Other Assets
G	Fixed Assets @ Cost
H	Accumulated Depreciation
G - H = I	Net Fixed Assets
E + F + I = J	Total Assets
K	Accounts Payable
L	Accrued Expenses
M	Current Portion of Debt
N	Income Taxes Payable
K + L + M + N = O	Current Liabilities
P	Long-Term Debt
Q	Capital Stock
R	Retained Earnings
Q + R = S	Shareholders' Equity
O + P + S = T	Total Liabilities & Equity

Table 3.1: Balance Sheet in [34]

How are these three statements related one another, then? Their relationships are best described as follows:

> The balance sheet reports the aggregate effect of transactions at a **point in time**, whereas the income statement, statement of retained earnings, and statement of cash flows report the effect of transactions over a **period of time**. [5, page 35].

The relationship of three financial statements thus can be best understood in terms of the above stock-flow relation of system dynamics as follows:

Principle 4 (Stock-flow relation of financial statements) Balance sheet is a collection of stocks only, while income statement and cash flow statement consist of inflows and outflows of the stocks in balance sheet.

Balance sheet in Table 3.1 is now best illustrated as a collection of stocks as in Figure 3.2. One remark may be needed on Net Fixed Assets. It is defined in

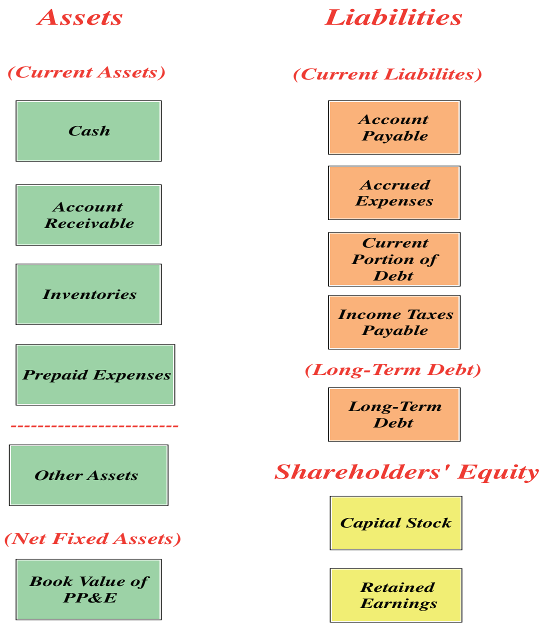

Figure 3.2: Balance Sheet in [34] as a Collection of Stocks

Table 3.1 as Fixed Assets @ Cost less Accumulated Depreciation. In Figure 3.2, it is renamed as Book Value of PP&E (Property, Plant and Equipment) and illustrated as the only stock for the net fixed assets. This is because, with the introduction of stock-flow relation, net fixed assets can be better represented as a book value relation as illustrated in Figure 3.3 below.

There are 13 stocks in the balance sheet of Figure 3.2. From the Principle 2, they all need to be illustrated together with inflows and outflows. However, from the Principle 4, only inflows and outflows of Retained Earnings and Cash can be illustrated from the figures in Income and Cash Flow Statements. Specifically, inflows and outflows of Retained Earnings are obtained from the Income Statement in Table 3.2. That is, its inflow is revenues or net sales, while its outflows consist of costs of goods sold, operating expenses, net interest income, and income taxes. These stock-flow relations are illustrated in Figure 3.4 below.

On the other hand, inflows and outflows of Cash could also be illustrated from Cash Flow Statement in Table 3.3. Its inflow is basically cash receipts and its outflow is cash disbursements. Cash flows, however, are better classified in detail into three activities; that is, operating activities, investing activities and financing activities, and accordingly stock-flow relations of Cash are usually described with additional inflows and outflows. They will be thus illustrated in Figure 3.14 after cash-related transactions are examined in Section 3.5.

To illustrate stock-flow relations of the remaining 11 stocks, we need to add

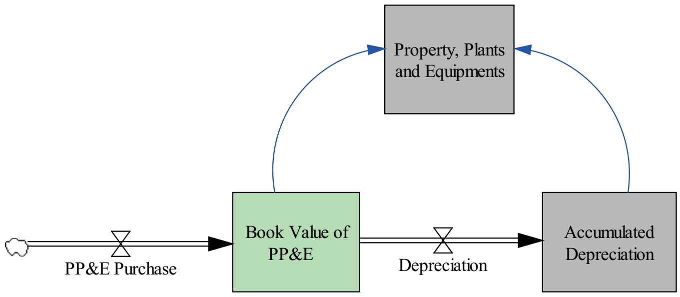

Figure 3.3: Net Fixed Assets (Book Value) Relation

Income Statement

1	Net Sales
2	Cost of Goods Sold
1 -2 = 3	Gross Margin
4	Sales & Marketing
5	Research & Development
6	General & Administrative
4+5+6=7	Operating Expenses
3-7=8	Income From Operations
9	Net Interest Income
10	Income Taxes
8+9-10=11	Net Income

Table 3.2: Income Statement in [34]

inflows and outflows to them by newly defining their names. A generic naming rule is employed here to define them as long as no other appropriate names are found in the existing accounting system. For instance, inflow and outflow of Accounts Payable are named *Accounts Payable Incurred* and *Accounts Payable Paid*. In this way, stock-flow relations of all stocks in the balance sheet are constructed.

How can the levels of these 13 stocks in the balance sheet be changed, then, by the changes in inflows and outflows? In the accounting system, they are changed by a so-called bookkeeping rule of *double entry*. Accounting system has a long history of more than several hundred years, and become a well-established and complete system. Its success has been attained by the introduction of this *double entry* principle. The double entry rule, however, has also been a major source of confusions for the students of accounting.

With the introduction of stock-flow relation, the double entry principle is now very intuitively illustrated as in Figure 3.5, in which all stocks in the Balance Sheet are collectively described as Assets and Liabilities, while Shareholders' Equity is described with its original stock names of Capital Stock and Retained Earnings. All inflows to Assets and all outflows from Liabilities and Equity are

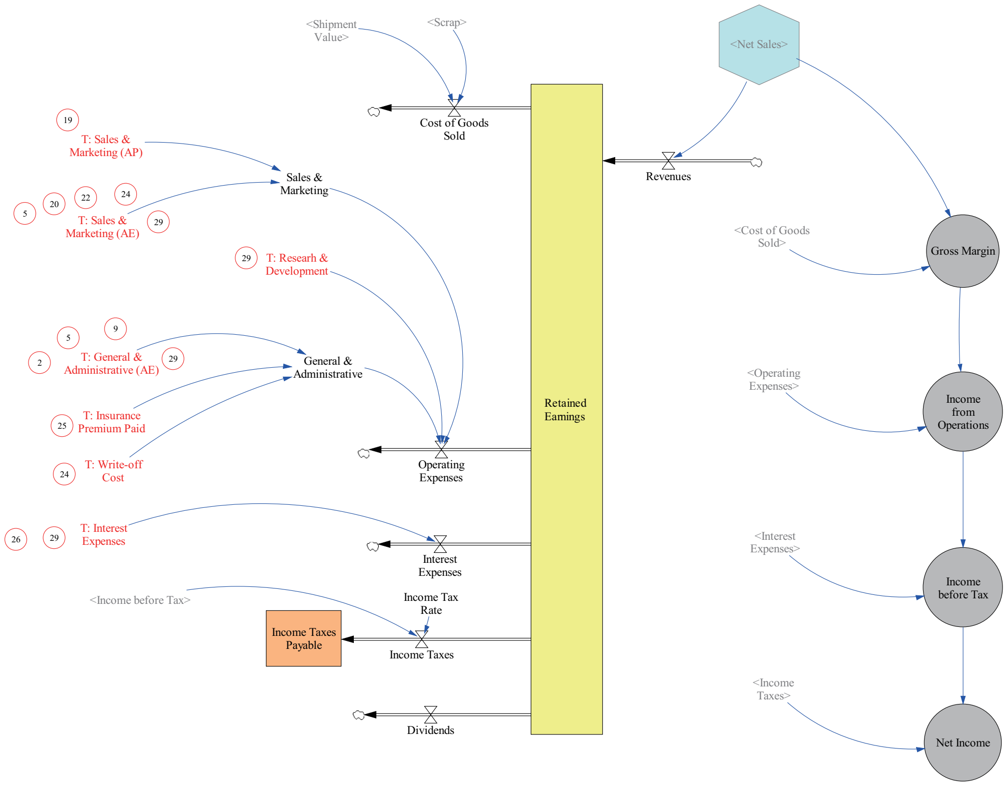

Figure 3.4: Income Statement as Stock-Flow Relation

booked on the left side of debit, while all outflows from Assets and all inflows to Liabilities and Equity are booked on the right side of credit. That is to say, each transaction has to be booked simultaneously on both sides of debit and credit to keep the balance sheet in balance – a very simple rule!. It is formally summarized as follows:

Principle 5 (Double entry rule of bookkeeping) All transactions in the accounting system are recorded as inflows and/or outflows of stocks in the balance sheet so that each transaction causes two corresponding stocks to change simultaneously in balance. For this purpose, each transaction is booked twice on both debit and credit sides. Inflows of assets and outflows of liabilities and shareholders' equity are booked on the debit side, while outflows of assets and inflows of liabilities and shareholders' equity are booked on the credit side.

Cash Flow Statement

a	Beginning Cash Balance
b	Cash Receipts
c	Cash Disbursements
b-c=d	Cash Flow From Operations
e	PP&E Purchase
f	Net Borrowings
g	Income Taxes Paid
h	Sale of Capital Stock
a+d-e+f-g+h=i	Ending Cash Balance

Table 3.3: Cash Flow Statement in [34]

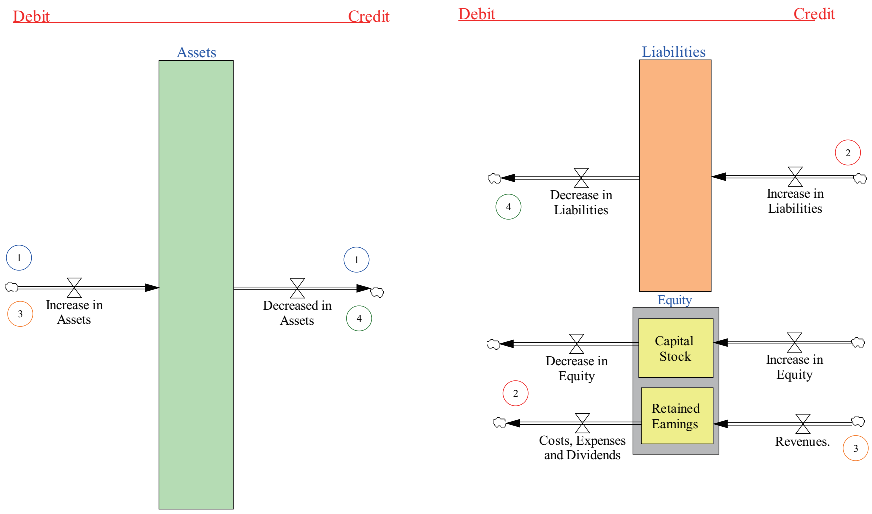

Figure 3.5: Double Entry Rule of Bookkeeping as Debit and Credit

3.4 Principles of Accounting System Dynamics

We have now obtained five principles from system dynamics and accounting system. Let us call them collectively Principle of Accounting System Dynamics (PASD).

Principle of Accounting System Dynamics Principles 1 through 5 obtained from system dynamics and accounting system constitutes the Principle of Accounting System Dynamics.

From the principle, four major categories of bookkeeping practices are easily classified as follows.

(1) Debit:inflow ↔ Credit:outflow Transactions within assets are classified in this category. For example, an increase in Fixed Assets by the purchase of PP&E is balanced by the decrease in Cash by its payment.

Figure 3.6 illustrates the example of bookkeeping (1). Right-hand diagram is the combined way to describe the left-hand diagram.

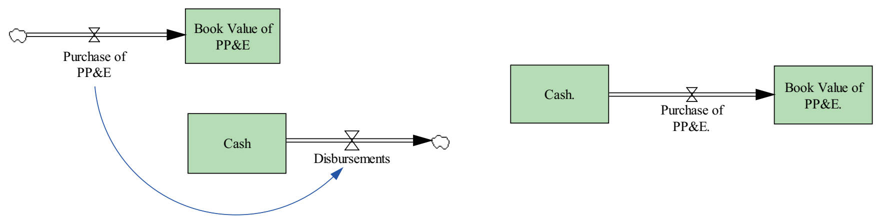

Figure 3.6: Double Entry Rule of Bookkeeping (1)

(2) Debit:outflow ↔ Credit:inflow Transactions within liabilities and equity are classified here. For example, a decrease in Retained Earnings caused by an increase in operating expenses such as sales & marketing expenses is balanced by the increase in Accrued Expenses.

Figure 3.7 illustrates the example of bookkeeping (2).

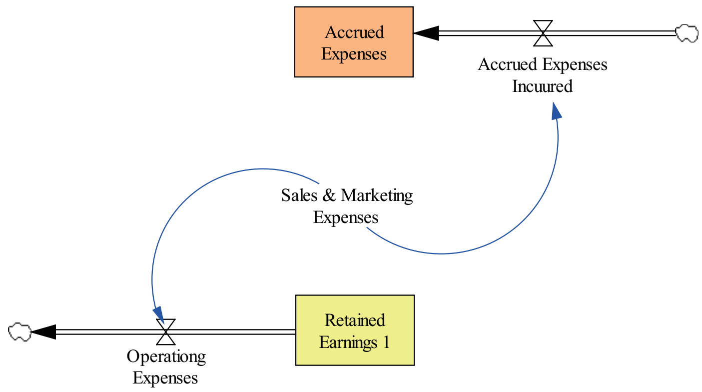

Figure 3.7: Double Entry Rule of Bookkeeping (2)

(3) Debit:inflow ↔ Credit:inflow Transactions in this category cause both Assets and Liabilities/Equity to increase. For instance, an increase in net sales causes both Accounts Receivable and Retained Earnings to increase.

Figure 3.8 illustrates the example of bookkeeping (3).

(4) Debit:outflow ↔ Credit:outflow Transactions here cause both Assets and Liabilities/Equity to decrease. For instance, payment of Accounts Payable causes both Cash and Accounts Payable to decrease.

Figure 3.9 illustrates the example of bookkeeping (4).

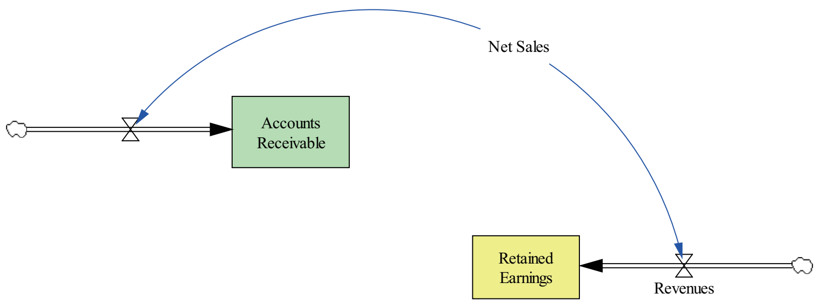

Figure 3.8: Double Entry Rule of Bookkeeping (3)

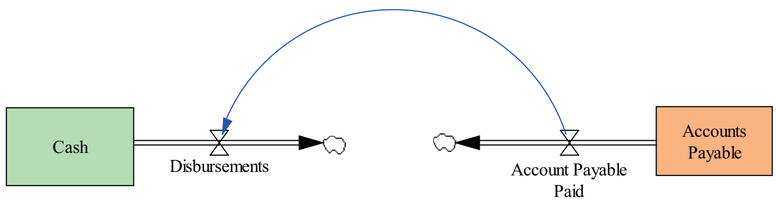

Figure 3.9: Double Entry Rule of Bookkeeping (4))

3.5 Accounting System Dynamics in Action

On the basis of the PASD, we are now in a position to model corporate financial statements by identifying all corresponding flows and stocks that are affected by transactions. Examples of transactions in the book [34] are used for this purpose. That is, all transactions below are quoted from the book, so this section can be better followed with the book and our accounting SD model at hand simultaneously.

According to PASD, all transactions have to be booked on both debit and credit sides simultaneously. This bookkeeping rule is formally described here as follows:

⇒ Transaction
  ⇒ [Stock 1 ±] ↔ [Stock 2 ±]

Stock 1 is a primary stock that is changed by the inflow or outflow of a transaction, and Stock 2 is its corresponding stock to be changed to keep the balance sheet in balance. For example, if an item in the accrued expenses is paid in cash, this transaction decreases both Cash and Accrued Expenses, and it is described as follows:

⇒ T: Accrued Expenses Paid
  ⇒ [Cash -] ↔ [Accrued Expenses -]

This formula implies that payment of accrued expenses lowers both the levels of cash and accrued expenses. Such an identifications of a primary stock affected by the transaction and its corresponding stock is essential for modeling financial statements.

In the book [34], 31 transactions are explained to describe the start-up business activities of AppleSeedd Enterprises, Inc. In our modeling here each transaction is assumed to be taken in a week, so that 31 transactions are done in 31 weeks. There are 28 transaction items, starting with suffix T:, that are used as model parameters.

Transaction 1 A group of investors is willing to exchange their \$1.5 million in cash for stock certificates representing 150,000 common shares of AppleSeed Enterprises, Inc.
Note: When you formed the company you bought 50,000 shares of "founder's stock" at \$1 per share for a total investment of \$50,000 in cash. Thus after this sale to the investor group there will be 200,000 shares outstanding. They will own 75% of AppleSeed and you will own the rest.

⇒ T:New Issue of Shares (= 150,000 common shares)
⇒ [Capital Stock +] ↔ [Cash +]

Transaction 2 Book all payroll-associated company expenses totaling \$6,230 including salary, employer's contribution to FICA (Social Security) and various insurance expenses. Issue yourself a payroll check for \$3,370 (your \$5,000 monthly salary minus \$1,250 in federal and state withholding tax and \$380 for your own contribution to FICA).

⇒ T: General & Administrative (= \$6,230)
⇒ [Accrued Expenses +] ↔ [Retained Earnings - : Operating Expenses]

⇒ T: Accrued Expenses Paid (= \$3,370)
⇒ [Accrued Expenses -] ↔ [Cash -]

Transaction 3 Borrow \$1 million to purchase an all-purpose building. This term note will run for 10 years, calling for yearly principal payments of \$100,000 plus interest at a rate of 10% per annum.

⇒ T: Long-Term Borrowing (= \$1 million)
⇒ [Long-Term Debt +] ↔ [Cash +]

⇒ T: Principal Payments (= \$100,000)
⇒ [Long-Term Debt -] ↔ [Current Portion of Debt +]

Transaction 4 Purchase 100,000 square foot building and land for \$1.5 million in cash. This facility will serve as AppleSeed Enterprises' headquarters, manufacturing facility and warehouse.

⇒ T: Property, Plant & Equipment Purchase (= $ 1.5 million)
⇒ [Book Value of PP & E +] ↔ [Cash -]

Transaction 5 Book this month's payroll-associated expenses of $14,790, (that is, $7,680 for Sales & Marketing and $7,110 for General & Administrative). These expenses include salaries, wages, insurance and other fringe benefits. Issue payroll checks totaling $7,960 to SG & A (sales, general and administrative) employees.

⇒ T: Sales and Marketing (AE)) (= $ 7,680)
⇒ [Retained Earnings -] ↔ [Accrued Expenses +]

⇒ T: General & Administrative (= $ 7,110)
⇒ [Retained Earnings -] ↔ [Accrued Expenses +]

⇒ T: Accrued Expenses Paid ((= $7,960)
⇒ [Accrued Expenses -] ↔ [Cash -]

Transaction 6 Pay all the payroll-associated expenses that were accrued in Transaction 2 and Transaction 5, including FICA, withholding tax and unemployment insurance due the government. Also pay to private insurance companies the workmen's compensation and health and life insurance premiums.

⇒ T: Accrued Expenses Paid (= $9,690)
⇒ [Accrued Expenses -] ↔ [Cash -]

Transaction 7 Place an order for $250,000 worth of applesauce-making machinery. Make a prepayment of $125,000 with the balance due upon successful installation.

⇒ T: Other Assets Purchase (= $125,000)
⇒ [Other Assets +] ↔ [Cash -]

Transaction 8 Make final payment of $125,000, the balance due on the applesauce-making machinery.

⇒ T: Other Assets Purchase (= $125,000)
⇒ [Other Assets +] ↔ [Cash -]

After the completion of payment and the delivery of machinery, it is now recorded as PP&E. It may be written in our transaction format as follows.

⇒ Installation (= $250,000)
⇒ [Book Value of PP&E +] ↔ [Other Assets -]

Transaction 9 Book supervisor's salary and associated payroll expenses as a General & Administrative expense since we have not yet started production. Issue first month's salary check. Make no entries for hourly workers since they have not yet reported for work.

⇒ T: General & Administrative (= $4,880)
⇒ [Retained Earnings -] ↔ [Accrued Expenses +]

⇒ T: Accrued Expenses Paid (= $2,720)
⇒ [Accrued Expenses -] ↔ [Cash -]

Transaction 10 Order and receive 1 million applesauce jar labels at a cost of $0.02 each for a total of $20,000 to be paid 30 days after delivery.

⇒ T: Raw Material Purchase (= $20,000)
⇒ [Raw Material Inventory +] ↔ [Accounts Payable+]

From this transaction on, production activities are booked under the stock account of inventory, which is, accordingly, separated from the balance sheep of assets here, though it is still its part. Moreover, inventories are further broken down as illustrated in Figure 3.10.

Transaction 11 Receive a two months' supply of all raw materials (apples, sugar, cinnamon, jars, caps, boxes) worth $332,400 in total. (That is, $8.55 total materials per case less $0.24 for the already received labels times 40,000 cases.)

⇒ T: Raw Material Purchase (= $332,400)
⇒ [Raw Material Inventory +] ↔ [Accounts Payable+]

Transaction 12 Pay production workers' wages and supervisor's salary for the month. Book associated fringe benefits and payroll taxes. (Now that we are manufacturing product, these salary and wages are costs that increase the value of our product, and are shown as an increase in inventory.)

⇒ T(Cash-): Wages (= $9,020)
⇒ [Work in Process Inventory +] ↔ [Cash -]

⇒ T: Payroll-associated Fringes and Taxes (= $8,160)
⇒ [Work in Process Inventory +] ↔ [Accrued Expenses +]

Transactions	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11	T12	T13	T14	T15	T16	T17	T18	T19	T20	T21	T22	T23	T24	T25	T26	T27	T28	T29	T30	T31
Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
T: Account Receivable Paid																							234,900							1,404,000	
T: Long-Term Borrowing			1,000,000																												
T: New Issue of Shares	150,000																														
T: Wages												9,020						9,020													
T: PP&E Purchase				1,500,000																											
T: Other Assets Purchase							125,000	125,000																							
T: Account Payable Paid														20,000			150,000											150,000	82,907		
T: Accrued Expenses Paid		3,370			7,960	9,690			2,720														4,698				18,480				
T: Current Debt Paid																										25,000					
T: Income Taxes Paid																															
T: Insurance Premium																									26,000						
T: Interest Expenses																										25,000			75,000		
T: Write-off Cost																								15,900							
T: Scrapped Cases																500		150													
T: Insurance Premium Paid																									6,500						
T: Raw Material Purchase										20,000	332,400							166,200													
T: All Other Overhead													8,677					8,677													
T: Sales & Marketing (AP)																			103,250												
T: Researh & Development																													26,000		
T: Payroll-associated fringes and taxes												8,160						8,160											26,435		
T: Sales & Marketing (AE)					7,680															318		4,698		-318					212,895		
T: General & Administrative (AE)		6,230			7,110				4,880																				162,900		
T: Production												20,000						20,000											20,000	20,000	20,000
T: Completion															19,500			19,000											19,500	19,500	19,500
T: Depreciation													7,143					7,143											64,287		
T: Principal Payments			100,000																							25,000			75,000	75,000	
T: Par Share Dividend																															0.375
T: Customer Order																				1,000		15,000							176,400		
Data: Price Change																					-0.24							0.24			

Table 3.4: All Transaction Data

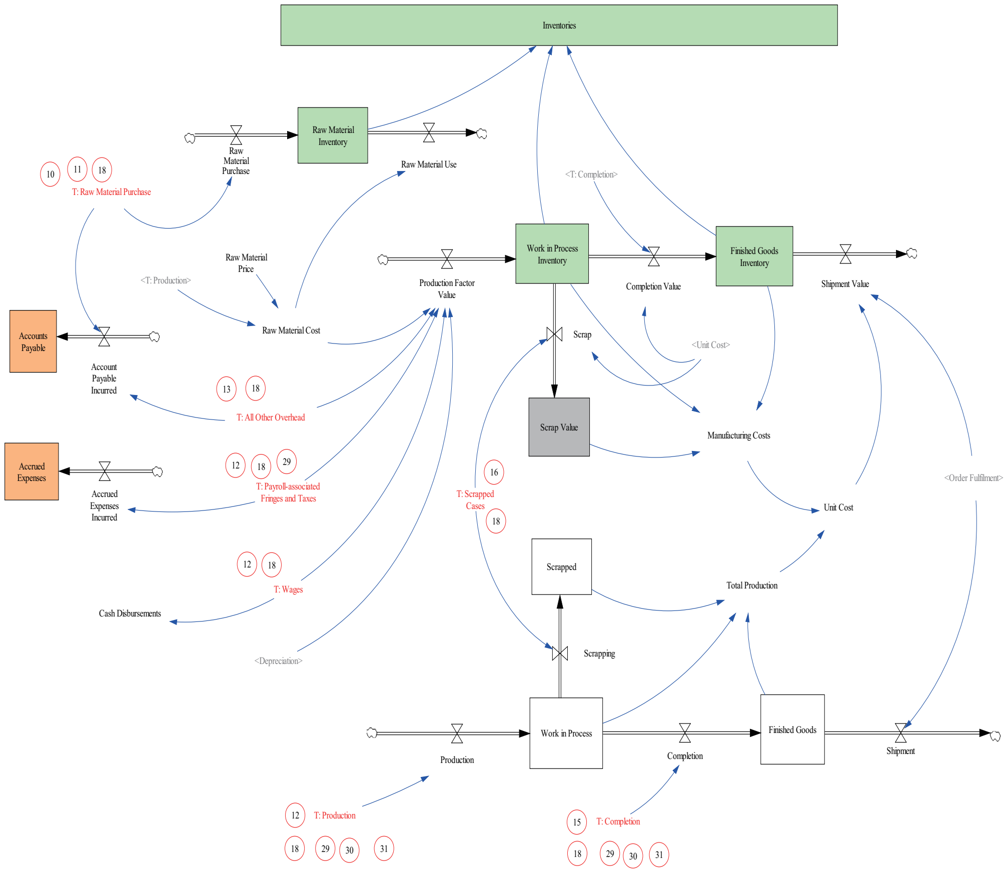

Figure 3.10: Inventories with Production

As production starts, raw material use may be written as follows.

⇒ T: Production (= 20,000 cases)
(⇒ Raw Material Use (= \$171,000 = 20,000 * \$8.55))
⇒ [Raw Material Inventory -] ↔ [Work in Process Inventory +]

Transaction 13 Book this month's manufacturing depreciation of \$7,143 and \$8,677 covering "all other" overhead costs. Note that depreciation is not a cash expense and will not lower our cash balance. But, the "all other" overhead we will eventually have to pay with cash.

⇒ T: All Other Overhead (= \$8,677)
⇒ [Work in Process Inventories +] ↔ [Account Payable]

⇒ T: Depreciation (= \$7,143)

⇒ [Work in Process Inventories +] ↔ [Book Value of PP&E -]

Transaction 14 Pay for 1 million labels received in Transaction 10. Issue a check to our vendor for $20,000 as payment in full.

⇒ T(Cash-): Accounts Payable Paid (= $20,000)
⇒ [Accounts Payable -] ↔ [Cash -]

Transaction 15 Finish production of 19,500 cases of our applesauce. Move product from work-in-process ("WIP") Inventory into Finished Goods. This movement of inventory into a different class is really just an internal management control transaction as far as the financial statements are concerned. There is no effect on the three major financial statements of AppleSeed. INVENTORIES on the *Balance Sheet* remains the same. Our Inventory Valuation Worksheet, as shown below, reflects the change in inventory status.
This may be written as follows.

⇒ T: Completion (= 19,500 cases)
(⇒ Completion Value (= 19,500 * 10.2 dollar = $198,900))
⇒ [Work in Process Inventory -] ↔ [Finished Goods Inventory +]

Transaction 16 Scrap the value of 500 cases of applesauce from the work-in-process inventory. Take a loss on the *Income Statement* for this amount.

⇒ T: Scrapped Cases (= 500 cases)
⇒ [Work in Process Inventory -] ↔ [Retained Earnings -: Cost of Goods Sold]

Transaction 17 Pay a major supplier a portion of what is due for apples and jars. Cut a check for $150,000 in partial payment.

⇒ T: Accounts Payable Paid (= $150,000)
⇒ [Account Payable -] ↔ [Cash +]

Transaction 18 Make entries in the *Income Statement, Cash Flow Statement* and *Balance Sheet* as shown in the total column at below right. Note that for each worksheet entry (K through Q below), the change in Assets equals the change in Liabilities.

⇒ T: Raw Material Purchase (= $166,200)
⇒ [Raw Material Inventory +] ↔ [Accounts Payable +]

⇒ T(Cash-): Wages (= $9,020)
⇒ [Work in Process Inventory +] ↔ [Cash -]

K. Receive a month's raw material supply less labels. (see T10)	\$166,200
L. Move a month's supply of raw materials into WIP. (see T12).	\$171,000
M. Pay hourly workers/supervisor for another month. (see T12)	\$17,180
N. Book manufacturing depreciation for the month. (see T13)	\$7,143
O. Book "all other" mfg. overhead for another month. (see T13)	\$8,677
P. Move 19,000 cases to finished goods standard cost.	\$193,800
Q. Scrap 150 cases from WIP. (see T16)	\$1,530

Table 3.5: Inventory Valuation Worksheet for Transaction 18

⇒ T: Payroll-associated Fringes and Taxes (= \$8,160)
⇒ [Work in Process Inventory +] ↔ [Accrued Expenses +]

⇒ T: Depreciation (= \$7,143)
⇒ [Work in Process Inventory +] ↔ [Book Value of PP&E -]

⇒ T: All Other Overhead (= \$8,677)
⇒ [Work in Process Inventory +] ↔ [Account Payable +]

⇒ T: Scrapped Cases (= 150 cases)
⇒ [Work in Process Inventory -] ↔ [Retained Earnings -: Cost of Goods Sold]

In addition, raw material use and completion of work in process may be written as follows.

⇒ T: Production (= 20,000 cases)
(⇒ Raw Material Use (= \$171,000 = 20,000 * \$8.55))
⇒ [Raw Material Inventory -] ↔ [Work in Process Inventory +]

⇒ T: Completion (= \$19,00)
⇒ [Work in Process Inventory -] ↔ [Finished Goods Inventory +]

⇒ T: Completion (= 19,000 cases)
(⇒ Completion Value (= 19,000 * 10.2 dollar = \$193,800))
⇒ [Work in Process Inventory -] ↔ [Finished Goods Inventory +]

Transaction 19 Our advertising agency submits a bill for designing, printing and mailing 4,500 very fancy brochures for a \$38,250 total cost.The T-shirts cost \$6.50 each for a total of \$65,000 for 10,000 shirts. Book these amounts (totaling \$103,250) as an AppleSeed Enterprises marketing and selling expense.

⇒ T: Sales & Marketing (= $103,250)
⇒ [Account Payable +] ↔ [Retained Earnings -: Operating Expenses]

Transaction 20 Receive order for 1,000 cases of applesauce at a selling price of $15.90 per case. Ship product and send a $15,900 invoice to the customer. Book on the *Income Statement* the 2% commission ($318) for our broker as a SALES & MARKETING expense.

⇒ T: Customer Order (= 1,000 cases)
⇒ [Retained Earnings +] ↔ [Accounts Receivable +]

⇒ Shipment Value (= 1,000 cases * $ 10.2 per case = $10,200)
⇒ [Finished Goods Inventory -] ↔ [Retained Earnings -: Costs of Goods Sold]

This transaction of sales invokes two different changes in stocks as illustrated in Figure 3.11. First, sales value increases both retained earnings and account receivable. Second, shipping value obtained by the unit cost has to be subtracted from finished goods inventory and simultaneously booked as costs of goods sold.

⇒ T: Sales & Marketing (= $318) ⇒ [Retained Earnings -] ↔ [Accrued Expenses +]

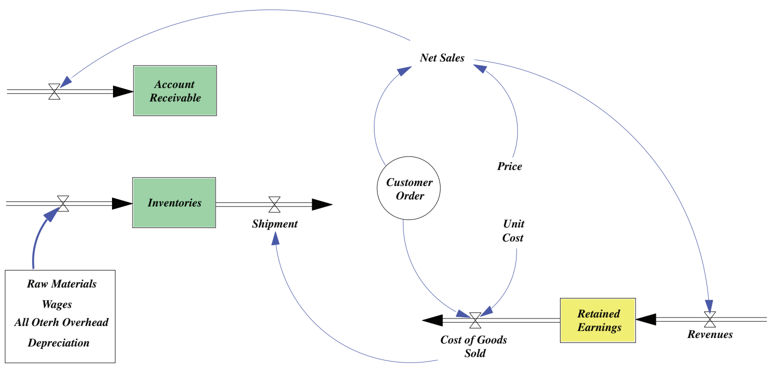

Figure 3.11: Double Transactions caused by Customer Order

Transaction 21 Receive an order for 15,000 cases of applesauce at a selling price of $15.66 per case, $234,900 for the total order.
Note: Receiving an order has no effect on the three major financial statements. Only when the product ordered is shipped to customers do you

record a SALE and the associated COST OF GOODS SOLD. Yet, this could be recorded as an increase in backlog order.

Transaction 22 Ship 15,000 cases of applesauce and send a $234,900 invoice to the customer.

⇒ T: Customer Order (= 15,000 cases)
⇒ [Retained Earnings +] ↔ [Accounts Receivable +]

⇒ Shipment Value (= 15,000 cases * $ 10.2 per case = $153,000)
⇒ [Finished Goods Inventory -]
↔ [Retained Earnings -: Costs of Goods Sold]

⇒ T: Sales & Marketing (= $4,698)
⇒ [Retained Earnings -] ↔ [Accrued Expenses +]

Transaction 23 Receive payment of $234,900 for shipment that was made in Transaction 22. Pay the broker his $4,698 selling commission.
Note: A customer's cash payment for goods in no way changes the *Income Statement.* The *Income Statement* recorded a sale when first, we shipped the goods, and second, the customer incurred the obligation to pay (our accounts receivable).

⇒ T: Accounts Receivable Paid (= $234,900)
⇒ [Cash +] ↔ [Accounts Receivable -]

⇒ T(Cash-): Accrued Expenses Paid (= $4,698)
⇒ [Accrued Expenses -] ↔ [Cash -]

Transaction 24 Write off the $15,900 accounts receivable that was entered when you made the 1,000 case shipment. Also, reduce the amount payable to our broker by what would have been his commission on the sale. If we don't get paid, he doesn't either !

Note: Our out-of-pocket loss is really just the $10,200 inventory value of the goods shipped. Remember that in Transaction 20 we booked a profit from this sale of $5,382 − the $15,900 sale minus the $10,200 cost of goods minus the $318 selling commission. Thus, if you combine the $15,582 drop in RETAINED EARNINGS booked in this transaction plus the $5,382 increase in RETAINED EARNINGS from Transaction 20, you are left with our loss of $10,200 from this bad debt.

⇒ T: write-off (= $15,900)
⇒ [Retained Earnings -: Operating Expenses]
↔ [Accounts Receivable -]

⇒ T: Sales & Marketing (= $-318)
⇒ [Retained Earnings -: Operating Expenses]
↔ [Accrued Expenses +]

Transaction 25 With this transaction we will pay a full year's insurance premium of $26,000, giving us three months' prior coverage (the amount of time we have been in business) and also coverage for the remaining nine months in our fiscal year.

Note: As time goes by, we will take this remaining $19,500 as an expense through the *Income Statement*. The transaction at that time will be to book the expense in the *Income Statement* and at the same time lower the amount of PREPAID EXPENSE in the *Balance Sheet*.

⇒ T: Insurance Premium (= $26,000)
⇒ [Prepaid Expenses +] ↔ [Cash -]

⇒ T: Insurance Premium Paid (= $6,500)
⇒ [Prepaid Expenses -] ↔ [Retained Earnings -: Operating Expenses]

Transaction 26 Make a quarterly payment of $25,000 in principal and also a $25,000 interest payment on the building mortgage.

⇒ T: Current Debt Paid (= $25,000)
⇒ [Current Portion of Debt -] ↔ [Cash -]

⇒ T: Principal Payment (= $25,000)
⇒ [Long-Term Debt -] ↔ [Current Portion of Debt +]

⇒ T: Interest Expenses (= $25,000)
⇒ [Retained Earnings -] ↔ [Cash -]

Transaction 27 Pay payroll taxes, fringe benefits and insurance premiums. Write checks to the government and to insurance companies totaling $18,480 for payment of withholding and FICA taxes and for payroll associated fringe benefits.

Note: The Income Statement and RETAINED EARNINGS are not affected by this payment transaction. Because AppleSeed runs its books on an accrual basis, we already "expensed" these expenses when they occurred – not when the actual payment is made.

⇒ T: Accrued Expenses Paid (= $18,480)
⇒ [Accrued Expenses -] ↔ [Cash -]

Transaction 28 Pay suppliers a portion of what is due for apples and jars. Cut a check for \$150,000 in partial payment.

⇒ T: Accounts Payable Paid (= \$150,000)
⇒ [Accounts Payable -] ↔ [Cash -]

Transaction 29 Book a series of entries in the Income Statement, Cash Flow Statement and the Balance Sheet summarizing transactions that take place in the remaining nine months of AppleSeed Enterprises' first fiscal year.

(⇒ Transaction Items beyond this transaction are not specified in the book [34]) The reader who followed our description up to this point can easily fill in the transactions given in Table 3.4.

Transaction 30 On a pretax income of \$391,687 AppleSeed owes 34% in federal income taxes (\$133,173), and \$6,631 in state income taxes for a total income tax bill of \$139,804. We will not actually pay the tax for several months.
Income tax is calculated as Income before tax times Income tax rate of 34%, and built in the program.

⇒ [Income Tax Payable +] ↔ [Retained Earnings -]

Transaction 31 Declare and pay a \$0.375 per share dividend to AppleSeed's shareholders. (With 200,000 shares outstanding, this dividend will cost the company \$75,000.)

⇒ T: Par Share Dividend (= \$0.375 per share)
⇒ [Cash -: Dividends Paid to Stockholders]
↔ [Retained Earnings -: Dividends]

Modeling corporate financial statements are now completed. They consists of Income Statement (Figure 3.4) and Balance Sheet (Figure 3.12). Inventories (Figure 3.10) is a part of the balance sheet and Cash Flow Statements (Figures 3.14 is a part of balance sheet.

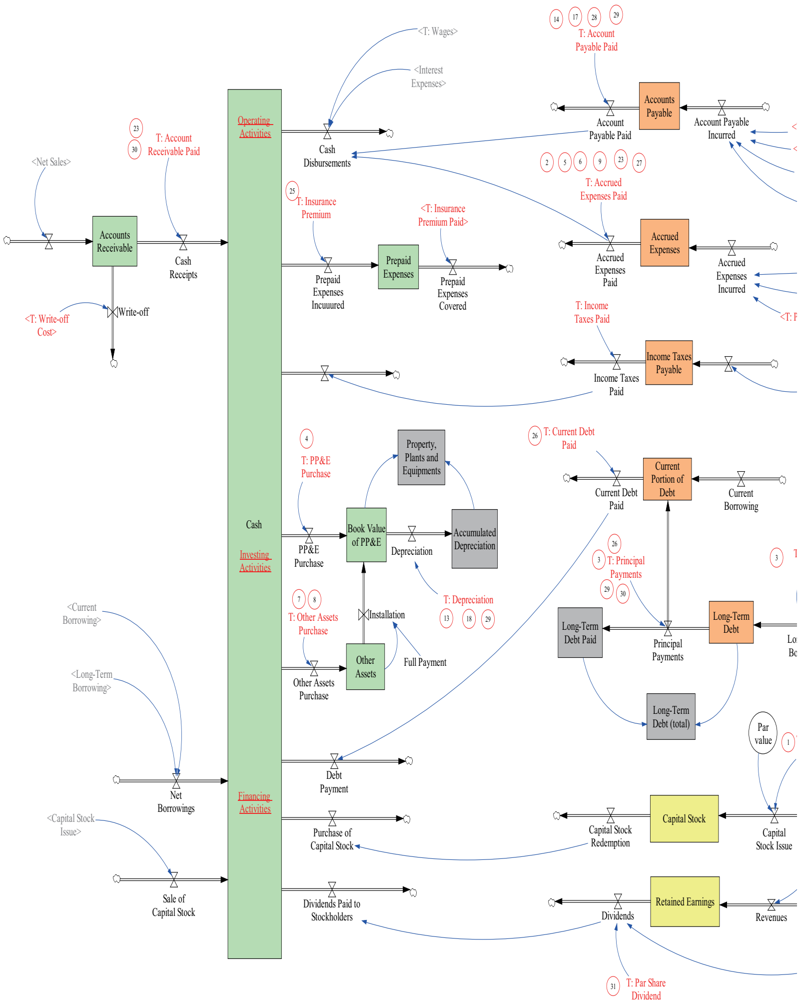

Figure 3.12: Balance Sheet

3.6 Making Financial Statements

Transactions given in the book [34] are not arranged as monthly data. To run the SD model, those data need to be reinterpreted as monthly data. For instance, Transaction 5 has to be regarded as the one in the 5th month. This is what is assumed here for the SD modeling.

There are two methods to import transaction data into the model. They could be put in the table functions, whose names are given in the list of Figure 3.13.

Almost half of the names in the list are related with the stock: Cash. This is

Figure 3.13: List of Transaction Data

because all transactions in a market economy need to be eventually paid in cash, and cash-related transactions constitute a large portion of transactions. Hence, Cash becomes the largest group among the primary stocks to be changed. The other major groups are related with the transactions by credits such as Accounts (Receivable or Payable), and (Prepaid or Accrued) Expenses. Transaction data in Figure 3.13 are arranged to reflect these facts.

Alternatively, transaction data could be prepared outside the model as those of spreadsheet such as Excel, then imported to the model. There are quite a few accounting software on the market that enable to keep recording daily transactions. These booked data are later classified as ledgers of items to construct balance sheet. Using spreadsheet such as Excel, therefore, it may not be hard to import them to the SD model as the data of inflows and outflows as shown in the list in Figure 3.13. The SD model could then become an alternative accounting software. Moreover, it could become a better one as a financial analysis tool as shown in the next section.

In fact, balance sheet in Table 3.6 is constructed by using the data given in Figure 3.13 (or alternatively by importing them as spreadsheet data). Due to a limitation of space, only figures of five different months among 31 months are shown here. Income statement and cash flow statement can be procured in a similar fashion.

Time(Month)	1	5	10	15	20
Cash	50,000	1.046M	776,260	747,240	588,220
Accounts Receivable	0	0	0	0	0
Inventories	0	0	0	385,400	577,970
Prepaid Expenses	0	0	0	0	0
Current Assets	50,000	1.046M	776,260	1.132M	1.166M
Other Assets	0	0	0	0	0
Book Value of PP&E	0	1.5M	1.75M	1.742M	1.735M
Assets	50,000	2.546M	2.526M	2.875M	2.901M
Accounts Payable	0	0	0	341,077	469,204
Accrued Expenses	0	2,860	2,160	10,320	18,480
Current Portion of Debt	0	100,000	100,000	100,000	100,000
Income Taxes Payable	0	0	0	0	0
Current Liabilities	0	102,860	102,160	451,397	587,684
Long-Term Debt	0	900,000	900,000	900,000	900,000
Capital Stock	50,000	1.55M	1.55M	1.55M	1.55M
Retained Earnings	0	-6,230	-25,900	-25,900	-135,780
Shareholders' Equity	50,000	1.543M	1.524M	1.524M	1.414M
Liabilities & Equity	50,000	2.546M	2.526M	2.875M	2.901M

Table 3.6: Balance Sheet Table

3.7 Ratio Analysis of Financial Statements

Structure of the corporate financial model developed above is very static in the sense that accounting system is merely to keep records of all transactions of the past business activities. In other words, transaction data are just imported to the inflows and outflows of the model as the outside parameters. In this sense, accounting system is not a SD system. To be a truly dynamic SD system, information for dynamic decision-making needs to be obtained within the system through the information feedback loops as depicted in Principle 3.

In the accounting system, balance sheet could become a main source of information from which many important feedback loops originate for management strategies and policies. Traditional method of obtaining such feedback information is a so-called financial ratio analysis. In the book [34], eleven such ratios are defined and grouped into four types as follows.

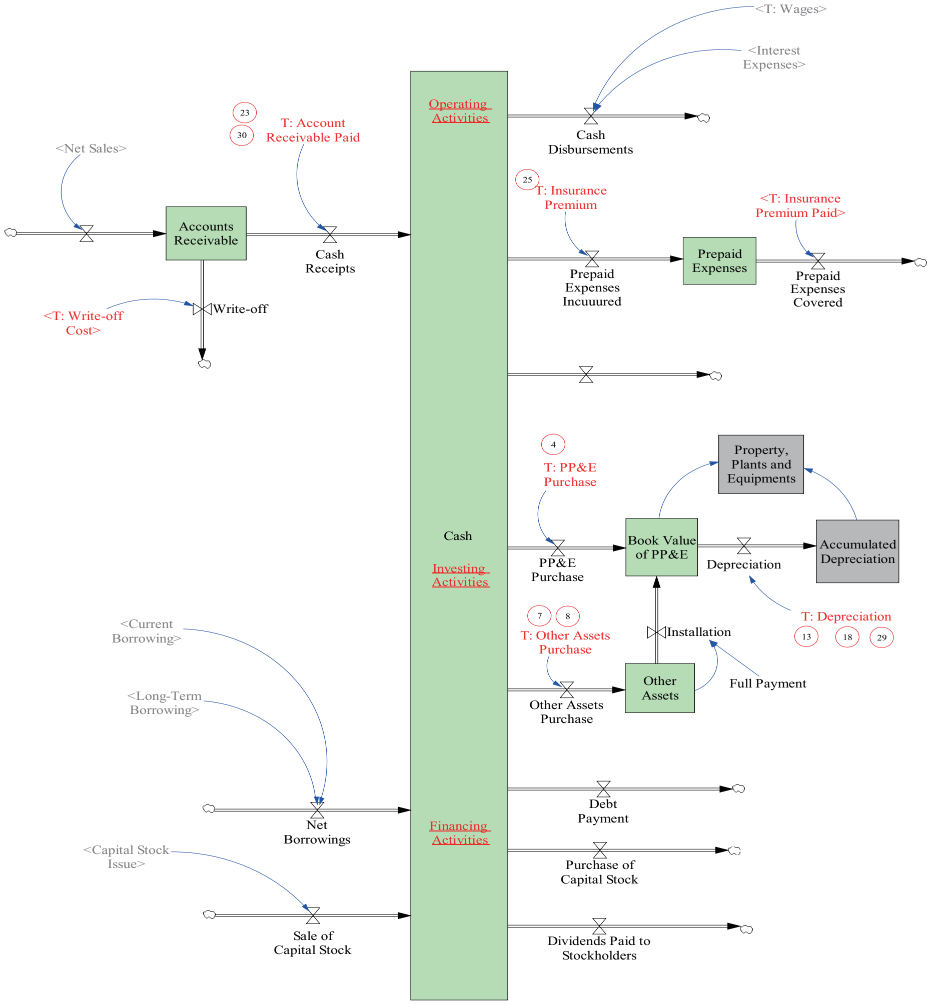

Figure 3.14: Cash Flow Statement

Liquidity Ratios

$$\text{Current Ratio} = \frac{\text{Current Assets}}{\text{Current Liabilities}}$$

$$\text{Quick Ratio} = \frac{\text{Cash} + \text{Accounts Receivable}}{\text{Current Liabilities}}$$

Asset Management Ratios

$$\text{Inventory Turnover} = \frac{\text{Cost of Goods Sold}}{\text{Inventories}}$$

$$\text{Asset Turn Ratio} = \frac{\text{Net Sales}}{\text{Assets}}$$

$$\text{Accounts Receivable Turnover} = \frac{\text{Net Sales}}{\text{Accounts Receivable}}$$

Profitability Ratios

$$\text{Return on Assets (ROA)} = \frac{\text{Net Income}}{\text{Assets}}$$

$$\text{Return on Equity (ROE)} = \frac{\text{Net Income}}{\text{Shareholders' Equity}}$$

$$\text{Return on Sales (Profit Margin)} = \frac{\text{Net Income}}{\text{Net Sales}}$$

$$\text{Gross Margin (Gross Profits)} = \frac{\text{Gross Margin}}{\text{Net Sales}}$$

Leverage Ratios

$$\text{Debt-to-Equity} = \frac{\text{Current Portion of Debt + Long-Term Debt}}{\text{Shareholders' Equity}}$$

$$\text{Debt Ratio} = \frac{\text{Current Portion of Debt + Long-Term Debt}}{\text{Assets}}$$

In SD modeling, these ratios can be easily calculated for financial analysis as illustrated in Figure 3.15. For instance, Returns on Assets (ROA) and Equity (ROE) are illustrated as in Figures 3.16.

3.8 Toward A Corporate Archetype Modeling

Balance sheet represents a whole system of financial activities for corporations, and managers have to rely on the information obtained within the system for their strategies and policies. Liquidity ratios, asset management ratios, profitability ratios and leverage ratios presented in the previous section provides essential indices of management strategies and financial policies. In other words, stocks in the balance sheet provide very important sources of information for corporations. From system dynamics viewpoint, the use of such information is nothing but establishing feedback loops from the sources of information (that is, stocks in the balance sheet) to the inflows and outflows. In this sense, 11 ratios illustrated in Figure 3.15 could be important parts of system feedback loops. With the introduction of such feedback loops, our corporate financial

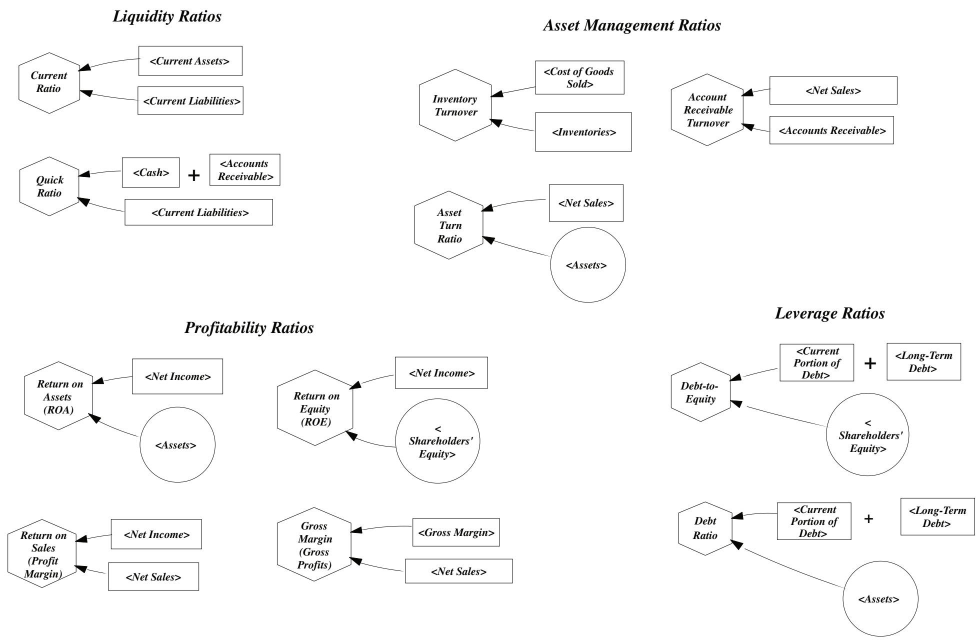

Figure 3.15: Ratio Analysis Diagram

model could become a relatively closed system and provide a wholistic picture of corporate dynamics.

It could be inferred, however, that such traditional ratio analysis is not the only method for managers to extract managerial information. For instance, a discrepancy between net cash flow and net income, as illustrated in Figure 3.17, could be another important source of information for better liquidity management. In this way, a lot of essential information could be derived within the SD accounting system, depending on the objectives of management.

What kind of information feedback loops, then, need to be built and how? Learning the current accounting system merely gives us no clue. In order to incorporate information feedback loops, we have to know how decisions on transactions such as the ones considered in section 3.5 are made. The introduction of appropriate feedback loops, in this sense, depends on the types of business activities of corporations. Only when such decision-making processes are specifically incorporated into our corporate financial model, it becomes a truly SD accounting model.

Even so, as long as modern corporations are part of the global market economic system, there could be generally accepted rules of drawing financial information feedback loops to make our SD model a truly corporate financial model. Such a model, if constructed, could be a corporate business archetype. In this sense, our research here is nothing but a beginning, though an important start, toward a truly corporate archetype modeling. This will be our task to be

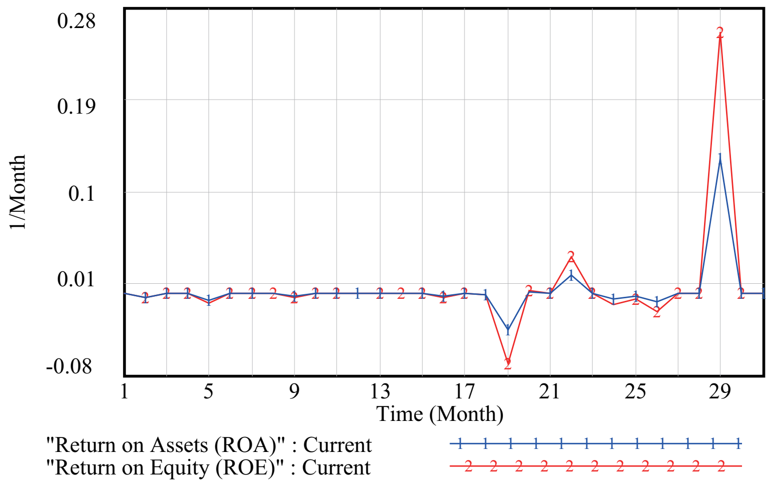

Figure 3.16: Returns on Assets and Equity

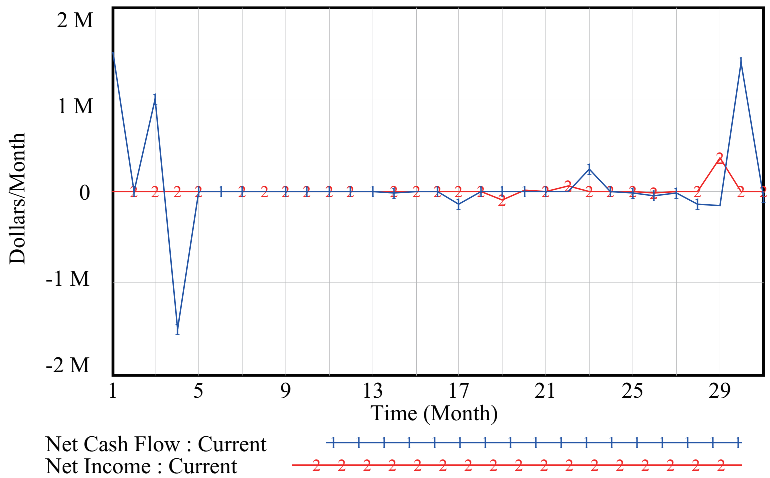

Figure 3.17: Net Cash Flow and Net Income

challenged in the near future.

Conclusion

We have demonstrated how to construct a SD model of corporate financial statements such as given in the book [34], by establishing the principle of accounting system dynamics (PASD) that consists 5 principles obtained from system dynamics and accounting system. It is shown that cash flow statement is indispensable, contrary to the practice that it has been long neglected in the Japanese financial statements. The model is shown to be static in the sense that all transaction data are given as parameters outside the system and no information obtained from the stocks in the balance sheet is utilized for better management practices - a limitation of the current accounting system. To make it a truly dynamic SD model, information feedback loops have to be incorporated in it.

Part II

Macroeconomic System

Chapter 4

Macroeconomic System Overview

This chapter tries to apply the method of accounting system dynamics developed in the previous chapters to the macroeconomic modeling. We start with the description of a simple capitalist market economy with the traditional budget equations. Then it is shown how they are constructed into an accounting system dynamics model of simple macroeconomy, which will be a fundamental framework of the macroeconomic models in the following chapters.

4.1 Macroeconomic System

Macroeconomics is one of the core economic subjects which has been widely taught, with the use of standard textbooks, all over the world by many macroeconomists. Under such circumstances, are there still something remaining to which system dynamics can contribute, I posed. An affirmative answer to this question has led me to work on the series of macroeconomic modeling in [64, 65, 66, 67, 68]. For instance, macroeconomic variables such as GDP, inventory, investment, price, money supply, interest rate, etc, could be more precisely presented by using a basic concept of stock and flow in system dynamics. Moreover, using SD modeling methods, determination of GDP and creation process of credits and money supply - two essential ingredients of macroeconomics- could be more precisely described as dynamic macroeconomic adjustment processes, compared with a traditional static approach.

System dynamics approach requires to capture macroeconomy as a wholistic system consisting of many parts that are interacting with one another. Specifically, macroeconomic system is viewed here as consisting of six sectors such as the central bank, commercial banks, consumers (households), producers (firms), government and foreign sector. Figure 4.1 illustrates an overview of such macroeconomic system and shows how these macroeconomic sectors interact with one another and exchange goods and services for money.

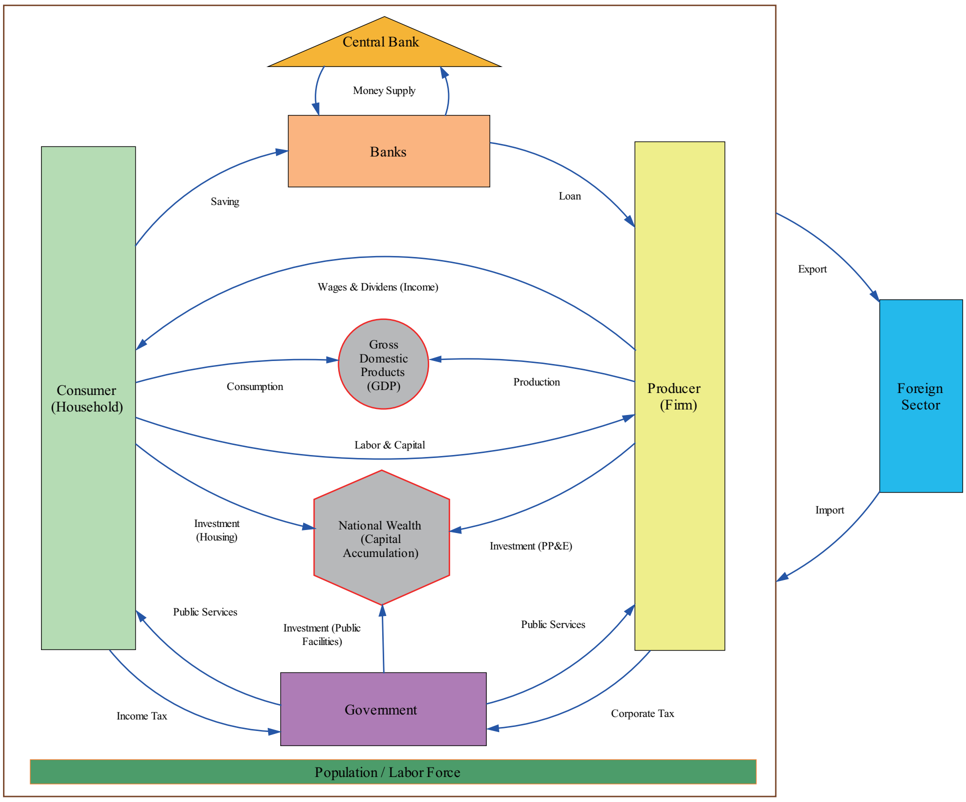

Figure 4.1: Macroeconomic System Overview

In this chapter, we show how to model the macroeconomic system illustrated in the Figure by constructing a simple macroeconomy consisting only consumers and producers in a capitalist market economy.

4.2 A Capitalist Market Economy

Market economy is an economic system in which goods and services are traded in the markets. A market economy we are currently living in is not the only market economy. For instance, a self-sufficient community, if any, may partly exchange goods and services with another community, or former socialist economies used to trade with another socialist economies. Accordingly, if we extend our concept of economic activities to cover all communities or international economies, their economy also form a kind of market economy. Or the MuRatopian economy consisting of co-workers I proposed in [58] as a most suitable economy to the information age is also a market economy.

To distinguish our market economy from other types of market economy mentioned above, let us call it a capitalist market economy. It is defined as having the following features. It is an economic system which allows private ownership of factors of production such as labor, capital and land. Specifically, workers are allowed to own their labor (thus no longer slaves), shareholders or capitalists can own capital or shares, and landowners can own land and housing for rent. Producers have to organize production activities by purchasing those factors of production owned by their owners in the markets in exchange for wages, profits (or dividends) and rents. The markets where those transactions are made are called labor market, financial capital market, and real estate market. On the other hand, owners as consumers have to purchase goods and services in the commodity market. In this way, in a capitalist market economy, all factors of production and goods and services are exchanged in the markets. To make these transactions easy, money as a medium of exchange is invented, whose unit of value is used as a price.

Desired Budget Equations

To describe our market economy as simple as possible without losing generalization, let us consider the economy consisting of workers, shareholders (or capitalists) and producers. Workers and shareholders need not be mutually exclusive. Workers who own corporate shares can also be classified as shareholders. Consumers consist of those workers and shareholders. Their desired budget equations are formally written as follows:

First, workers (W) expect to receive wages against their labor supply and spend them as their income on consumption. The remaining is to be saved. Thus, their desired budget equation becomes

$$pC_W + S_W = wL^s \tag{4.1}$$

where p is price, C_W is their consumption, S_W is their savings, w is a wage rate, and L^s is labor supply.

Next, shareholders (O) expect to receive profits (dividends) and spend them as their income on consumption. The remaining is to be saved. Then, their desired budget equation becomes

$$pC_O + S_O = \Pi(= pY - wL^d) \tag{4.2}$$

where Π is profits (dividends), C_O is their consumption, S_O is their savings, Y is output (or GDP, Gross Domestic Products, whose concept is assumed to be familiar to the reader), and L^d is their demand for labor.

Finally, producers organize production activities and are assumed to make investment I to expand their production capacity on behalf of shareholders. Since all revenues have to be distributed to workers as wages and shareholders as dividends in a private ownership economy, no fund is left available for new investment. Accordingly, in a capitalist market economy producers are destined to constantly raise fund I^d for investment. Thus, their desired budget equation becomes

$$pI = I^d \tag{4.3}$$

When all of these desired budget equations are added, the following equation is obtained. Since it holds all the time, it becomes an identity, and called Walras law.

$$p(C_W + C_O + I - Y) + w(L^d - L^s) + (S_W + S_O - I^d) \equiv 0 \tag{4.4}$$

The first item implies an excess demand for goods and services in commodity market, the second item is an excess demand for labor, and the third item is an excess demand for money in financial capital market. Once a capitalist market economy is formalized as above, the major question is whether there exit market prices which clear excess demand in all markets. To be precise, from Walras law, whenever two markets are in equilibrium, the remaining market attains equilibrium automatically. This problem is called the existence of general equilibrium. As already discussed in chapter 2, it is proved by Arrow and Debreu.

The next major question is how to find the equilibrium prices. Such a finding process is said to be globally stable if any initial prices can eventually attain the equilibrium through tâtonnement processes. As already discussed in chapter 2, the existence of chaos makes it impossible under some circumstances. It is worth noting again that under the neoclassical framework of price adjustment, transactions can only start when equilibrium is attained. Until that moment, their budget equations are not the actual ones based on the actual receipts and payments. That is why above budget equations are called desired budget equations.

4.3 Modeling a Market Economy

Our method of economic analysis is to allow off-equilibrium transactions on a historical time. And the accounting system dynamics developed in the previous chapters enables to model the off-equilibrium transactions. Accordingly we are now in a position to model the above capitalist market economy as a generic macroeconomy.

Let us start with producers' balance sheet. Whenever output is produced it becomes their revenues and at the same time booked as inventory. In an actual booking practice of companies, it is usually booked as accounts receivable.

Producers pay wages and dividends to consumers consisting of workers and shareholders, who in turn spend their income on consumption, and the remaining is saved. Consumption thus becomes part of sales which reduces inventory and increase producers' stock of cash. Producers also make investment, which in turn becomes sales of other producers. In our integrated stock of producers, these bookings are done in a same stock-flow diagram. Figure 4.2 illustrates our first macroeconomic modeling.

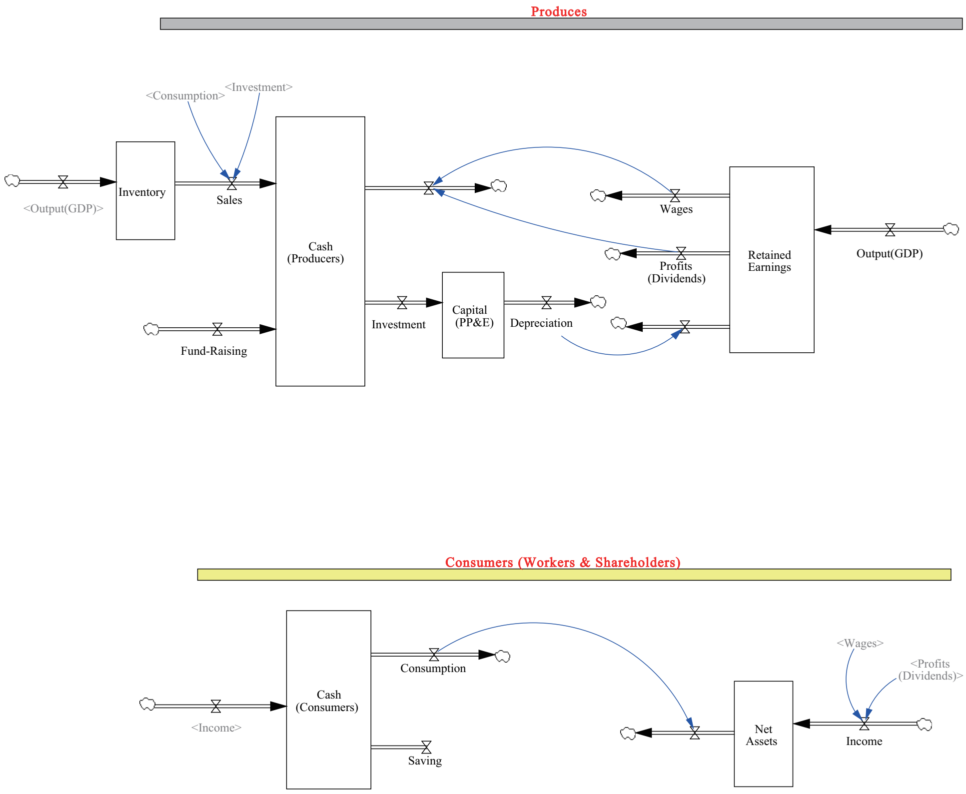

Figure 4.2: Macroeconomic System Flow Chart

At this point, one remark may be needed. In the model, capital depreciation is added to make our modeling precise. Accordingly, investment in the model has to be interpreted as gross investment consisting of net investment and depreciation. Thus, income that consumers receive are also interpreted as net income; that is, output less depreciation.

Cash Flow of Producers

Let us now calculate net cash flow of producers. It is shown as inflow and outflow of producers' cash stock in Figure 4.3. Thus, it is obtained as follows:

$$
\begin{aligned}
\text{Net Cash Flow} &= \text{Cash Inflow - Cash Outflow} \\
&= \text{Consumption + Investment} \\
&\quad \text{- Wages - Profits (Dividends) - Investment} \\
&= \text{Consumption - National Income at Factor Cost} \\
&= \text{- Saving}
\end{aligned} \tag{4.5}
$$

The net cash flow of producers becomes equal to the negative amount of saving. In other words, in a capitalist market economy, producers are always in a state of cash deficiency. Accordingly, to make new investment, they are obliged to raise funds. This becomes a fundamental framework of our macroeconomy.

Theoretically, there are four ways to raise funds as follows:

- Borrowing from banks (bank loans)
- Issuing corporate bonds (borrowing from the public)
- Issuing corporate shares (sharing ownership)
- Retaining earnings for investment (retained saving)

4.4 Bank Loans

Let us consider the fund-raising by bank loans. In this economy, consumers are supposed to deposit their savings with banks, which, in turn, make loans to producers as illustrated in Figure 4.4

In this fund-raising system, banks are used to facilitate the circulation of money as a means of exchange. Historically, however, usury evolved into banks, and interests are being imposed on producers. Accordingly, producers are forced to seek for economic growth incessantly to pay interests as well as principals. Remember from the previous argument of reinforcing feedback of banking system in Chapter 1. Loans grow exponentially. To repay this increasing amount of loans, production has to grow exponentially If economic growth is not attained, those who cannot repay are forced to collapse. Apparently, this incessant growth is not possible under limited resources.

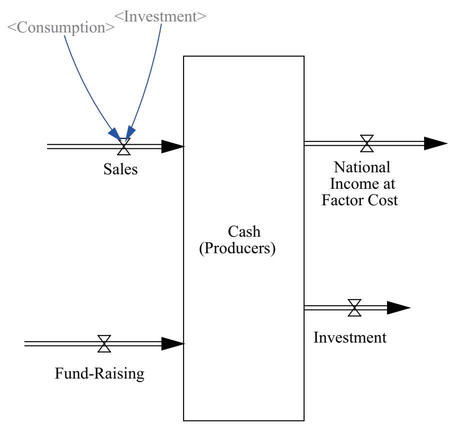

Figure 4.3: Cash Flow of Producers

Accordingly, this system of fund-raising has built-in mechanism of repeated collapses and business cycles. Eventually this interest-paying system creates unfair income distribution (the rich becomes richer due to the exponential growth), and has to be reset by triggering wars, as history tells us.

Moreover, forced economic growth causes environmental destructions. In conclusion, this interesting-bearing banking system is not sustainable. These points are discussed in detain in chapter 6

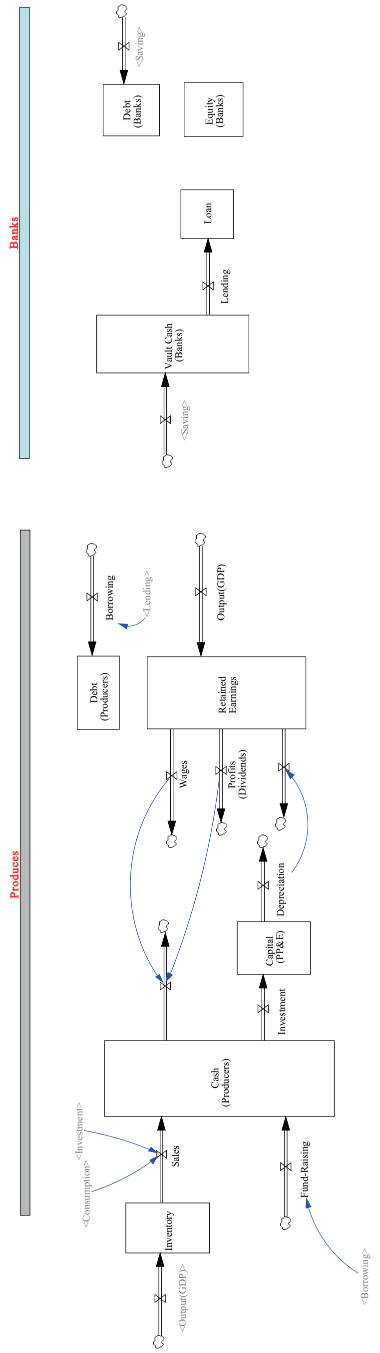

Figure 4.4: Macroeconomic System Flow Chart with Banks

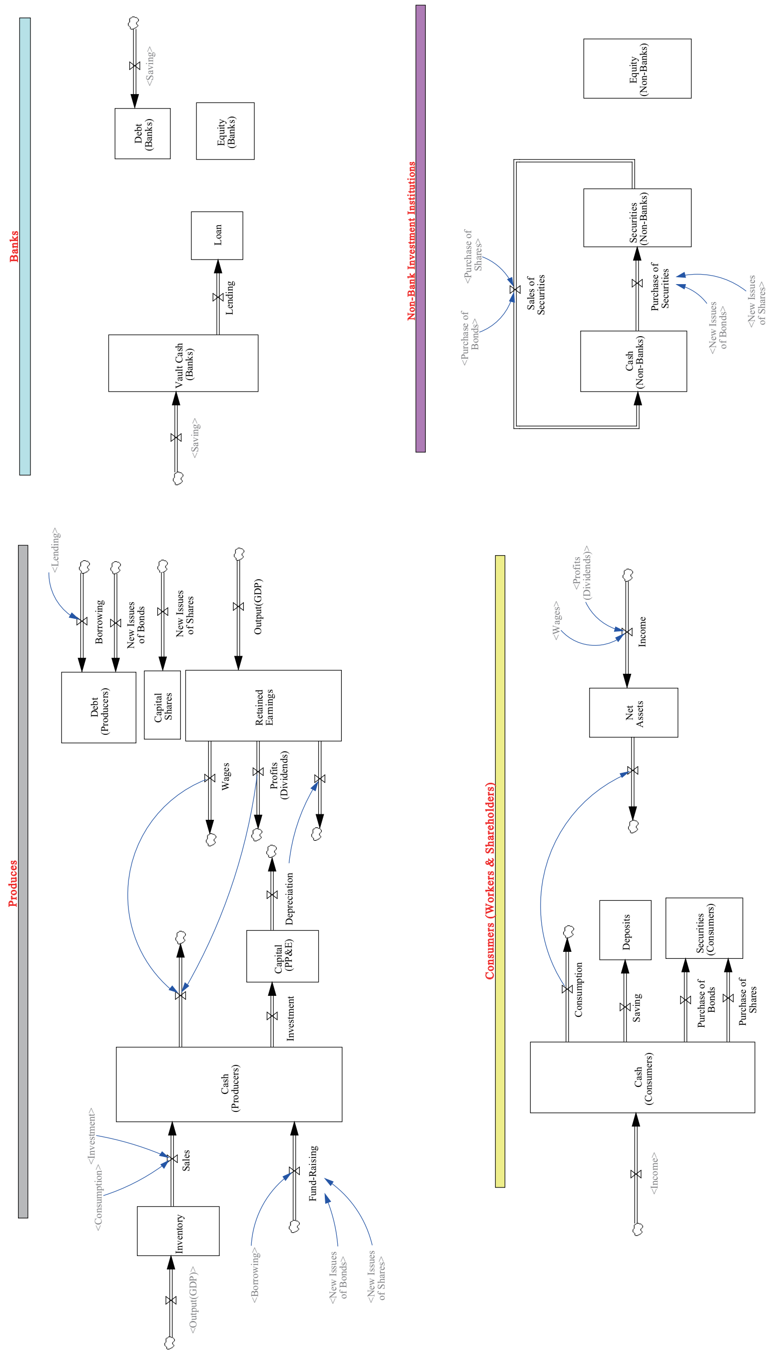

Figure 4.5: Macroeconomic System Flow Chart with Investment Institutions

4.5 Securities

Next fund-raising system is through the use of bonds and stocks (shares), which are called securities. We need non-bank investment institutions that handle these transactions as illustrated in Figure 4.5.

Problem with this fund-raising system is that there exists no way of creating money within the system that is needed to match the increasing demand for money in a growing economy.

Historically, the above two fund-raising systems with banks and non-bank investment financial institutions co-evolved. And consumers have a portfolio choice among savings, bonds, and shares, while producers have three sources of fund-raising: loans, bonds and shares.

However, the role of banks and non-bank investment institutions has been separated by laws; for in stance in the United States by Glass-Stengall Act in 1933. The Act was overruled in 1999 by Gramm-Leach-Bliley Act, and no clear distinction of financial transactions is made between banks and investment institutions under the deregulation forces of free financial activities. This excessive freedom became the cause of global financial crisis starting in 2007.

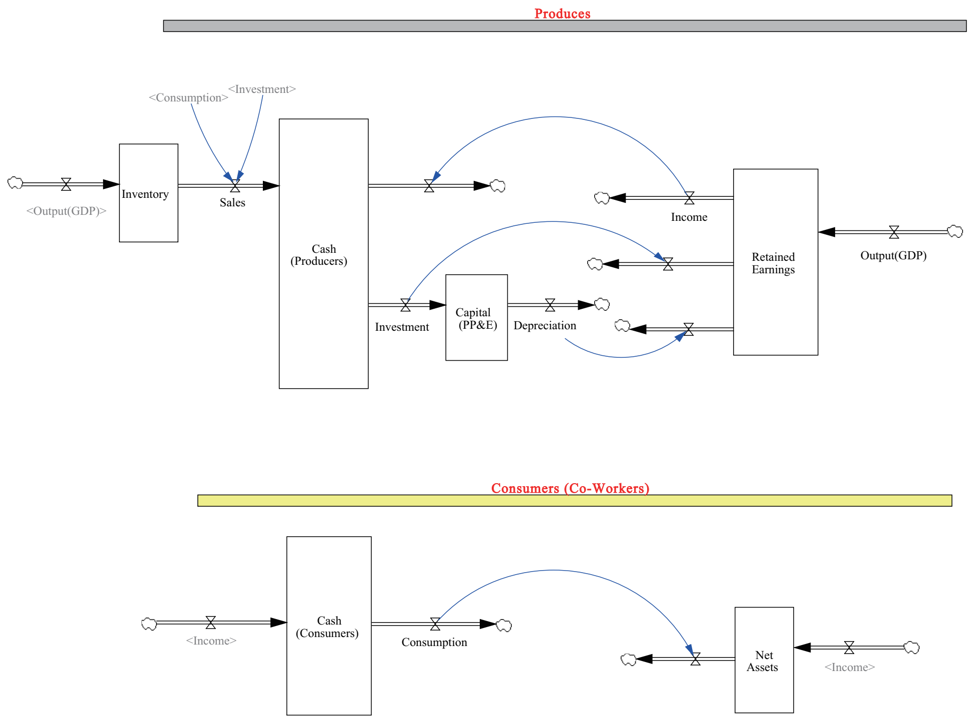

Figure 4.6: Macroeconomic System Flow Chart of MuRatopian Economy

4.6 Retained Earnings

Finally, producers may be allowed to save retained earnings for future's investment. Japanese auto maker, Toyota, is known for this self-sustained financial management.

System dynamics is the method not only for solving problems, but designing better systems. An ideal economic system of fund-raising would be the one in which producers are possessed by consumers, and no distinction is made between workers and shareholders. Accordingly, retained earnings become main source of fund. It is called MuRatopian economy in [58]. In this economy, investment is made first, and the remaining is distributed for consumption, as illustrated in Figure 4.6.

Chapter 5

Money and Its Creation

This chapter[1] tries to model money supply and its creation on a basis of the principle of accounting system dynamics developed in the previous chapters. For this purpose, a simple model based on gold standard is first constructed with the introduction of high-powered money and monetary base as two different stocks, contrary to the macroeconomic tradition that treat them identically. Then three different expressions of money supply based on these two stocks are presented, and it is shown how they differ each other. The model is further expanded to a complete money supply model consisting of loans by the central bank and government securities that allow the central bank to exercise a discretionary control over money supply through open market operations.

5.1 Analytical Methods

Three Sectors for Money Creation

For the analysis of money supply and its creation, six macroeconomic sectors illustrated in Figure 4.1 in the previous chapter need to be reorganized into three sectors: the central bank, commercial banks and non-financial sector (consisting of producers, consumers, and government). Foreign sector is excluded in this analysis. Figure 5.1 shows the reorganized three sectors among which money circulates and is created.

This does not imply that three sectors are always required to understand a process of money creation. Historically, there was a time when central bank did not exist, yet money has been created for economic activities. This suggests that for designing a new financial system for sustainable macroeconomies, central bank needs not be necessarily required.

The reason why it is included in our modeling here is to reflect the currently

[1]This chapter is based on the paper: Money Supply and Creation of Deposits – SD Macroeconomic Modeling (1) – in "Proceedings of the 22nd International Conference of the System Dynamics Society", Oxford, U.K. , July 25-29, 2004, ISBN 0-9745329-1-6.

existing macroeconomic sectors in our model. Yet, it does not justify its existence for sustainable macroeconomy, whose analysis will be explored in chapter 12: Designing a new macroeconomic system.

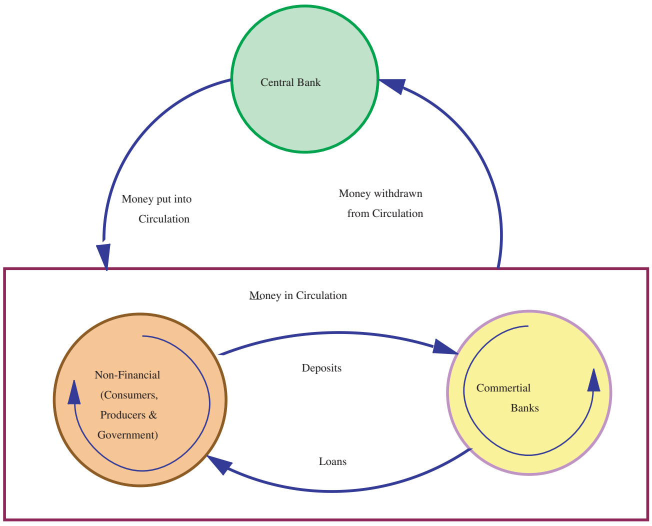

Figure 5.1: Three Sectors for Money Supply

Twofold Double Entry Rule

How can we describe economic transactions and circulation of money among three sectors? The method we employ here is based on the accounting system dynamics in which balance sheet plays a key role. Balance sheet is an accounting method of keeping records of all transactions in both credit and debit sides so that they are kept in balance all the time as follows:

$$\text{Assets} = \text{Liabilities} + \text{Equity} \tag{5.1}$$

As already discussed as the principle 5 in chapter 3, a modeling method of corporate balance sheet is based on the double entry rule of bookkeeping. In system dynamics modeling, this principle is simply illustrated as Figure 5.2.

Hence, all transactions of the central bank, commercial banks and non-financial sector are modeled respectively as inflows and outflows of money in their balance sheets. Moreover, macroeconomic transactions of money among three sectors not only influence their own balance sheets, but also other's balance sheets simultaneously. For instance, whenever a commercial bank makes loan to a consumer, it affects the balance sheets of both bank and consumer, simultaneously. In other words, one transaction in macroeconomy activates twofold

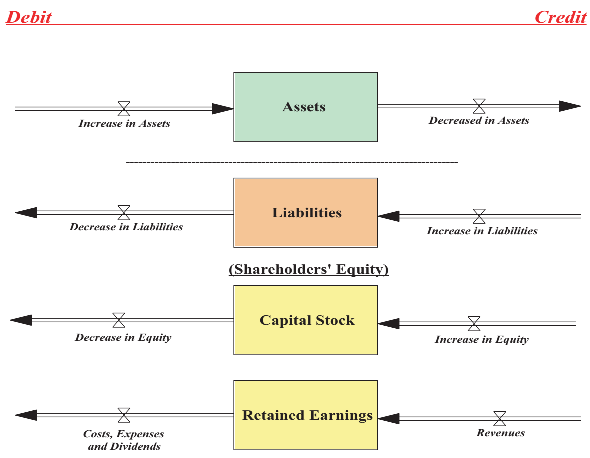

Figure 5.2: Double Entry Rule of Bookkeeping as Debit and Credit

double entries of bookkeeping among two sectors. In this sense, macroeconomic transactions can be said to be governed by *twofold double entry rule*. This makes our modeling a little bit more complicated compared with the case of corporate balance sheet in which we only need to focus on the balance of credit and debit sides of a specific company.

5.2 What is Money?

In this book, money, money supply and money stock are synonymously used. What is money, then? Most economists agree that anything, tangible or intangible, could be money if it can play the following three roles.

- Medium of Exchange
- Unit of Account
- Store of Value

In short, anything playing these roles becomes money that is generally accepted and has a purchasing power. Money we can immediately imagine, in this sense, is coinage and bank notes. Coins are minted by the government as subsidiary currency, while bank notes are issued by the central bank as main currency. In this book currency in circulation is defined as consisting of coinage and bank notes. In addition, its deposits held by commercial banks can also play similar roles, and thus become money.

Money as Commodity Currency

Let us examine money as currency in circulation first. It could be classified into commodity and fiat currencies. Historically, commodities such as shell, silk, and stones, and precious metals such as silver and gold have been used as currency, thus, money. Accordingly, these are also called commodity money.

Gradually, precious metals such as silver and gold have been replaced with paper currencies that guarantee a convertibility into them.

Money as Fiat Currency

Then, non-convertible paper and coin currencies are forced to be used as legal tender by the government. It is called fiat currency whose acceptance as a medium of exchange is authorized and no one can reject it for payment.

Money as Deposits

Whenever, people own more than enough currencies for daily transactions, they tended to deposit them with goldsmith in case of gold and with commercial banks in case of fiat money in order to store them in safe. Those deposits can be withdrawn any time, at request, for transactions. Thus, deposits are regarded as an another form of money.

Money Supply as Stock

The amount of money that is available at a certain point of time is called money supply. In system dynamics terminology, it is nothing but a money stock. Hence, money supply as stock can be defined as follows[2]:

$$\text{Money Supply} = \text{Currency in Circulation} + \text{Deposits} \tag{5.2}$$

5.3 A *Fractional Reserve* Banking System

Where does money, or currency in circulation, come from, then? In a modern capitalist market economy, only the central bank is allowed to create currency. The reader may wonder why the government cannot issue its currency? Indeed, it can mint a small portion of currency as coins, yet paper currencies can only be issued by the central bank as its bank notes. We are not in a position to argue the legitimacy of the power of central bank to issue the currency in this chapter.

Currencies thus created by the central bank is called currency outstanding. It constitutes liabilities in its balance sheet. To balance the account, the central bank needs corresponding assets. In this chapter, we consider three major assets

[2]Similar but a detailed definitions of money supply are widely used in macroeconomic textbook such as Frederic S. Mishkin [41]. In our simple model here, however, it may not be needed to classify monetary aggregates further into M1, M2 and M3.

such as gold, discount loans and government securities against which currency outstanding is issued.

In any case, this amount of currency outstanding becomes monetary base, out of which currency begins to circulate among macroeconomic sectors. Once currency outstanding is put into circulation, it begins to be used for transaction payments. If more than enough currency is in circulation, it will be deposited with commercial banks. Money in this way begins to be used as currency in circulation and deposits.

Does its different use affect money supply, then? To see this, let us define currency ratio (α) as follows:

$$\text{Currency Ratio } (\alpha) = \frac{\text{Currency in Circulation}}{\text{Deposits}} \tag{5.3}$$

Then, one dollar put into circulation is divided into currency in circulation and deposits according to the following proportion:

$$1 \Rightarrow \begin{cases} \frac{\alpha}{\alpha+1} & : \text{Currency in Circulation} \\ \frac{1}{\alpha+1} & : \text{Deposits wit Banks} \end{cases} \tag{5.4}$$

Once commercial banks receive deposits, what do they do with them? Their role was to keep the deposits at a safe place to meet the request of depositors for withdrawal in the future. However, they gradually realized that only a portion of deposits were to be withdrawn. Accordingly, they started to make loans out of deposits at interest. In this way, once-prohibited usury became a dominating practice - a start of modern banking.

To meet an insufficiency of deposits against a sudden withdrawal, private banks secretly formed a cartel. One such example is the Federal Reserve System in the United States that was created in 1913. Fascinating story about its birth was described in [29]. Though it is a privately owned bank, it pretends to be the American public central bank.

Once the central bank is established in many capitalist economies, it begin to request a portion of deposits from commercial banks to protect their liquidity shortage problem. Specifically, they are required by law to have an account with the central bank and keep some portion of their deposits in it in order to meet unpredictable withdrawal by depositors. These deposits of commercial banks at the central bank are called required reserves. Now commercial banks can freely make loans out of their deposits (less required reserves) without any risk. This modern banking system is called a *fractional reserve* banking system.

Money *out of Nothing!*

It is this fractional reserve banking system that enables to create money *out of nothing!*. To see this process, let us define reserve ratio as follows.

$$\begin{aligned}
\text{Reserve Ratio } (\beta) &= \frac{\text{Reserves}}{\text{Deposits}} \\
&= \frac{\text{Required Reserves}}{\text{Deposits}} + \frac{\text{Excess Reserves}}{\text{Deposits}} \\
&= \beta_r + \beta_e. \qquad (5.5)
\end{aligned}$$

That is, reserve ratio β becomes the sum of required reserve ratio β_r and excess reserve ratio β_e.

Let us now consider how one dollar put into circulation is transacted. From the equation (5.4), $1/(\alpha+1)$ dollars are deposited first, out of which commercial banks are allowed to make maximum loans of $(1-\beta)/(\alpha+1)$ dollars. This amount will be put into circulation again as a loan to non-financial sector. In a capitalist market economy, producers in a non-financial sector is always in a state of liquidity deficiency as discussed in chapter 4. In this way, one dollar put into circulation creates additional dollars to be put into circulation. Total sum of currency created by the original dollar and put into circulation is calculated as follows.

$$\begin{aligned}
&\text{Total currency put into circulation} \\
&= 1 + \frac{1-\beta}{\alpha+1} + \left(\frac{1-\beta}{\alpha+1}\right)^2 + \left(\frac{1-\beta}{\alpha+1}\right)^3 + \cdots \\
&= \frac{1}{1-\frac{1-\beta}{\alpha+1}} \\
&= \frac{\alpha+1}{\alpha+\beta} \qquad (5.6)
\end{aligned}$$

This is a process of creating money *out of nothing* by commercial banks, in which one dollar put into circulation is increased by its multiple amount. It is called money multiplier (m); that is,

$$\text{Money Multiplier } (m) = \frac{\alpha+1}{\alpha+\beta} = \frac{\alpha+1}{\alpha+\beta_r+\beta_e} \qquad (5.7)$$

Since $1 \geq \beta \geq 0$, we have

$$1 + \frac{1}{\alpha} \geq m \geq 1 \qquad (5.8)$$

Hence, money multiplier can be easily calculated if currency ratio and required reserve ratio as well as excess reserve ratio are given in a macroeconomy. Three sectors in Figure 5.1 play a role of determining these ratios. Depositors in the non-financial sector (consumers & producers) determine the currency ratio: how much money to keep at hand as cash and how much to deposit. Central bank sets a level of required reserve ratio as a part of its financial policies, while commercial banks decide excess reserve ratio: how much extra reserves to hold against the need for deposit withdrawals.

In this way, an additional dollar put into circulation will eventually create its multiple amount of money supply, which are being used as currency in circulation and deposits with banks as follows:

$$1 \Rightarrow \begin{cases} \frac{\alpha}{\alpha+1}\frac{\alpha+1}{\alpha+\beta} = \frac{\alpha}{\alpha+\beta} & : \text{Currency in Circulation} \\ \frac{1}{\alpha+1}\frac{\alpha+1}{\alpha+\beta} = \frac{1}{\alpha+\beta} & : \text{Deposits wit Banks} \end{cases} \tag{5.9}$$

In a real economy, then, how much real currency or cash is actually being put into circulation? It is the sum of currency in current circulation and reserves that commercial banks withhold at the central bank. This sum indeed constitutes a real part of money supply issued by the central bank through which creation of deposits and money supply are made as shown above. In this sense, it is called high-powered money.

$$\text{High-Powered Money} = \text{Currency in Circulation} + \text{Reserves} \tag{5.10}$$

To see the amount of money supply created by high-powered money, let us calculate a ratio between money supply and high-powered money as follows:

$$\begin{aligned} &\frac{\text{Money Supply}}{\text{High-Powered Money}} \\ &= \frac{\text{Currency in Circulation + Deposits}}{\text{Currency in Circulation + Reserves}} \\ &= \frac{\text{Currency in Circulation/Deposits + 1}}{\text{Currency in Circulation/Deposits + Reserves/Deposits}} \\ &= \frac{\alpha+1}{\alpha+\beta} \end{aligned} \tag{5.11}$$

This ratio becomes exactly the same as money multiplier calculated above in equation (5.7). Thus, money supply can be uniformly expressed as[3]

$$\text{Money Supply} = m * \text{High-Powered Money} \tag{5.12}$$

In the above definitions, currency in circulation appears both in money supply and high-powered money. However, it is hard to calculate it in a real economy and in practice it is approximated by the amount of bank notes (currency) outstanding which is recorded in the balance sheet of the central bank. Accordingly, high-powered money is also approximated by the sum of currency outstanding and reserves. It is called

[3]When money multiplier is calculated as the equation (5.7) and applied to the equation (5.12) to obtain money supply, it turns out that money suddenly jumps from the money supply defined in (5.2) as the currency ratio and reserve ratio are changed during a simulation. To avoid this problem, currency and reserve ratios need be constantly recalculated during the simulation. In the money supply model below, they are done as “actual currency and reserve ratios”.

$$\text{Monetary Base} = \text{Currency Outstanding} + \text{Reserves}, \quad (5.13)$$

because this is the amount of currency that the central bank can control. And most macroeconomic textbooks treat high-powered money equivalently as monetary base. For instance, a well-established textbook says: "This is why the monetary base is also called high-powered money" [41, p. 394]. However, SD modeling below strictly requires them to be treated differently.

If high-powered money is approximated by the monetary base, money supply could also be estimated similar to the equation (5.12) and it is called here money supply(base).

$$\text{Money Supply (Base)} = m * \text{Monetary Base} \quad (5.14)$$

It could be used as a reference amount of money supply with which true money supply is compared (or to which true money supply converges, as it turns out below).

In a real economy, however, money supply is calculated from the existing data as follows:

$$\text{Money Supply (Data)} = \text{Currency Outstanding} + \text{Deposits} \quad (5.15)$$

It is called money supply(data) here to distinguish it from the money supplies previously defined in equations (5.2) (or 5.12) and (5.14).

In this way, we have now obtained three different expressions of money supply in the equations (5.2), (5.14), and (5.15) . It is one of the purposes of this chapter to investigate how these three expressions of money supply behave one another.

5.4 Money under Gold Standard

To examine a dynamic process of money supply and its creation, let us now construct a simple money supply model [Companion model: Money(Gold).vpm]. Without losing generality it is assumed from now on that excess reserve ratio is zero, $\beta_e = 0$ so that reserve ratio β becomes equal to required reserve ratio β_r. Vault cash of commercial banks in the model could be interpreted as excess reserves. The model is then built by assuming that the only currency available in our macroeconomic system is gold, or gold certificates (convertibles) issued by the central bank against the amount of gold. In short, it is constructed under gold standard. By doing so, we could avoid complicated transactions of discount loans by the central bank and government securities among three sectors, and focus on the essential feature of money supply per se. This assumption will be dropped later and discount loans and government securities will be introduced into the model.

Figures 5.3 and 5.4 illustrate our simple money supply model under gold standard. In the model, currency outstanding in the central bank and currency

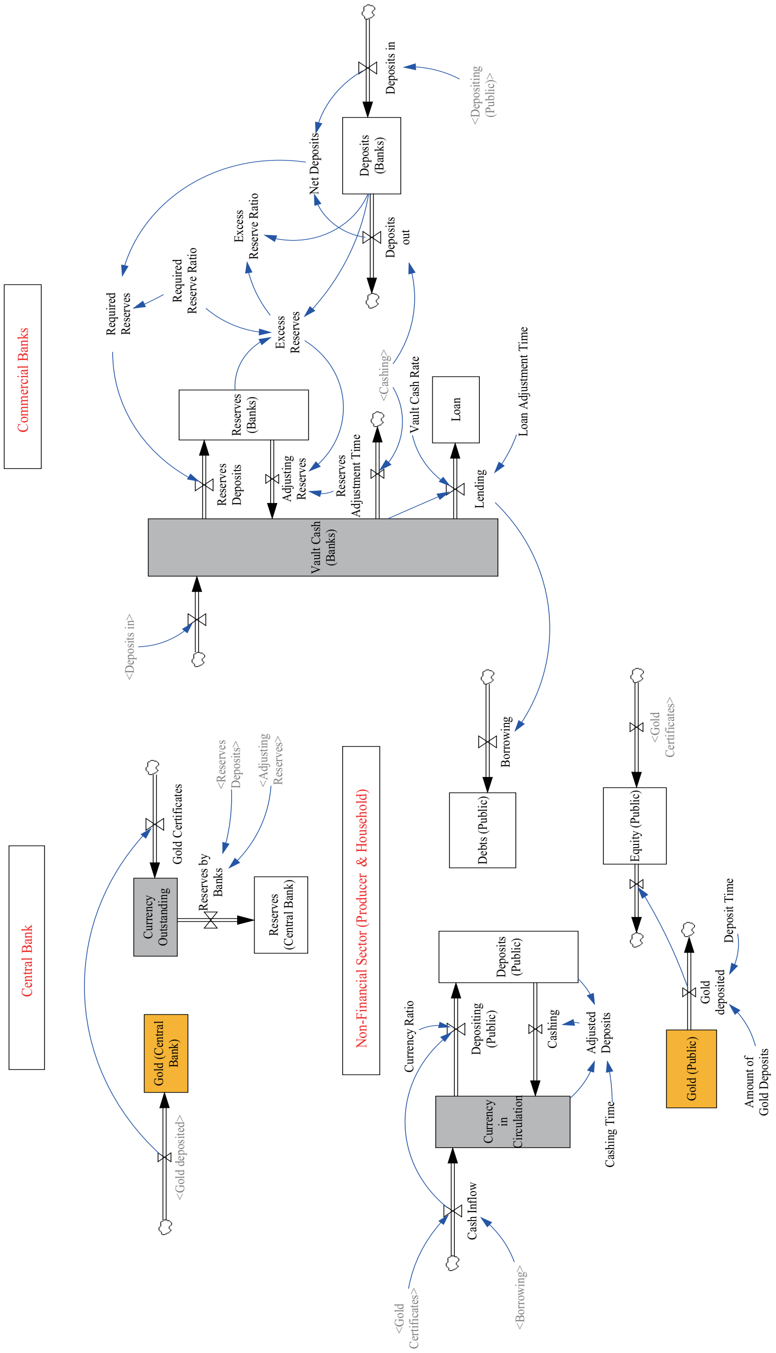

Figure 5.3: Money Supply Model under Gold Standard

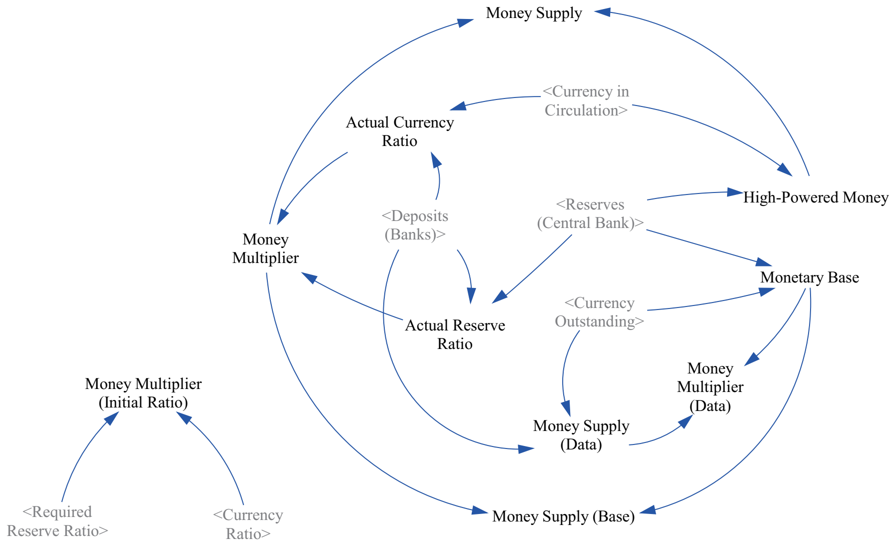

Figure 5.4: Money Supply Model under Gold Standard (Continued)

in circulation in non-financial sector are illustrated as two different stocks. Thus, they need not be identically equal as most macroeconomics textbooks treat so. This is one of the features that system dynamics modeling can precisely differentiate itself from traditional macroeconomic modeling. It is interesting to observe how these two differentiated stocks of currency and three expressions of money supply derived from them will behave in the economy.

Monetary Base vs High-Powered Money

Let us now run the model and see how it works. In the model, currency ratio and required reserve ratio are set to be $(\alpha, \beta) = (0.2,\ 0.1)$. Hence, money multiplier becomes $m = (0.2 + 1)/(0.2 + 0.1) = 4$. Meanwhile, from the balance sheet of the central bank monetary base under gold standard is always equal to the fixed amount of gold, the only assets held by the central bank, which is here set to be equal to 200 dollars. This amount of gold is also equal to the gold assets by the public. In other words, the central bank is assumed to be trusted to start its business with the gold owned by the public and issue gold certificates as bank notes against it.

From the equation (5.14), money supply(base) can be easily calculated as 800 (= 4 * 200) dollars without running the model. Meanwhile, true money supply based on high-powered money in equation (5.12) cannot be obtained without running a simulation.

Figure 5.5 illustrates our simulation result in which money supply(base), money supply(data), and money supply are represented by the lines numbered 1, 2 and 3, respectively. Monetary base and high-powered money are illustrated

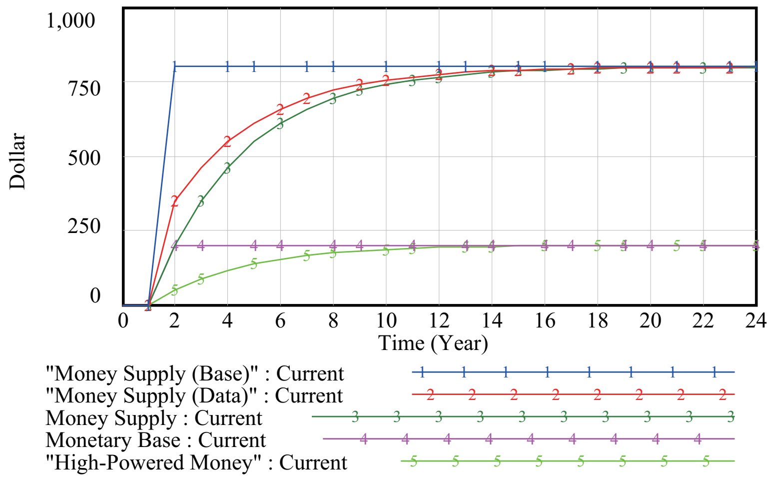

Figure 5.5: Simulation of Money Supply under Gold Standard

by the lines 4 and 5.

From the figure, two features are easily observed. First, three expressions of money supply appear to have the following orderly relation.

$$\text{Money Supply (Base)} > \text{Money Supply (Data)} > \text{Money Supply} \quad (5.16)$$

Latter part of the inequality implies that actual money supply(data) overestimates true money supply. Since money supply(data) is the only figure actually obtained by using real data of the currency outstanding (liabilities of the central bank) and deposits (liabilities of commercial banks), the overestimation of true money supply might mislead economic activities in the real economy.

Second, monetary base turns out to be greater than high-powered money.

$$\text{Monetary Base} > \text{High-Powered Money}, \quad (5.17)$$

which then leads to the following inequality from the definitions in equations (5.12) and (5.14):

$$\text{Money Supply (Base)} > \text{Money Supply}. \quad (5.18)$$

It also leads to

$$\text{Currency Outstanding} > \text{Currency in Circulation}, \quad (5.19)$$

which in turn implies

$$\text{Money Supply (Data)} > \text{Money Supply}. \quad (5.20)$$

In other words, actual money supply(data) (line 2) calculated by the central bank always overestimates true money supply(line 3) available in the economy, which, however, tends to approach to the money supply(data) eventually.

To understand the above features observed from the simulation, specifically the difference between currency outstanding and currency in circulation, let us consider the amount of currency that exists outside the central bank. From the money supply model, it is the sum of cash in circulation in the non-financial sector (consumers and producers) and cash held in the vaults of all commercial banks. All other remaining currencies in the economy would be either held by the central bank as reserves or further put into circulation through the activities of making loans by commercial banks and borrowing debts by non-financial sector. Hence, the following relation holds:

$$\text{Cash outside Central Bank} = \text{Currency in Circulation} + \text{Vault Cash(Banks)} \tag{5.21}$$

Furthermore, cash outside the central bank should be equal to cash outstanding in the balance sheet of the central bank; that is, the amount of cash that the central bank owes to its outside world (non-financial sector and commercial banks).

$$\text{Currency Outstanding} = \text{Cash outside Central Bank} \tag{5.22}$$

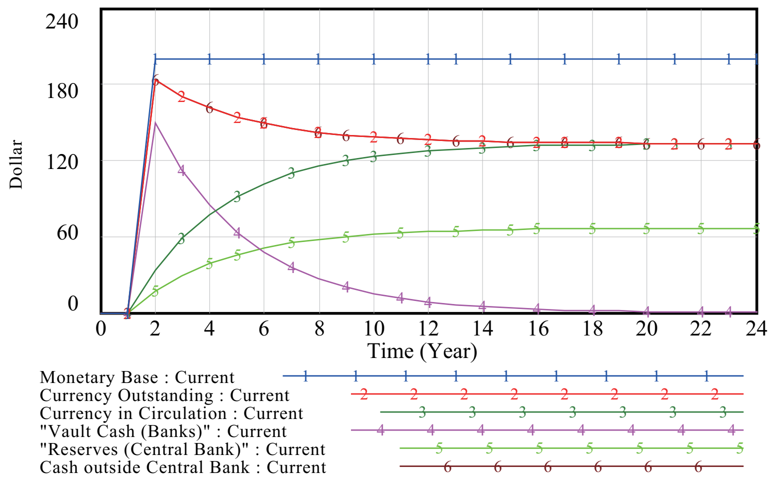

Figure 5.6: Simulation of Currency Outstanding, Cash outside Central Bank etc.

Figure 5.6 confirms that currency outstanding (line 2) is equal to the cash outside the central bank (line 6). Hence, we have correctly arrived at the equation:

$$\begin{aligned}
&\text{Vault Cash(Banks)}\\
&\qquad = \text{Currency Outstanding} - \text{Currency in Circulation}\\
&\qquad = \text{Monetary Base} - \text{High-Powered Money} > 0 \qquad (5.23)
\end{aligned}$$

which in turn leads to the above inequality relations of equation (5.17) so long as vault cash is positive.

Furthermore, it is easily proved that

$$\text{Money Supply} > \text{Money Supply (Data)}. \qquad (5.24)$$

Hence, we have

$$\text{Money Supply (Base)} > \text{Money Supply (Data)} > \text{Money Supply}. \qquad (5.25)$$

All three expressions of money supply are shown to converge as long as vault cash tends to diminish, and overestimation of money supply will be eventually corrected.

How to Create Money

Since currency ratio of 0.2 cannot be controlled, money multiplier can take the range of $6 \geq m \geq 1$. When monetary base is \$200 in our example, this implies that money supply can take the range between \$1,200 and \$200.

Loan Adjustment Time

There is a case in which such a convergence becomes very slow and overestimation of money supply remains. In Figure 5.7 loan adjustment time is assumed to triple and become 3 periods. This is a situation in which a speed of bank loans becomes slower, or commercial banks become reluctant to make loans. Accordingly, money supply might converge to money supply (data), but extremely slow. In other words, money supply will not converge to the money supply(data) for a foreseeable future and overestimation of money supply remains. Specifically, money supply(data) (line 2) is always greater than money supply (line 3) during the simulation of 24 periods.

Excess Reserves / Vault Cash

How can the amount of money supply be changed or controlled by the central bank? Under the gold standard, monetary base is always fixed, and the central bank can only influence money supply by changing a required reserve ratio. Even so, money supply may not be under the control of the central bank in a real economy. It could be affected by the following two situations. First, commercial banks may be forced to hold excess reserves in addition to the required

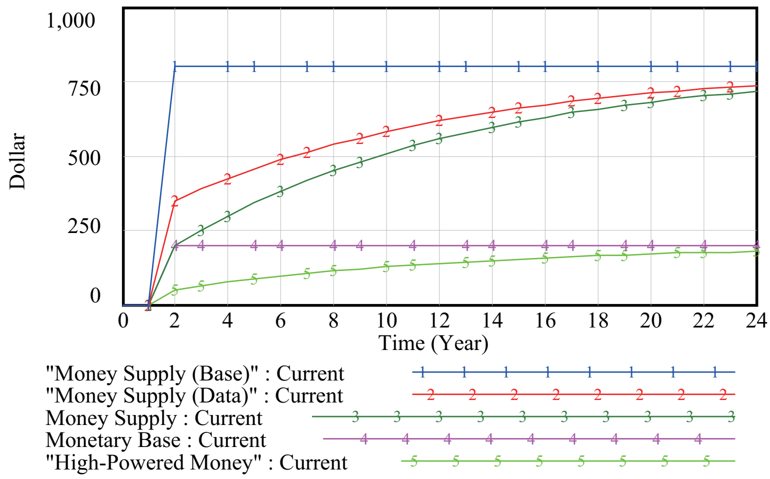

Figure 5.7: Money Supply when Loan Adjustment Time triples.

reserves due to a reduced opportunity of making loans. Second, depositors in the non-financial sector may prefer to hold cash or liquidity due to a reduced attractiveness of financial market caused by lower interest rates. Money supply will be reduced under these situations.

Let us consider the situation of excess reserves first. In our model excess reserves are stored as vault cash in the asset of commercial banks. Excess reserves are needed to an imminent demand for liquidity. Thus, commercial banks may additionally need to keep excess reserves as vault cash in their vaults. To see how excess reserves affect money supply, let us increase a vault cash rate to 0.5 from zero, so that 50% of available vault cash is constantly reserved.

As Figure 5.8 illustrates, the effect of keeping a portion of vault cash is similar to the above case of loan adjustment time. That is, three expressions of money supply converges eventually as the amount of vault cash diminishes.

Currency Ratio

Let us now consider the second situation in which non-financial sector prefers to hold more liquidity. To analyze its effect on money supply, let us assume that at t = 8 consumers suddenly wish to withhold cash by doubling currency rate from 0.2 to 0.4. Money multiplier is now calculated as $m = (0.4+1)/(0.4+0.1) = 2.8$ and money supply(base) becomes 560 dollars (= 2.8 * 200).

Figure 5.9 illustrates how money supply is reduced due to a sudden increase in liquidity preference in the non-financial sector. Three expressions of money supply tend to converge again.

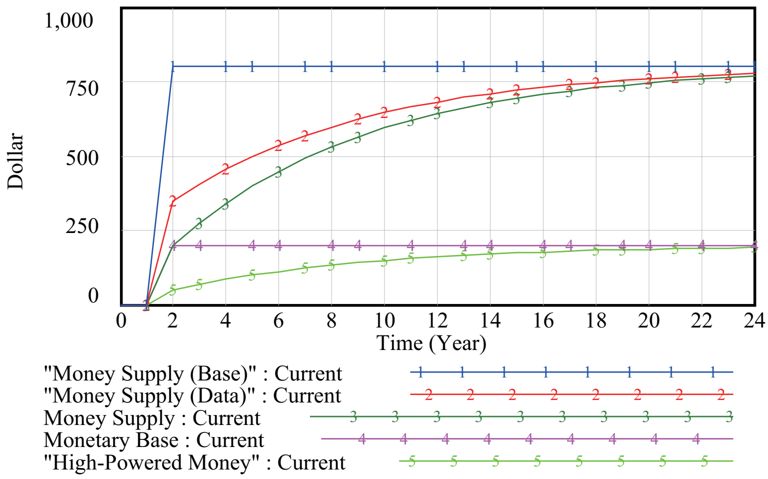

Figure 5.8: Money Supply when Excess Reserve Ratio is 0.04.

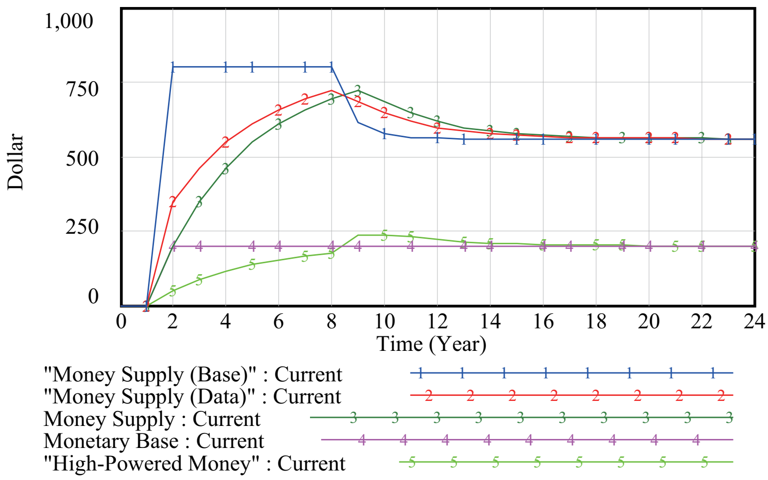

Figure 5.9: Money Supply when Currency Ratio doubles at t = 8

Assets, Equity and Money as Debts

When money is created, though *out of nothing*, non-financial sector's assets also increase from the original equity (or gold assets) of \$200 to \$800; that is, assets is increased by \$600. Does this mean that non-financial sector becomes wealthy out of the process of money creation under a *fractional reserve* banking system? Apparently, if 100% fractional reserve is required; that is, $\beta = 1$, commercial banks have to keep the same amount of deposits as deposited by the non-financial sector. Accordingly, money supply remains the same as the original gold certificates of \$200. Thus, equity, assets and money supply remain the same amount as Figure 5.10 illustrates.

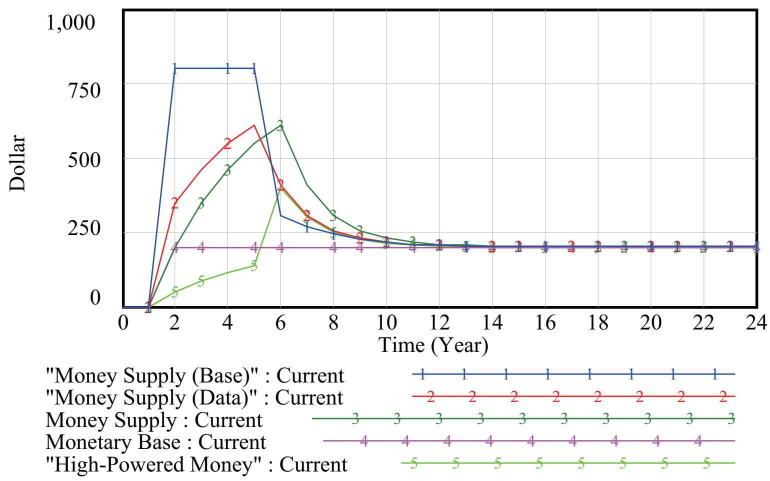

Figure 5.10: 100% Fractional Reserve

Where does the money of \$600 come from when a required reserve ratio is $\beta = 0.1$, then? There is no magic. Nothing cannot be created *out of nothing!* It comes from the non-financial sector's debts of \$600, as Figure 5.11 indicates. In other words, no debts, no money creation. Non-financial sector's *wealth* is its gold assets of \$200 as its equity. It has never been increased through this process of money creation under a fractional reserve banking system.

5.5 Money out of Bank Debt

Limit to A Gold Standard System

As already mentioned above, monetary base is fixed under gold standard. How can we increase money supply under such circumstances to meet the need for increasing transactions as our economy continues to grow? Let us ask differently. What's the maximum amount of money supply the gold standard system can provide?

From equation 5.14, under the fixed amount of monetary base, only money multiplier can change the money supply(base), and money supply accordingly.

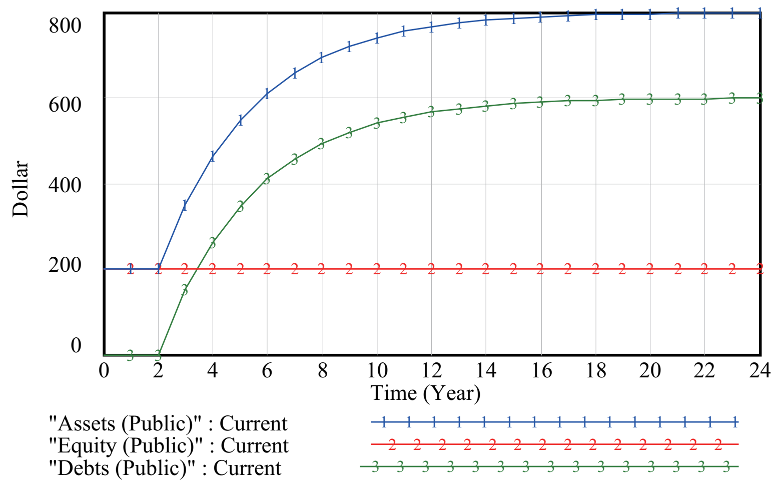

Figure 5.11: Money Supply, Assets, Equity and Debt

Since currency ratio is not under the direct control of the central bank, the only discretionary policy the central bank can exercise is a change in required reserve ratio, as already shown above. Hence, money multiplier could be maximized if required reserve ratio is set to be zero (!), and commercial banks are allowed to fully make loans of all deposits. In this case, money multiplier becomes $m = (0.2 + 1)/0.2 = 6$ and the maximum money supply(base) increases from 800 to 1,200.

If our growing macroeconomy needs more money supply, gold standard system has to be eventually abandoned as a monetary system. This is what has historically happened. It was replaced with a paper money (legal tender) system in which the central bank has a free hand of issuing bank notes as long as there exist some institutions which are eager to borrow; that is, commercial banks and the government.

A Money Creation Model with Discount Loans

Money cannot be money unless someone is willing to accept it, or someone is eager to borrow it for use as a medium of exchange. Who could be borrowers? Under the growing economy, commercial banks may be eager to borrow due to growing demand for money from non-financial sectors (producers and consumers) from the central bank at a lower interest rate, which is called a discount rate. Then they make loans to a non-financial sector at a higher interest rate to make arbitrage profits.

The other major borrower is government in the time of war and economic recession. Historically, government gave the right of printing money to the central bank, and as a result was forced to borrow from the central bank. A complete upside-down transaction.

Let us consider a money creation of the central bank out of bank debts. This process of money creation is easily modeled as an expansion of the gold standard model by adding a stock of discount loan in the assets account of the central bank, and that of debt in the liabilities account of commercial banks, as illustrated in Figure 5.12 [Companion model: Money(Loan).vpm].

Under the gold standard in the above model, the maximum amount of money supply to be created is limited to \$800 when $\beta = 0.1$. Suppose the demand for money from the economic activities is \$1,000. To meet this additional demand for money of \$200, the amount of \$50 worth of gold is further needed under the gold standard, since the economy's money multiplier is 4. Line 1 and 2 in Figure 5.13 illustrates how money supply is increased under the amount of gold deposited increases to \$250 from \$200.

Under the limitation of gold production, it eventually becomes impossible to meet the increasing demand for money by adding more gold to the currency system. Historically, currency system of gold standard has been abandoned repeatedly. To save the currency system, fiat money is forced to be introduced into circulation by giving the central bank an exclusive authority to print money.

In this case, it now becomes very easy to increase money supply. The central bank just print a paper money worth of \$50 and make loans to commercial banks, which in turn make loans to non-financial sector. Line 3 in Figure 5.13 illustrates how money supply is increased to \$1,000 when the central bank makes a loan of \$50 at the period of 6. Vice versa, money supply is contracted when the central bank retrieves loans from commercial banks. In this way, by abandoning the gold standard, the central bank has an almighty power to create money by just making a discount loan to commercial banks; a process of money creation as commercial bank debt.

For instance, according to Richard A. Werner [54], the bank of Japan used to exercise a so-called window guidance - a hidden monetary policy, in which previous presidents of the bank intentionally assigned the amount of discount loans to the commercial banks according to their own preferences. Moreover, money can buy military weapons, economic hit mans, drags, and media information to control political and economic activities in favor for those who control money supply. That is, a free-hand control of the economy is endowed to the central bank, so long as people are willing to accept printed money of the central bank notes. This implies the central bank has to be under the control of democratic government and people in exchange for getting the right to create money out of nothing.

Finally, let us examine that non-financial sector's equity or wealth remains the same at \$200, though its assets increases to \$1,000. As already examined under gold standard, the increased amount of \$800 is made available by the same amount of increase in debt of \$800 as illustrated in Figure 5.14.

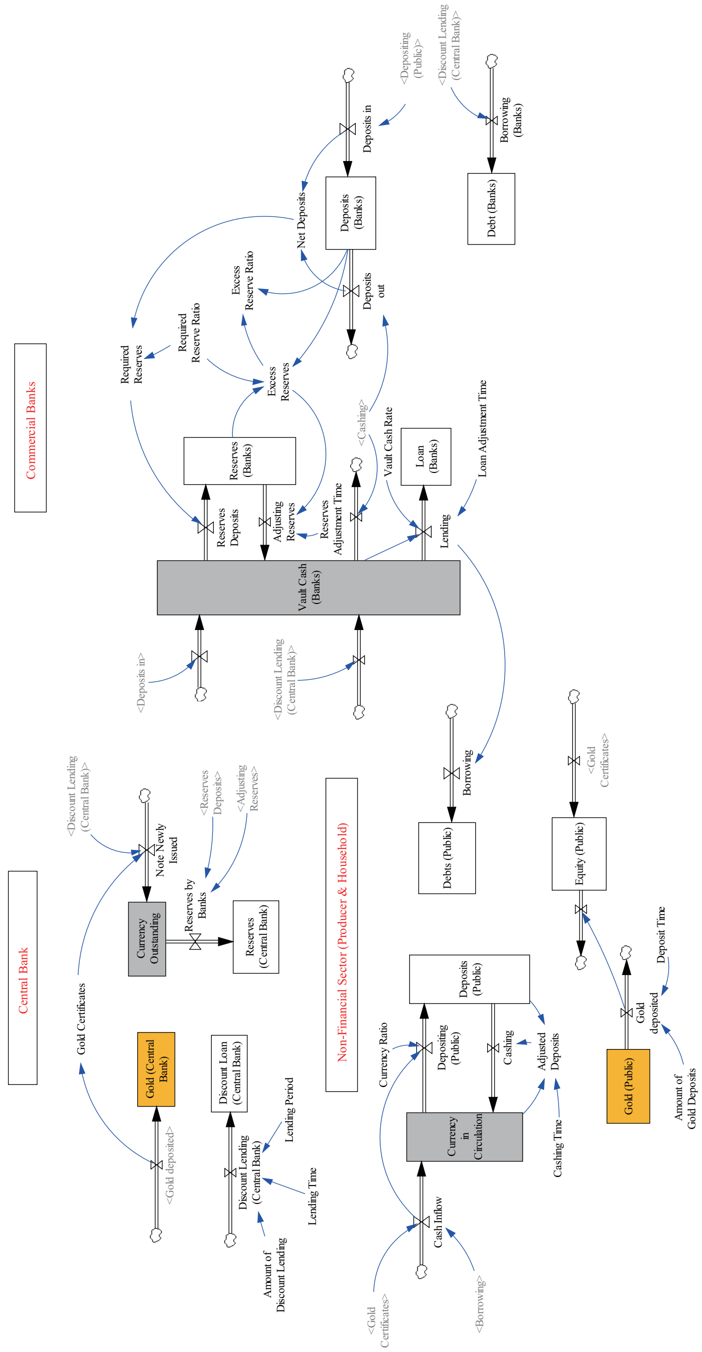

Figure 5.12: Money Creation Model out of Bank Debts

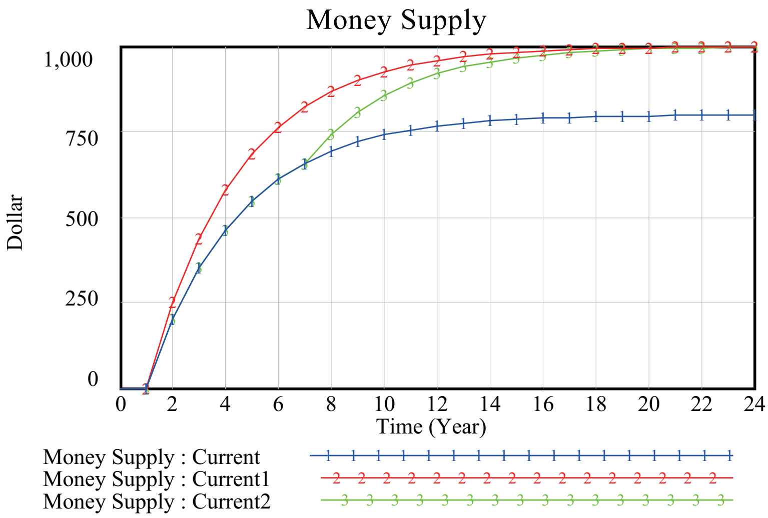

Figure 5.13: Money Creation out of Bank Debt

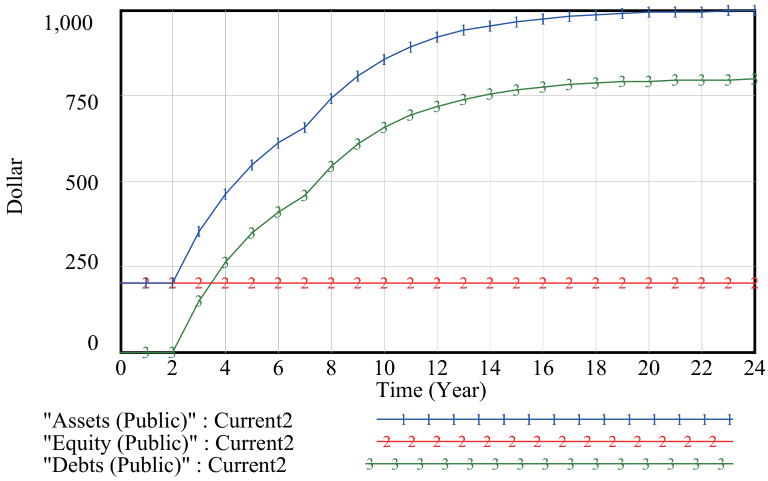

Figure 5.14: Assets, Equity and Debts

5.6 Money out of Government Debt

A Complete Money Creation Model

Let us now expand our money supply model to a further complete money supply model in which government securities are introduced and central bank can hold them as assets and issue its bank notes against them as liabilities [Companion model: Money(Fiat).vpm]. Under the introduction of government securities, our expanded money supply model becomes a little bit complicated. To avoid a further complication the model is now split into four submodels based on four different macroeconomic sectors as discussed below.

Non-Financial (Public and Government) Sectors

First, let us consider the balance sheet of non-financial sector. Due to the introduction of government securities, the non-financial sector is now split into public (consumers and producers) and government sectors.

Figure 5.15 illustrates a new non-financial sector (public), in which stock of government securities held by the public is newly added to the assets side of its balance sheet.

On the other hand, Figure 5.16 illustrates a new non-financial sector (government), in which stock of cash held by the government is added to the assets side of its balance sheet, and stocks of debts by the government and its equity(government) are added to its liabilities side.

Apparently money supply is not affected by the introduction of government securities as long as they are purchased by consumers and producers, and government spends the money it borrowed within the non-financial sector, as shown below.

Commercial Banks

Stock of government securities held by commercial banks has to be newly added to the assets side of the balance sheet of commercial banks. Now commercial banks have a portfolio choice of investment between loans and investment on government securities. Figure 5.17 illustrates a sector of commercial banks.

Money supply is not affected by the introduction of government securities as long as they are purchased by commercial banks and government spends the money it borrowed within the non-financial sector, as shown below.

Central Bank

Stock of government securities held by the central bank has to be newly added to the assets side of the balance sheet of the central bank. It can now purchase government securities with its newly issued bank notes in the case of securities held by the public, and by increasing the same amount of figure to the reserves account of the commercial banks in the case of securities held by the commercial banks[4]. These transactions increase the same amount of the central bank's liabilities such as currency outstanding and reserves, and accordingly monetary base.

[4] Direct purchase of government securities, or direct loan to government by the central bank is prohibited in Japan. Therefore, such purchases has to be indirectly done through market.

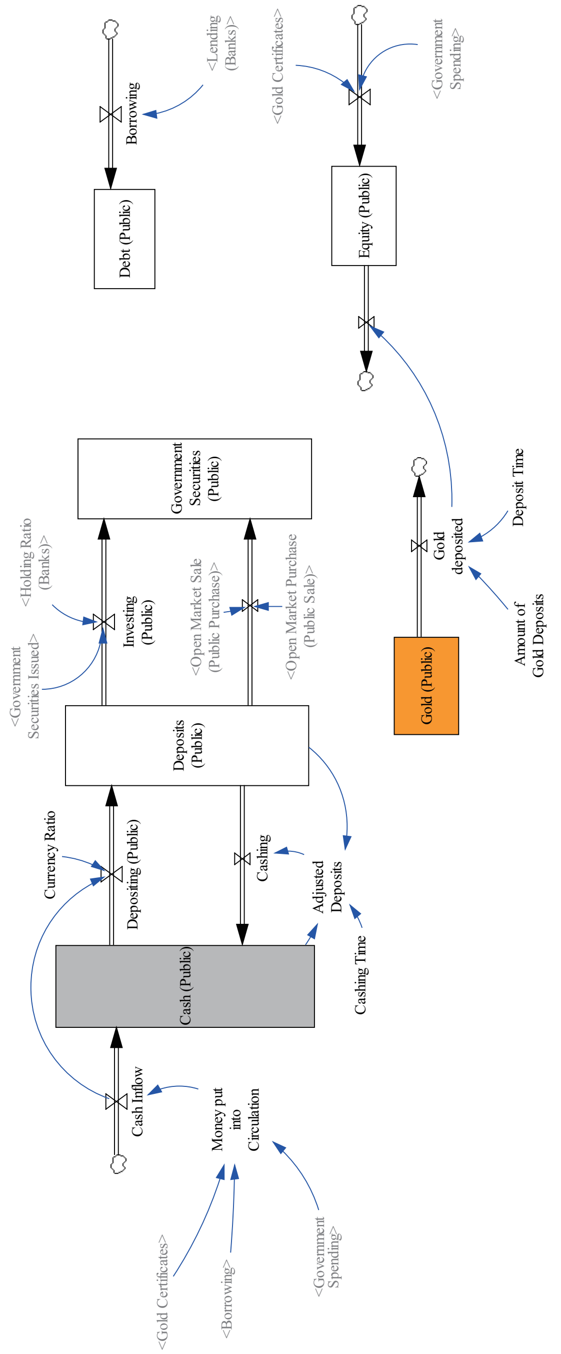

Figure 5.15: A Complete Money Creation Model: Non-Financial Sector (Public)

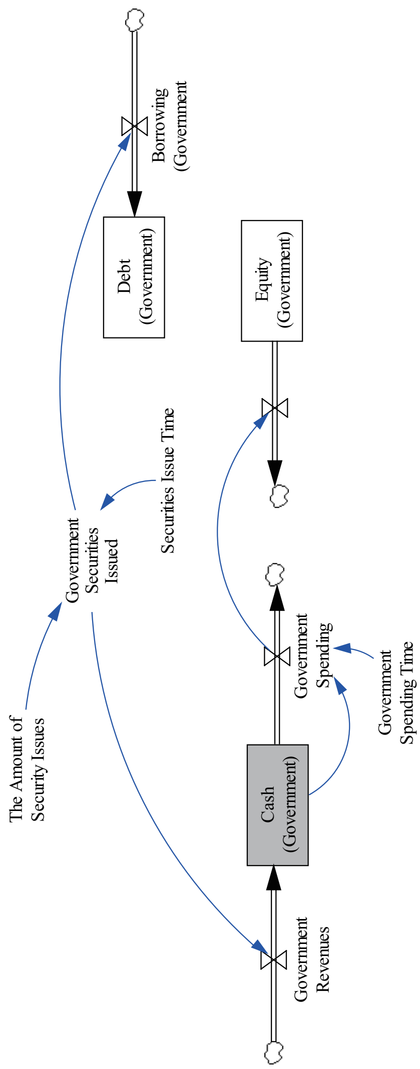

Figure 5.16: Non-Financial Sector (Government)

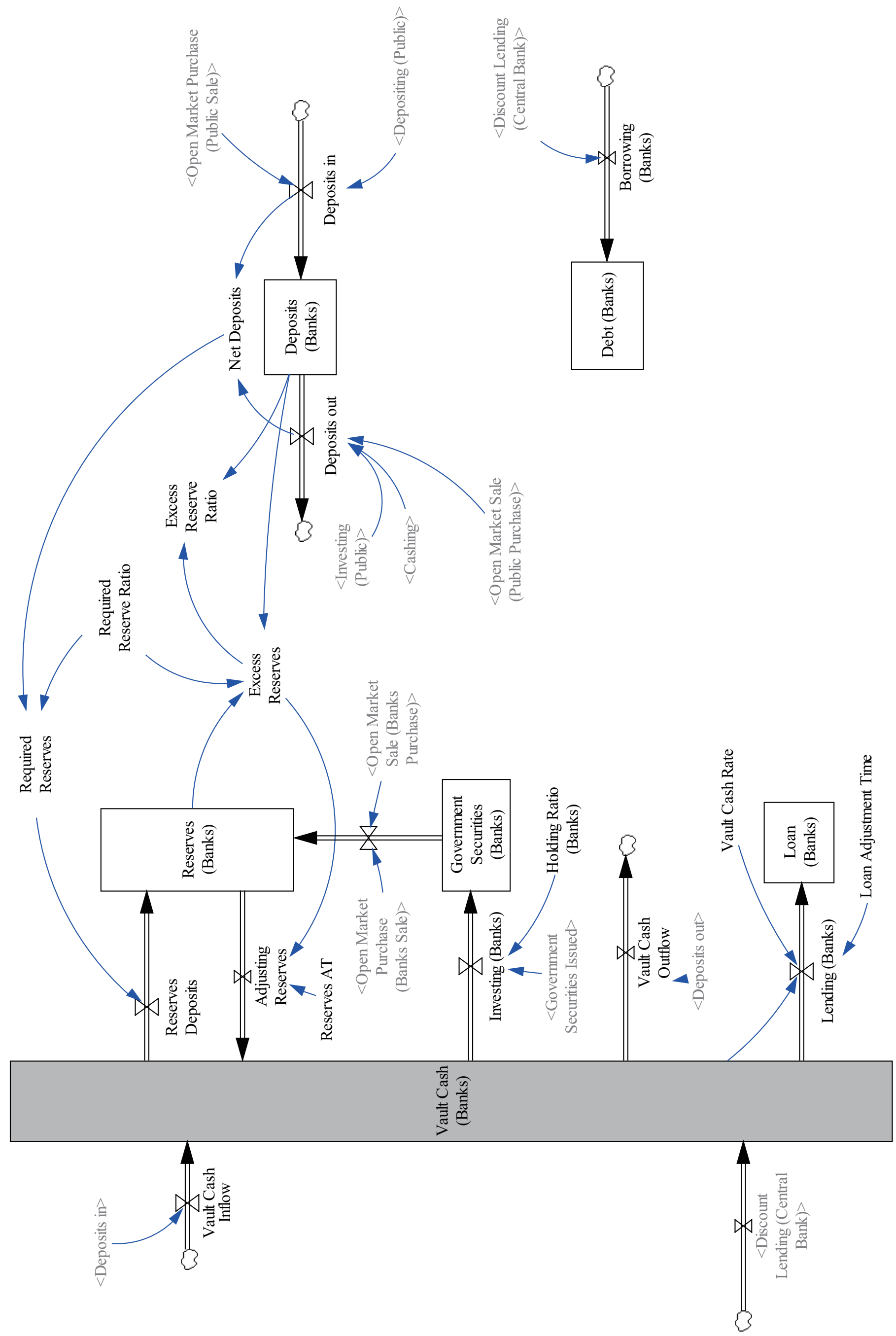

Figure 5.17: A Complete Money Creation Model: Commercial Banks

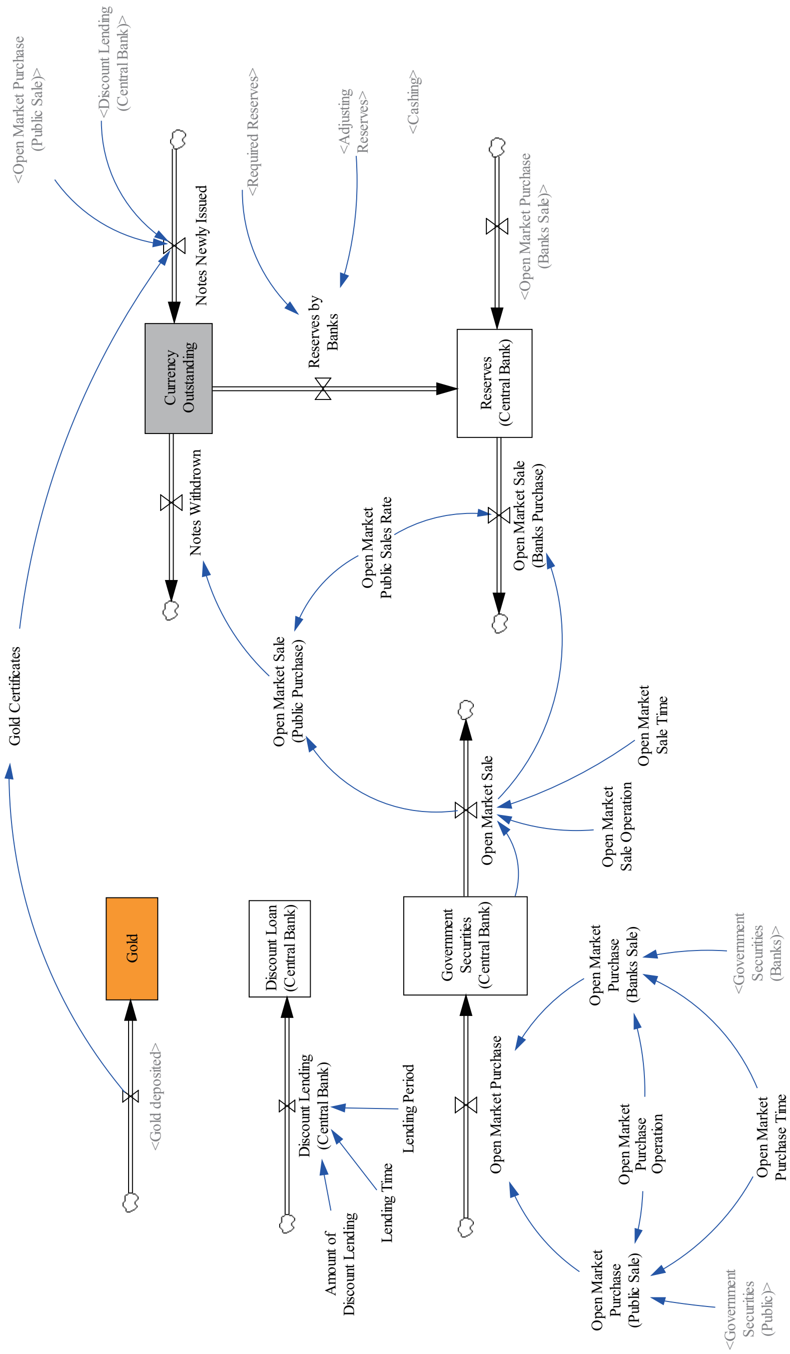

Figure 5.18: A Complete Money Creation Model: Central Bank

If it sells government securities to the non-financial sector (public) and commercial banks, currency in circulation is withdrawn back to the central bank, decreasing its currency outstanding, and reserves by commercial banks are reduced by the same amount, and hence monetary base as a whole. Purchase and sale of government securities by the central bank are known as open market operations. Figure 5.18 illustrates these operations of the central bank. In this way, with the introduction of government securities, the central bank has a discretionary control of monetary base and money supply.

Open Market Operations

Let us now see how our complete money creation model works. Figure 5.19 illustrates how open market operations affect the behavior of money supply. It is assumed that government issues securities (and borrow money) of 100 dollars at the period t = 3, 30% of which are assumed to be purchased by the public (consumers and producers) and 70% by commercial banks. Yet, this does not change money supply, which continues to converge to 800 dollars as before (lines 2 & 3), though money supply (base) in line 1 temporarily drops.

At the period t = 10 central bank purchases 50% of government securities held by the public and commercial banks through open market purchase operation. Accordingly, monetary base (line 4) is now increased from the original 200 dollars to 250 dollars. Since money multiplier is obtained as $m = (0.2+1)/(0.2+0.1) = 4$, money supply(base) (line 1) now starts to increase to 1,000 dollars (= 4 * 250), while money supply(data) and money supply (lines 2 & 3) also continue to grow.

At the period t = 20 the central bank sells 50% of the government securities it holds, and monetary base decreases to 225 dollars. Money supply(base) (line 1) is now reduced to converge to 900 dollars (= 4 * 225). Money supply(data) (line 2) slightly reduces at that period, but then continues to grow, while money supply (line 3) only continues to grow. Eventually three expressions of money supply converge to 900 dollars as shown in Figure 5.19.

In this way, the central bank can increase or decrease money supply by its discretionary monetary policy of open market operations, and theoretically there exists no ceiling or upper boundary of money supply.

Even so, there is a case in which the central bank cannot control money supply. Figure 5.20 illustrates the case in which a currency ratio is additionally doubled from 0.2 to 0.4 at t = 23. Money multiplier is now calculated as $m = (0.4 + 1)/(0.4 + 0.1) = 2.8$, and money supply(base) becomes 630 dollars (= 2.8 * 225); a reduction of money supply by 270 dollars. Three expressions of money supply all converge to this reduced amount.

Finally, let us examine how the above open market operations and a change in currency ratio affect assets (line 1), equity (line 2) and debt (line 3) of non-financial sector (public). As expected from the above analyses, it is again confirmed that the equity or wealth of non-financial sector is not affected at all as illustrated in Figure 5.21. All changes in assets are balanced by debt.

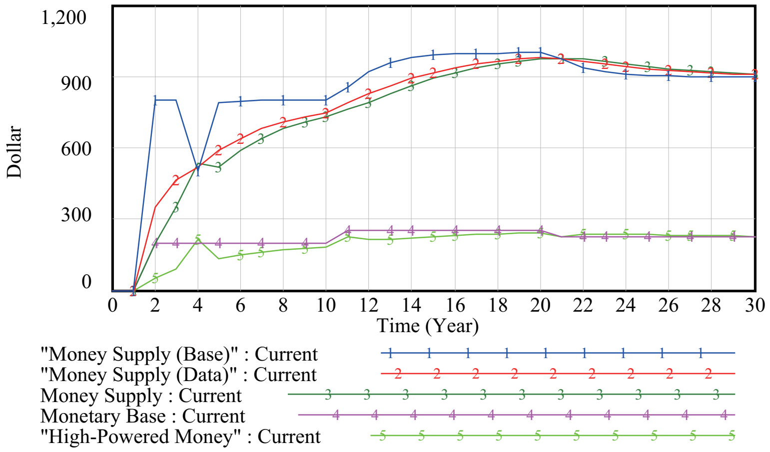

Figure 5.19: Simulation of Open Market Operation

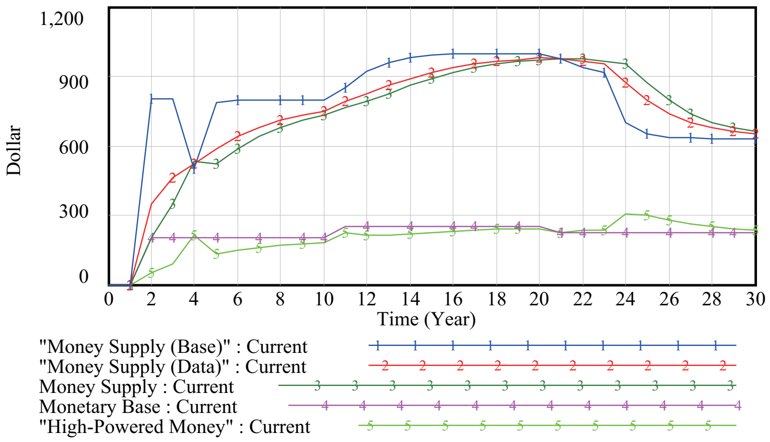

Figure 5.20: Open Market Operation with Currency Ratio Doubled

Conclusion

From the overview of macroeconomic system, six sectors are rearranged to three sectors: central bank, commercial banks and non-financial sector. Then a simple model of money supply under gold standard is constructed to examine some essential features of money supply and creation of deposits. This modeling process inevitably requires a distinction between currency in circulation and

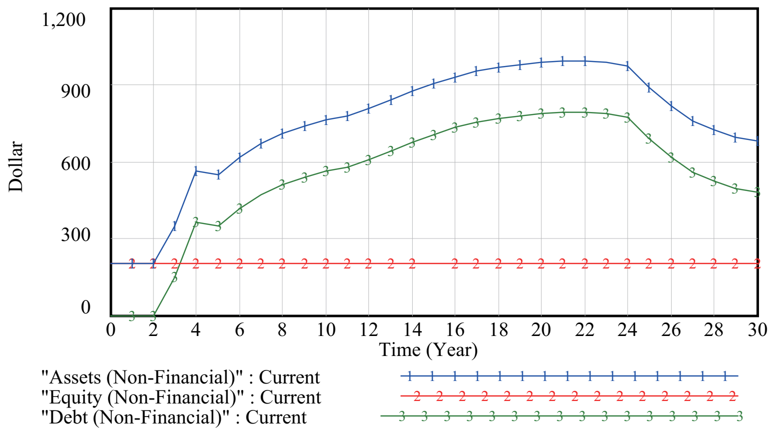

Figure 5.21: Assets, Equity and Debt of Non-Financial Sector (Public)

currency outstanding, and accordingly high-powered money and monetary base that are traditionally treated equivalently in macroeconomics.

Three expressions of money supply are derived from the above distinction, and it is shown that money supply(data) obtained from actual economic data tends to overestimate true money supply based on high-powered money. It is also shown that three expressions of money supply tend to converge one another as long as vault cash held by commercial banks diminishes.

The model is then expanded to create money out of debts by commercial banks and government by introducing the central bank's discount loan and government securities. Accordingly, the central bank can exercise a discretionary control over money supply through its lending and open market operations.

It is important to understand how non-financial sector's assets, equity and debts are being affect by the process of money creation. The equity or wealth of non-financial sector is not affected whether money is created under gold standard or as debt by commercial banks or government. Increasing assets due to money creation are always balanced by the same amount of increase in debts!

Questions for better monetary systems

- Who owns the central bank in your country?
- Creation of credits (a source of control) depends on the fractional baning system; that is, a reserve ratio $0 \leq \beta \leq 1$. If $\beta = 1$, what will happen?
- Do we really need the central bank to manage money supply and run the economies? Without the central bank, what problems will arise?

- If the central bank is owned by the government, what alternatives could be to the current open market operations? Do they work?

Chapter 6

Interest and Equity

This chapter tries to analyze how money supply is affected by the introduction of interest. It is assumed that bank loans, deposits and discount loans are no longer interest-free, and different interest rates are applied to them. As a result, money supplies turn out to be increased due to a change in currency ratio. More importantly, it is found that equity tends to be distributed in favor of commercial banks and the central bank.

6.1 What is Interest?

In the previous chapter, it is argued that money is created as debt by non-financial sectors such as public and government, and commercial banks under a fractional reserve banking system. If money is created this way to meet the growing demand for economic transactions as a medium of exchange, the banking system becomes essential sector for economic activities.

Debt is, however, not free in our economy. When non-financial sector borrows money from commercial banks, they have to pay interest. In other words, commercial banks charge interest to make loans. What is interest, then? According to a typical macroeconomic textbook, "Interest is the price for the use of money. It is the price that borrowers need to pay lenders for transferring purchasing power to the future (259 page in [40]).

If extra money is sitting idle at hand without a specific plan to be used in the near future, why can't we let someone in need of medium of exchange use it free of charge? As a matter of fact, usury has been historically prohibited. Yet greedy bankers began to charge interest when loan were made. Eventually, to secure more fund for loans from those who have extra money in non-financial sector, bankers began to attract those extra money at interest. And in a capitalist market economy, as the above quotation of the textbook justifies, no one now doubts that "interest is the price for the use of money."

Since system dynamics is a method for designing a better system, it's worth while to consider whether it's possible to design an economy free of interest charge. To examine this question, let us expand our models of money creation in the previous chapter one by one.

6.2 Money and Interest under Gold Standard

For the expansion of the models [Companion model: Money-Interest(Gold).vpm], let us introduce two types of interest rates. When commercial banks get deposits, they pay an *interest rate* per dollar deposit to non-financial sector. On the other hand, when they make loans to non-financial sector, they charge a higher interest rate called *prime rate* per dollar loan. The difference becomes a major source of income by the commercial banks. In this way, two different prices of interest rates begin to be introduced upon the introduction of commercial banks. Interest rate and prime rate are set here to be 2% and 3%, respectively

The receipts of interest become interest incomes and treated as inflows to the equity, while its payments become interest expenses and booked as outflows from the equity. Figure 6.1 reflects these transactions and becomes a revised model of money and interest under gold standard.

Under the introduction of interest, monetary base is not affected since gold held by the central bank does not change, Currency in circulation drops from $133 to $114, and non-financial sector's deposits increases from $665.8 to $764, resulting in the decrease in actual currency ratio from 0.2 to 0.15. Actual reserve ratio remains at 0.1. Accordingly, money multiplier increases from 4 to 4.6. and money supply increases from $799 to $878 at the period 24 as illustrated in the left-hand diagram of Figure 6.2, though high-powered money decreases slightly from $200 to $190. In this way, an introduction of interest has a positive effect to increase money supply through the decrease in actual currency ratio.

A more drastic change under the introduction of interest is observed in the distribution of equity between non-financial sector and commercial banks. The amount of equity in the non-financial sector begins to decline from $200 to $84.65, while that of the commercial banks increases from zero to $115.35 as illustrated in the right-hand diagram of Figure 6.2.

Since no production is assumed in this simple economy of gold standard, its only equity or net assets is the $200 dollars's gold held by the non-financial sector, which remains the same through the process of money creation. The introduction of interest causes the economy's equity to be redistributed between non-financial sector and commercial banks. In other words, the commercial banks can exploit non-financial sector's equity, no matter how positive interest payments please depositors in the non-financial sector. This is the essence of the introduction of interest to the monetary economy.

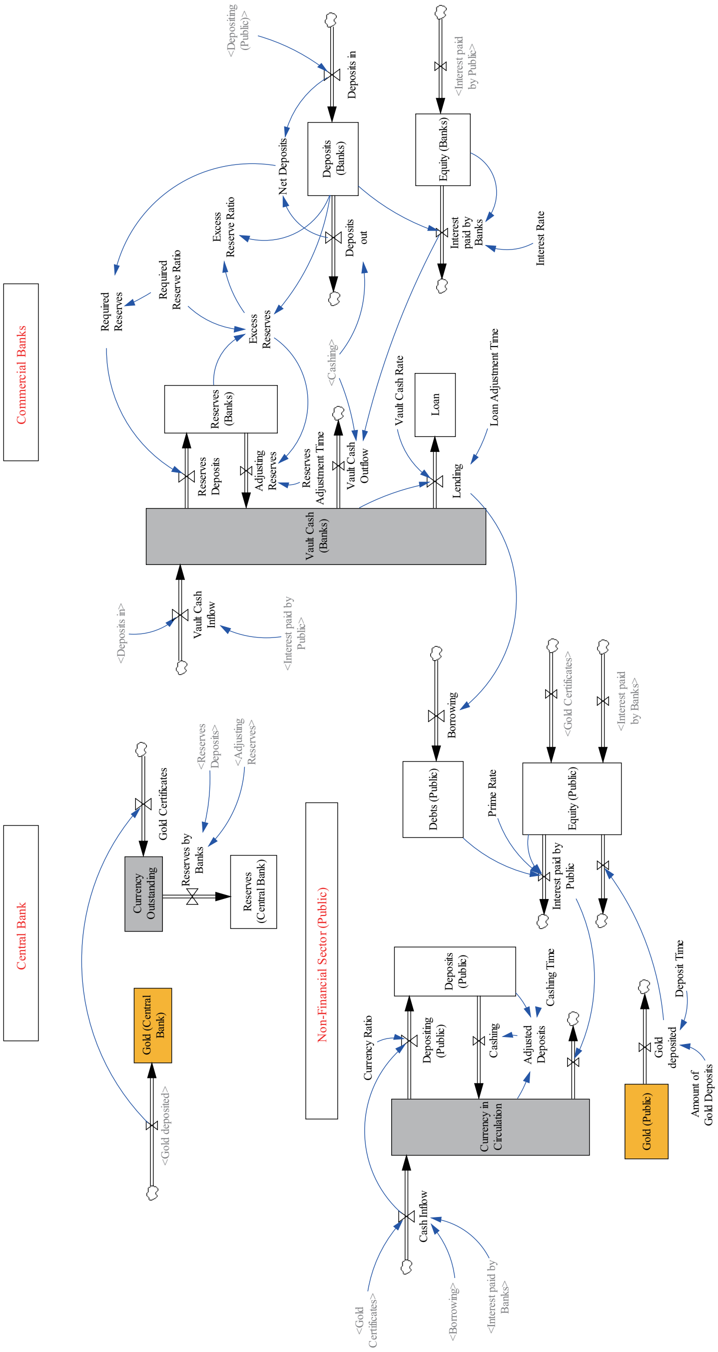

Figure 6.1: Money and Interest under Gold Standard

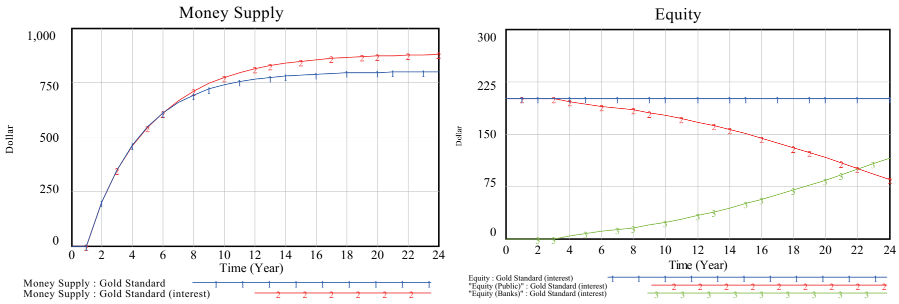

Figure 6.2: Money Supply and Equity under Gold Standard

6.3 Money and Interest under Bank Debt

What happens if the central bank make a discount loan of $50 to commercial banks, as in the previous chapter? To examine this effect, let us assume that the discount rate charged by the central bank is 0.01 [Companion model: Money-Interest(Loan).vpm]. In this case, money multiplier remains almost the same, yet high-powered money increases from $190 to $230. Accordingly, money supply increases from the above $878 to $1,068 (in the previous chapter it was $1,000) as illustrated in the left-hand diagram of Figure 6.3.

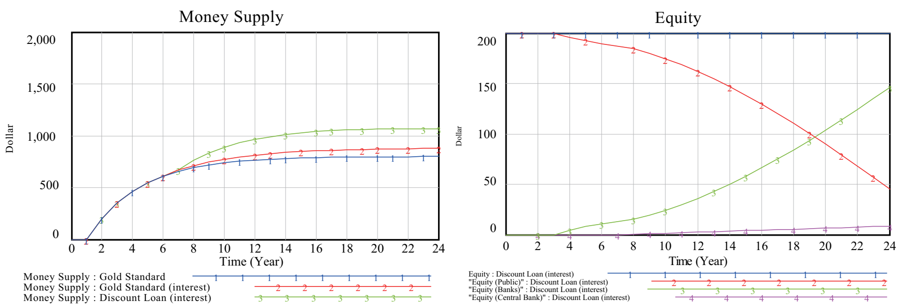

Figure 6.3: Money Supply and Equity under Banks Debt

The amount of equity in the non-financial sector begins to decline from the above $84.65 to $45.2 while that of the commercial banks increases from the above $115.35 to $146. Moreover, the equity of the central bank increases to $8.5 as illustrated in the right-hand diagram of Figure 6.3.

Left-hand diagram of Figure 6.4 shows how the original equity of $200 in the non-financial sector has been further reduced (line 3) when discount loan is made to commercial banks, compared with the case under gold standard (line 2). Right-hand diagram indicates how commercial banks, as well as the central bank, benefits (line 3) compared with the case under gold standard (line 2). In other words, the central bank can not only obtain a portion of the economy's equity through the discount loan, but also help commercial banks to exploit

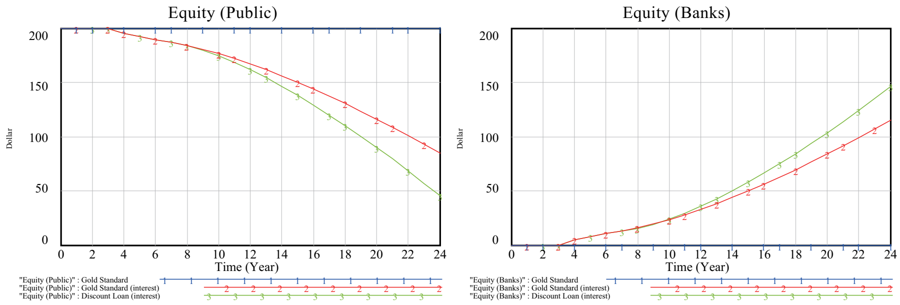

Figure 6.4: Equity Distribution between non-financial sector and Banks

non-financial sector's equity furthermore!

6.4 Money and Interest under Government Debt

Let us now expand the above model to the economy where government is allowed to borrow by newly issuing its securities [Companion model: Money-Interest(Fiat).vpm]. Security interest is assumed to be the same as the interest rate of 2% non-financial sector (public) receives for its deposits. Under this new environment, the same activities are assumed as in the section of "Open Market Operations" of the previous chapter. To repeat, government issues securities (and borrow money) of 100 dollars at the period t = 3, 30% of which are assumed to be purchased by the public (consumers and producers) and 70% by commercial banks. At the period t = 10 central bank purchases 50% of government securities held by the public and commercial banks through open market purchase operation. At the period t = 20 the central bank sells 50% of the government securities it holds, and monetary base decreases to 225 dollars.

Under such situation, money supply is illustrated as line 4 in the left-hand diagram of Figure 6.5. Lines 2 and 3 are money supplies under gold standard and bank debt as discussed above for comparison. Right-hand diagram shows the new distribution of equity among non-financial sectors (public and government), commercial banks and central bank.

Let us take a closer look at the equity distribution of non-financial sectors (public and government). Left-hand diagram of Figure 6.6 illustrates the changes of public sector's equity. Compared with the previous cases (lines 2 and 3), government borrowing jumps the public equity as illustrated in line 4, which is caused by the government spending in the non-financial sector (public). This seems that government borrowing increases the public equity. It is, however, balanced by the decrease in the government equity of -$100 as shown in the right-hand diagram. (Only government is allowed to suffer from negative equity!) To be worse, this increased public equity continues to decline as line 4 of the left-hand diagram indicates.

To sum, nothing new has happened to the distribution of equity when gov-

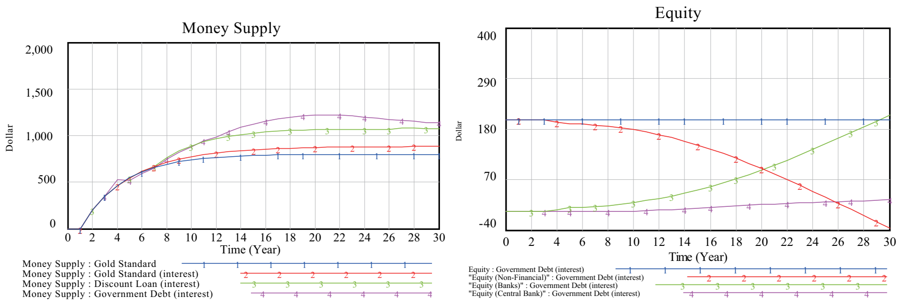

Figure 6.5: Money Supply and Equity under Government Debt

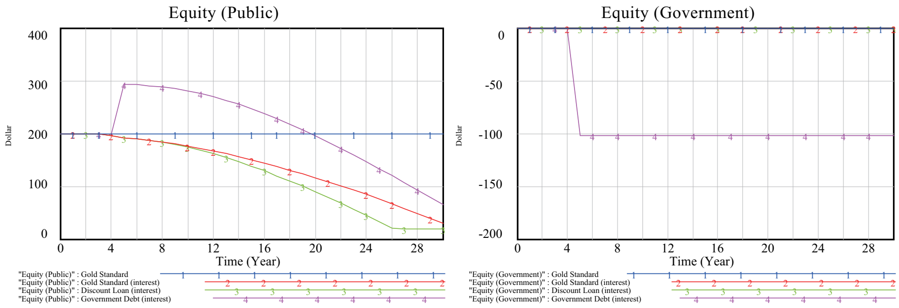

Figure 6.6: Public and Government Equities under Government Debt

ernment borrows. To see this, let us combine public and government equities as non-financial equity. Left-hand diagram of Figure 6.7 illustrates that consolidated non-financial sector's equity distribution (line 4) behaves similar to the line 3 under discount loan, except that this time non-financial sector suffer from the negative equity because of the government debt. Meanwhile, the central bank's equity (line 4) continues to rise more than the case under discount loan (line 3) as the possession of government securities as its assets increases as shown in the right-hand diagram.

6.5 Interest and Sustainability

The introduction of interest always plays in favor of commercial banks and the central bank in terms of the equity distribution. This is a negative side of the coin. The positive side is that through the banking system wit interest, non-financial sector obtains enough money for productive investment that enables economic growth and eventually an increase in non-financial sector's equity.

The fundamental question is whether this increase in the non-financial sector's equity is large enough to compensate the exploitation of its equity by banking system. In system dynamics, this financial (interest) system of deposits and debts can be described by a simple model illustrated in Figure 6.8.

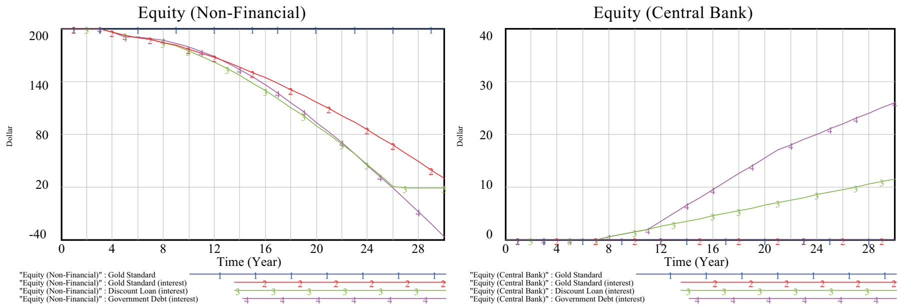

Figure 6.7: Non-Financial and Central Bank Equities under Government Debt

That is, this financial system guarantees the infinite inflow of interest to the owner of deposits and lenders.

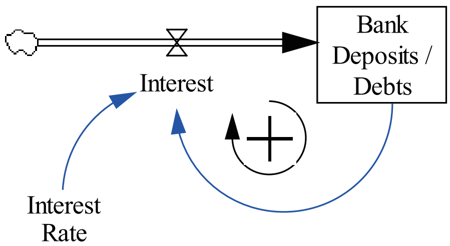

Figure 6.8: Financial (Interest) System of Deposits/ Debts

This is nothing but the example of exponential growth explained in chapter 1. And the reader can remember its power with a built-in doubling time. In other words, this financial system makes the haves richer and richer. Once we are enslaved with debt, we are forced to work indefinitely to attain endless economic growth for the payments of interest if we want to avoid the decline in our equity values. Otherwise, as we have seen above, our equity eventually will be totally exploited by bankers. In other words, considering the power of exponential growth, this financial system of distorted equity distribution does not work consistently, and its re-settings eventually need be necessary with financial and economic crises and wars as our economic history indicates.

This may lead to our ultimate question: Can the resettings together with indefinitely forced economic growth work well for attaining a sustainable economy under a finite world of resources? The answer seems to be negative. Accordingly it is always expedient to think about the option of designing an interest-free economy as a system designer. We'll challenge this in chapter 12. Until we arrive to that chapter, let us continue to model our capitalist market macroeconomy by focusing on the production side in the chapters to follow.

Conclusion

To create money supply out of a limited monetary base, banking system of deposits and debts play a crucial role. This system seems to be better managed if interest is introduced. We have examined how positive interest rate affect money supply under the systems of gold standard, discount loan and government debts. It is analyzed that in all cases money supplies are further increased. It is also found that in this process of money creation equity distribution is made in favor of the commercial banks and the central bank. In other words, non-financial sector's equity will be completely exploited unless economic growth is attained with debts as investment.

Chapter 7

Aggregate Demand Equilibria

This chapter [1] tries to model dynamic determination processes of GDP, interest rate and price level on the same basis of the principle of accounting system dynamics. For this purpose, a simple Keynesian multiplier model is constructed as a base model for examining a dynamic determination process of GDP. It is then expanded to incorporate the interest rate, whose introduction enables the analysis of aggregate demand equilibria as well as transactions of savings and deposits, and government debt and securities. Finally, a flexible price is introduced to adjust an interplay between aggregate demand equilibrium and full capacity output level. A somewhat surprise result of business cycle is observed from the analysis.

7.1 Macroeconomic System Overview

System dynamics approach requires to capture a system as a wholistic system consisting of many parts that are interacting with one another. Specifically, macroeconomic system has been viewed as consisting of six sectors such as the central bank, commercial banks, consumers (households), producers (firms), government and foreign sector, as illustrated in Figure 4.1 in chapter 4. It shows how these macroeconomic sectors interact with one another and exchange goods and services for money.

In the previous analysis of money and its creation, these six sectors are regrouped into three sectors: the central bank, commercial banks and non-financial sector consisting of consumers, produces and government. And government is separated in a later analysis. For the analysis of aggregate demand and supply in this chapter, we need at least four sectors such as producers,

[1]This chapter is based on the paper: Aggregate Demand Equilibria and Price Flexibility – SD Macroeconomic Modeling (2) – in "Proceedings of the 23rd International Conference of the System Dynamics Society", Boston, USA, July 17 - 21, 2005. ISBN 0-9745329-3-2.

consumers, banks and government. Since money supply is assumed to be exogenously determined in this chapter, central bank is excluded. Our analysis is also limited to a closed macroeconomic system and foreign sector is not brought to the discussion here. Figure 7.1 illustrates the overview of the standard macroeconomy in this chapter.

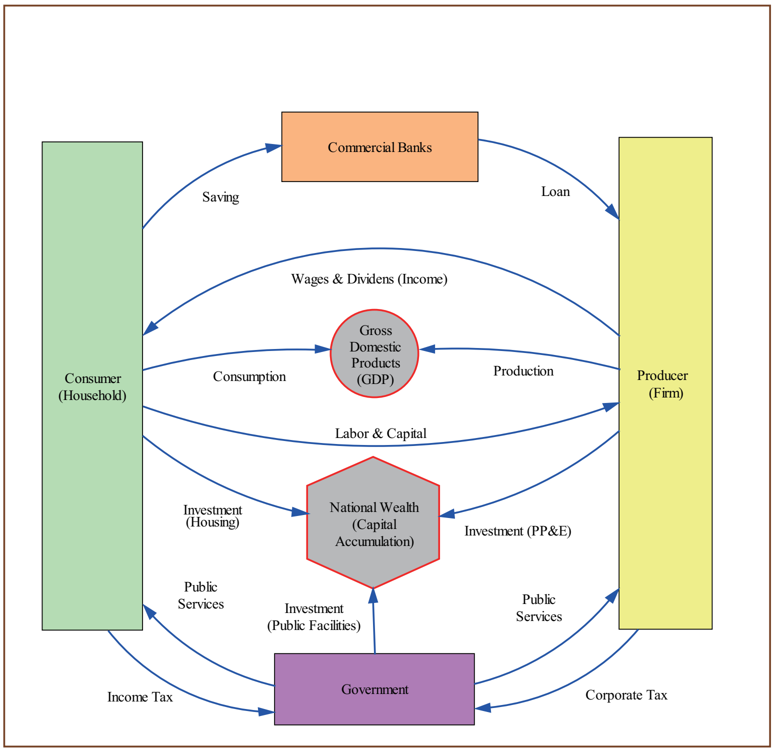

Figure 7.1: Macroeconomic System Overview

How can we describe transactions among four sectors, then? The method we employ here is the same as the one used in the previous chapters; that is, the use of financial balance sheet. Balance sheet is an accounting method of keeping records of all transactions in both credit and debit sides so that they are kept in balance all the time.

7.2 A Keynesian Model

Since macroeconomics is one of the major subjects in economics, many standard textbooks are in circulation. As references, textbooks such as [7], [39], [40], and [41] are occasionally used to examine a standard approach to macroeconomics.

A simple Keynesian macroeconomic model is described as follows.

$$Y = AD \quad \text{(Determination of GDP)} \tag{7.1}$$

$$AD = C + I + G \quad \text{(Aggregate Demand)} \tag{7.2}$$

$$C = C_0 + cY_d \quad \text{(Consumption Decisions)} \tag{7.3}$$

$$Y_d = Y - T - \delta K \quad \text{(Disposable Income)} \tag{7.4}$$

$$T = \bar{T} \quad \text{(Tax Revenues)} \tag{7.5}$$

$$I = \bar{I} \quad \text{(Investment Decisions)} \tag{7.6}$$

$$G = \bar{G} \quad \text{(Government Expenditures)} \tag{7.7}$$

$$\frac{dK}{dt} = I - \delta K \quad \text{(Net Capital Accumulation)} \tag{7.8}$$

$$Y_{full} = F(K, L) \quad \text{(Production Function)} \tag{7.9}$$

$$Y_{full} = Y \quad \text{(Equilibrium Condition)} \tag{7.10}$$

This macroeconomic model consists of 10 equations with 9 unknowns; that is, $Y_{full}, Y, K, AD, C, I, G, Y_d, T$, with 7 exogenously determined parameters $(L, C_0, c, \bar{T}, \bar{I}, \delta, \bar{G})$.[2] Obviously, one equation becomes redundant. A possible redundant equation is equations (7.1) or (7.10). Which equation should be deleted from the analysis of macroeconomic model?

According to the neoclassical view, supply creates its own demand in the long run, and in this sense the equation (7.1) becomes redundant. Left-hand diagram of causal loops in Figure 7.2 illustrates how full capacity supply and aggregate demand are separately determined without the equation(7.1). Therefore, in order to complete this neoclassical logic, we need to add another equation of price mechanism which adjusts discrepancies between Y_{full} and AD such as [3]

$$\frac{dP}{dt} = \Psi(AD/P - Y_{full}). \tag{7.11}$$

[2]In this model, demand for and supply of labor, L, is not analyzed. To do so we need to add another equation of population (labor) growth such as

$$\frac{dL}{dt} = nL$$

which will be done in the chapters to follow.

[3]Whenever price is explicitly introduced, all variables have to be expressed (or interpreted) as real values.

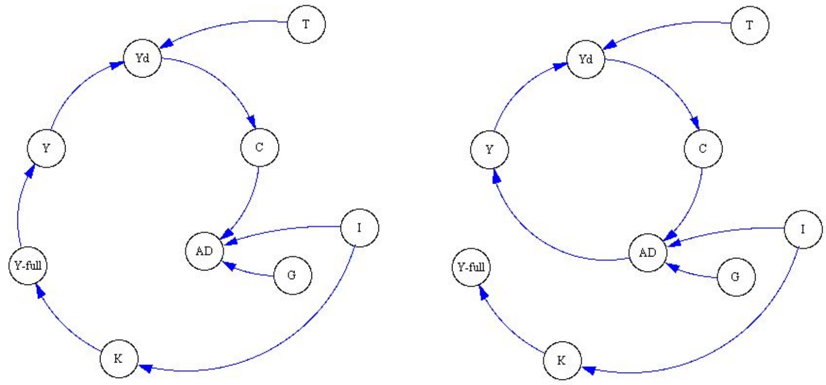

Figure 7.2: Causal loops of Neoclassical and Keynesian Models

In this way, we have 10 unknown variables and 10 equations. The equilibrium attained this way is called neoclassical long-run equilibrium.

On the other hand, according to a Keynesian view GDP is determined by the aggregate demand in the short-run. In this sense, the equation (7.10) becomes redundant. Right-hand diagram shows that GDP is determined by the aggregate demand without the equation (7.10). In this case, the level of GDP is nothing but equal to the level of aggregate demand, and needs not be the same as the amount of output produced by the economy's production function (7.9). Contrary to the neoclassical view, the economy has no autonomous mechanism to attain an equilibrium in which output produced by the equation (7.9) is equal to the aggregate demand; that is, a neoclassical long-run equilibrium. This is because price is regarded as sticky in the short-run, and cannot play a role to adjust a discrepancy between aggregate supply of output and aggregate demand. Hence, Keynesian economists argue that such a neoclassical long-run equilibrium could only be attained in the short run through changes in aggregate demand made possible by monetary and fiscal policies.

Can we create a synthesis model to deal with these controversies between neoclassical and Keynesian schools[4]? From a system dynamics point of view, macroeconomy is nothing but a system and different views on the behaviors of the system can be uniformly explained as structural differences of the same system. This is what we like to pursue in this book so that an effectiveness of system dynamics modeling can be demonstrated.

Keynesian Adjustment Process

Let us start with a Keynesian approach by deleting the equation (7.10). We have now 9 equations with 9 unknowns; that is, $Y, AD, C, I, G, Y_d, T, K, Y_{full}$

[4]I once posed this question in the book [58]. At that time, I was unaware of system dynamics and unable to model my general equilibrium framework for simulation.

with 7 exogenously determined parameters $(C_0, c, \bar{T}, \bar{I}, \delta, \bar{G}, L)$.

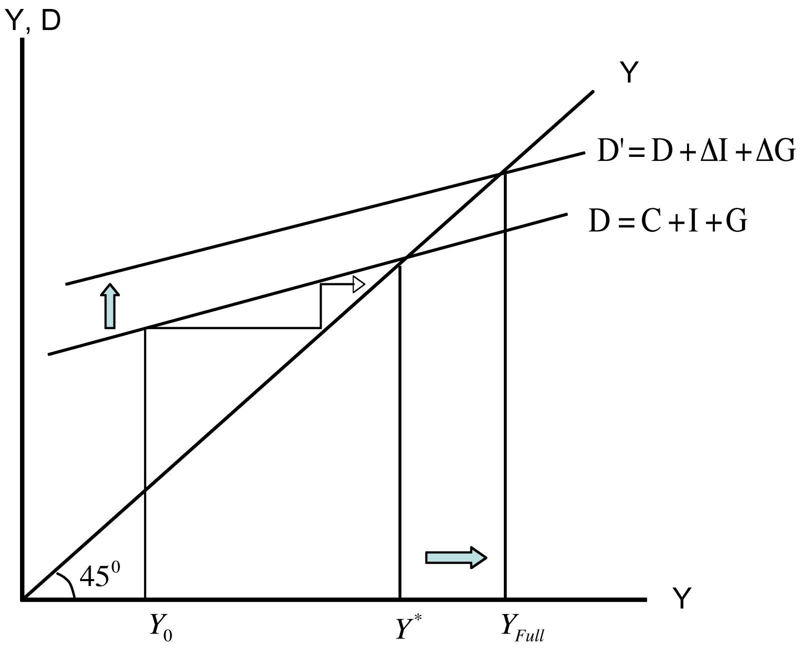

Figure 7.3: Keynesian Determination of GDP

A level of GDP that holds $Y = AD$ is obtained in terms of the parameters as follows:

$$Y^* = \frac{C_0 - c\bar{T} + \bar{I} + \bar{G}}{1 - c} \tag{7.12}$$

Let us assign some numerical values to these parameters $(C_0, c, \bar{I}, \bar{G}, \bar{T}) = (24, 0.6, 120, 80, 40)$, then we have $Y^* = 500$.

How can such a Keynesian equilibrium GDP be attained if aggregate supply and aggregate demand are not equal initially? The Keynesian model assumes that aggregate supply is determined by the size of aggregate demand. Fig 7.3 illustrates how an initial GDP of Y_0 continues to increase until it catches up with the aggregate demand, and eventually attains a Keynesian equilibrium Y^*. In this way the equilibrium can be always gained at a point where aggregate demand curve meets aggregate supply curve. Comparative statics is a well-known analytical method in standard textbooks to compare with two points of equilibria for two different levels of aggregate demand.

To model these static comparisons dynamically, the determination equation of GDP (7.1) has to be replaced with the following differential equation:

$$\frac{dY}{dt} = (AD - Y)/AT \tag{7.13}$$

where AT is an adjustment time.

In system dynamics this process is known as balancing feedback or goal-seeking dynamics in which aggregate demand plays a role of goal and GDP tries to catch up with it. Figure 7.4 illustrates a SD model of such Keynesian process, in which an aggregate demand forecasting mechanism is additionally introduced without changing an essential mechanism of Keynesian adjustment process [Companion model:1 Keynesian.vpm].

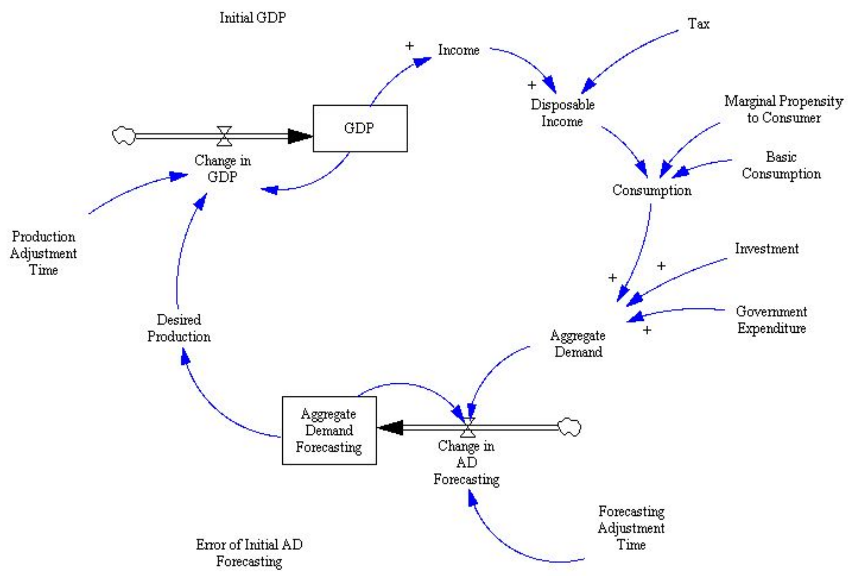

Figure 7.4: Keynesian SD Model of GDP

Left-hand diagram of Figure 7.5 illustrates how an initial GDP is smoothly increased to attain the Keynesian equilibrium GDP at $Y^* = 500$. In the right-hand diagram investment and government expenditures are respectively increased by 10 at the periods 5 and 10, while tax is reduced by 10 at the period 15. Again, GDP is shown to increase smoothly for attaining new equilibrium levels of aggregate demand.

From the production function (7.9) the maximum amount of output is produced by fully unitizing the existing capital stock K and labor force L.

$$Y_{full} = Y(K, L) \tag{7.14}$$

Obviously, there is no guarantee that the Keynesian equilibrium GDP of Y^* is equal to Y_{full}, and the equilibrium equation (7.10) is met. When it is less than the maximum output level, capital stock is under-utilized and some workers are unemployed; that is, the economy is in a recession. In other words, the

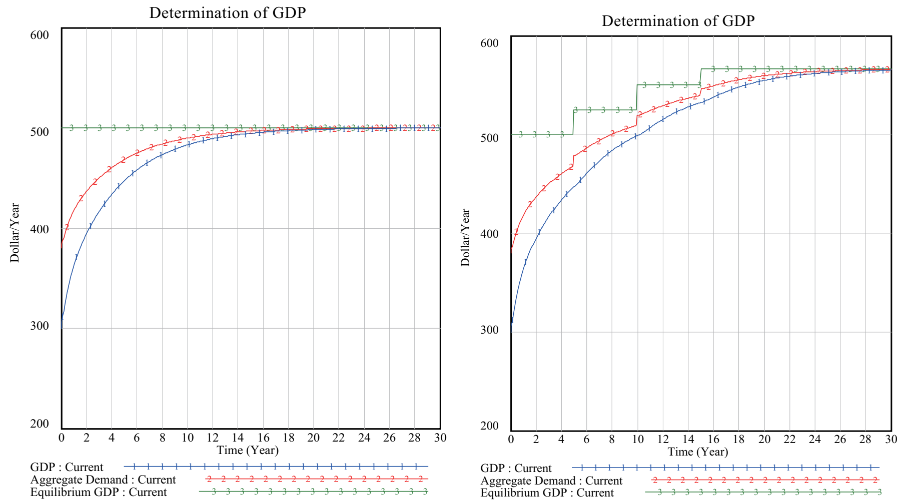

Figure 7.5: Keynesian Determination of GDP

Keynesian aggregate demand equilibrium is no longer an equilibrium in the sense that capital and labor are fully utilized.

According to the Keynesian theory, the underutilization is caused by deficiencies of effective demand, and to gain full capacity and full employment equilibrium, additional effective demand has to be created by increasing investment and government expenditures, or decreasing taxes.

How much increase in the effective demand is needed, then? The answer lies in the Keynesian multiplier process. From the equilibrium equation (7.12), we have

$$\Delta Y = \frac{-c\Delta T + \Delta I + \Delta G}{1-c} \tag{7.15}$$

Thus, multipliers for I, G and T are calculated as follows:

$$\frac{\Delta Y}{\Delta I} = \frac{\Delta Y}{\Delta G} = \frac{1}{1-c} = 2.5 \ ; \ \frac{\Delta Y}{\Delta T} = \frac{-c}{1-c} = -1.5 \tag{7.16}$$

Suppose that $Y_{full} = 560$. Then, to attain a full capacity level of GDP, we need to increase $\Delta Y = Y_{full} - Y^* = 60$. This could be done by increasing the investment or government expenditure by 24 (that is, $\Delta Y = 2.5 \cdot 24$), or decreasing tax by 40 (that is, $\Delta Y = (-1.5) \cdot (-40)$). Figure 7.3 illustrates how Y_{full} is attained by increasing aggregate demand such as investment and government expenditures.

System Dynamics Adjustment Process

The above Keynesian adjustment process is very mechanistic and does not reflect how actual production decisions are made by producers. More realistic decision-making process of production is to introduce an inventory adjustment management as explained in chapter 2 or in Chapter 18 of John Sterman's book [48]. In reality a discrepancy between production and shipment (or aggregate demand) is adjusted first of all as a change in inventory stock. Hence, the introduction of inventory as a stock is essential for SD modeling of macroeconomic system. The reason why inventory is not well focused in a standard macroeconomic framework may be because inventory is always treated as a part of (undesired) investment and output becomes in this sense identically equal to the aggregate demand.

Keynesian adjustment process (7.13) now needs to be revised as follows:

$$\frac{d\ I_{nv}}{dt} = (Y - AD) \tag{7.17}$$

with the introduction of inventory stock, I_{nv}. This adds another new unknown variable to the macroeconomic system. Accordingly, we need an additional equation to solve the amount of inventory. To do so, let us first define the amount of desired production as a sum of the amount of inventory replacement and aggregate demand forecasting:

$$Y^D = \frac{\text{Desired Inventory} - \text{Inventory}}{\text{Inventory Adjustment Time}} + \text{AD Forecasting} \tag{7.18}$$

where desired inventory is an exogenous parameter and set to be 30 dollars in our model. Then, redefine the aggregate supply as

$$Y = Y^D \quad \text{(Desired Production)} \tag{7.19}$$

Figure 7.6 illustrates our revised SD model of the Keynesian model [Companion model: 2 Keynesian(Revised).vpm]. When this model is run, we observe that aggregate demand and production overshoot an equilibrium as illustrated by the left-hand diagram of Figure 7.7. This overshooting behavior vividly contrasts with a smooth adjustment process of the Keynesian model.

In the right-hand diagram investment and government expenditures are respectively increased by 10 at the periods 5 and 10, while tax is reduced by 10 at the period 15 in the exactly same fashion as the right-hand diagram of Figure 7.5. However, output and aggregate demand do not catch up with new equilibrium levels smoothly here, instead they are shown to overshoot the equilibrium levels. This suggests that Keynesian adjustment process is intrinsically cyclical or fluctuating off equilibrium, rather than smoothly adjusting as illustrated by many standard textbooks. This behavior may be the first finding in our SD macroeconomic modeling against standard Keynesian smooth adjustment process.

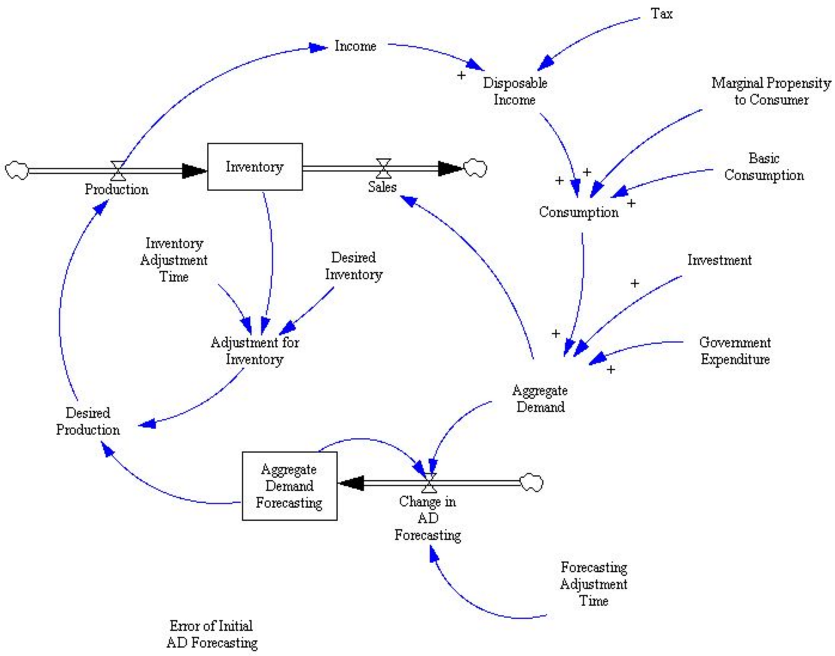

Figure 7.6: Revised Keynesian SD Model of GDP

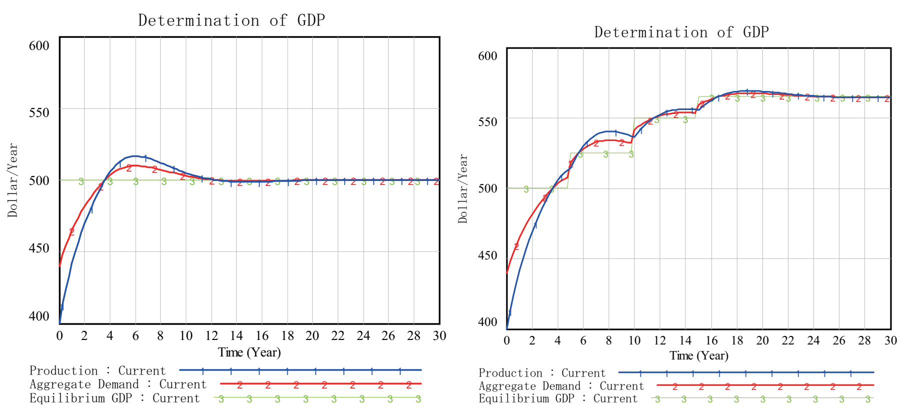

Figure 7.7: SD Determination of GDP

7.3 Aggregate Demand (IS-LM) Equilibria

In the above Keynesian macroeconomic model, taxes, investment and government expenditures are assumed to be exogenously determined. To make it more complete, we now try to construct these variables to be endogenously determined. Let us begin with government taxes by assuming that they consist of three parts: lump-sum taxes such as property taxes (T_0), income taxes that are proportionately determined by an income level, and government transfers such as subsidies (T_r):

$$T = T_0 + tY - T_r \tag{7.20}$$

where t is an income tax rate.

Next, investment is assumed to be determined by the interest rate:

$$I(i) = \frac{I_0}{i} - \alpha i \tag{7.21}$$

where α is an interest sensitivity of investment. We have now added a new unknown variable of the interest rate to the model, and hence an additional equation is needed to make it complete. According to the standard textbook, it should be an equilibrium condition in money market such that real money supply used in a year is equal to the demand for money:

$$\frac{M^s}{P}V = aY - bi \tag{7.22}$$

where V is velocity of money having a unit 1/year, a is a fraction of income for transactional demand for money, and b is an interest sensitivity of demand for money. P is a price level and it is treated as a sticky exogenous parameter.

From the equilibrium condition in the goods market, a relation between GDP and interest rate, which is called IS curve, is derived as follows:

$$Y = \frac{C_0 + I_0 + G + c(T_r - T_0)}{1 - c(1-t)} - \frac{\alpha}{1 - c(1-t)} i \tag{7.23}$$

On the other hand from the equilibrium condition in the money market, a relation between GDP and interest rate, called LM curve, is derived as

$$Y = \frac{1}{a}\frac{M^s}{P}V + \frac{b}{a} i \tag{7.24}$$

Equilibrium GDP and interest rate (Y^*, i^*) are now completely determined by the IS and LM curves. For instance, the aggregate demand equilibrium of GDP is obtained as

$$Y^* = \frac{C_0 + I_0 + G + c(T_r - T_0)}{1 - c(1-t) + \alpha(a/b)} + \frac{\alpha/b}{1 - c(1-t) + \alpha(a/b)}\frac{M^s}{P}V \tag{7.25}$$

This is a standard Keynesian process of determining an aggregate demand equilibrium of GDP in the short run in which price is assumed to be sticky. Figure 7.8 illustrates how IS and LM curves determine the equilibrium GDP and interest rate (Y^*, i^*).

As discussed in the previous section, GDP thus determined needs not be equal to the full capacity output level, Y_{full}. The Keynesian model only specifies

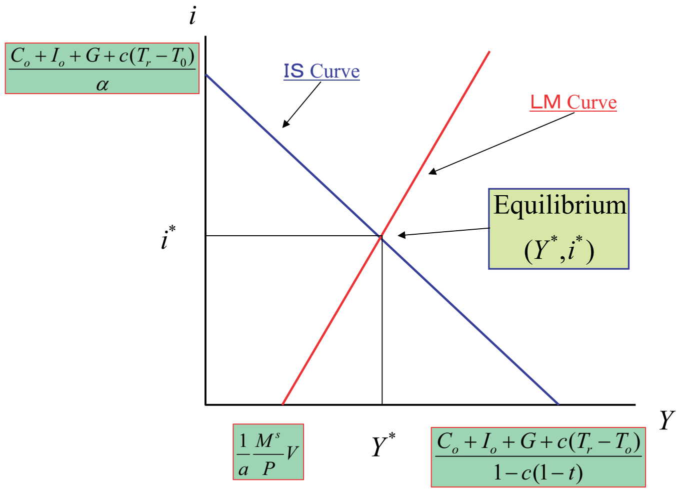

Figure 7.8: IS-LM Determination of GDP and Interest Rate

GDP as determined by the level of aggregate demand. This is why it is called aggregate demand equilibrium of GDP. To realize a full capacity equilibrium $Y^* = Y_{full}$, price needs to be flexibly changed in the long run. On the contrary the Keynesian model we presented so far lacks this price flexibility.

Endogenous Government Expenditures

We have successfully made variables such as T and I endogenous. The only remaining exogenous variable is government expenditures, G. They are usually determined by a democratic political process, and in this sense could be left outside the system as an exogenously determined parameter.

Instead, we try to make it an endogenous variable. First approach is to assume that the government expenditures are dependent on the economic growth rate, $g(t) = \Delta Y(t)/Y(t)$, such that

$$\frac{d\,G}{dt} = g(t)G. \tag{7.26}$$

This approach seems to be reasonable because many governments try to increase government expenditures proportionally to their economic growth rates so that a run-away accumulation of government deficit will be avoided.

Second approach is to assume that government expenditures are dependent on its tax revenues, since the main source of government expenditures is tax revenues which are endogenously determined by the size of output or income level. Then government expenditures become a function of tax revenues:

$$G = \beta T \tag{7.27}$$

where β is a ratio between government expenditures and tax revenues, called primary balance ratio here. When $\beta = 1$, we have a so-called balanced budget, while if $\beta > 1$, we have budget deficit.

With the introduction of the government expenditures in either one of these two ways, all exogenously determined variables such as T, I, and G are now endogenously determined within the macroeconomic system.

Let us analyze the second case furthermore. In this case IS curve becomes

$$Y = \frac{C_0 + I_0 + (\beta - c)(T_0 - T_r)}{1 - c - (\beta - c)t} - \frac{\alpha}{1 - c - (\beta - c)t} i \tag{7.28}$$

By rearranging, the aggregate demand equilibrium of GDP is calculated as

$$Y^* = \frac{C_0 + I_0 + (\beta - c)(T_0 - T_r)}{1 - c - (\beta - c)t + \alpha(a/b)} + \frac{\alpha/b}{1 - c - (\beta - c)t + \alpha(a/b)} \frac{M^s}{P} V \tag{7.29}$$

How does the introduction of tax-dependent expenditures affect behaviors of the equilibrium? Let us consider, as one special case, how a tax reduction in lump-sum taxes, T_0, affect the equilibrium GDP under a balanced budget; that is, $\beta = 1$. In this case, we have from the equation (7.29)

$$\frac{dY}{dT_0} = \frac{1 - c}{(1 - c)(1 - t) + \alpha(a/b)} > 0 \tag{7.30}$$

On the other hand, in the case of the exogenously determined expenditures, we have from the equation (7.25)

$$\frac{dY}{dT_0} = \frac{-c}{1 - c(1 - t) + \alpha(a/b)} < 0 \tag{7.31}$$

This implies that under a balanced budget a reduction in lump-sum taxes will discourage GDP, contrary to a general belief that it stimulates the economy. This counter-intuitive feature seems to be deemphasized in standard textbooks in which tax cut is usually treated as stimulating the economy.

7.4 Modeling Aggregate Demand Equilibria

Now we are in a position to construct our SD macroeconomic model of aggregate demand equilibria based on IS-LM curves [Companion model: 3 GDP(IS-LM).vpm]. To do so, the equilibrium condition (7.22) in the money market needs to be replaced with a dynamic adjustment process of interest rate as a function of excess demand for money:

$$\frac{d\ i}{dt} = \Phi\left((aY - bi) - \frac{M^s}{P}V\right) \tag{7.32}$$

Applying the formalization of adjustment processes discussed in the equations (2.8) and (2.10) in Chapter 2, adjustment process of the interest rate can be further specified as

$$\frac{d\ i}{dt} = \frac{i^* - i}{\text{DelayTime}} \tag{7.33}$$

where the desired interest rate i^* is obtained as

$$i^* = \frac{i}{\left(\frac{M^s}{P}V/(aY - bi)\right)^e} \tag{7.34}$$

Figure 7.9 illustrates the adjustment process of interest rate.

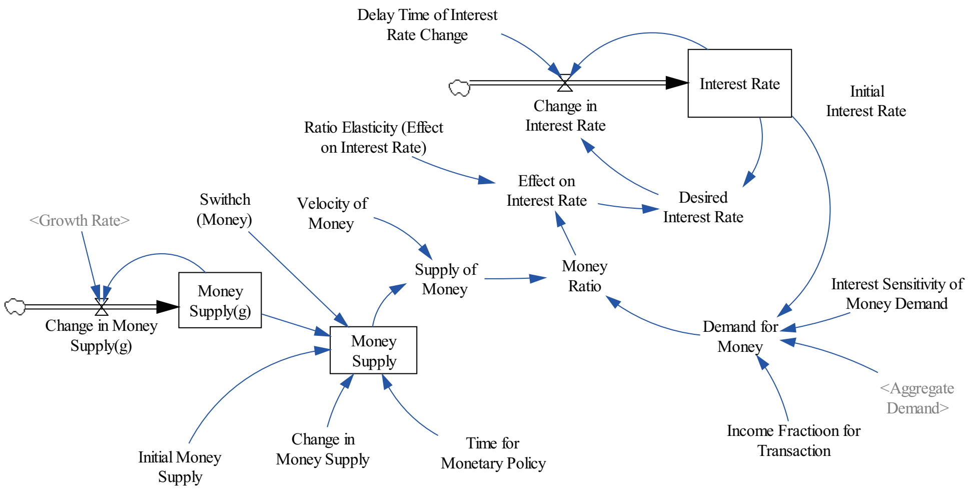

Figure 7.9: Interest Rate Adjustment Process

With this replacement, we could directly build a SD macroeconomic model of aggregate demand model in a mechanistic way such that IS and LM curves interact one another as developed in Figure 7.8 in the previous section. This could be a better approach than the comparative static analysis in which IS and LM curves are manually sifted to observe how aggregate demand equilibrium of (Y^*, i^*) is changed as usually done in the standard textbooks.

However, from a system dynamics point of view, this mechanistic approach of modeling aggregate demand equilibria may incur many causal loopholes. For instance, when consumers save, they receive interests from banks. If government spends more than receives, its deficit has to be funded by consumers as a purchase of government securities, against which they also receive interests. Whenever the macroeconomy is viewed as a wholistic economic system, these transactions play important feedback roles and such feedback effects need not be neglected. Therefore, as a complete system it should include those transactions among consumers, producers, banks and government from the beginning.

Due to the existence of these causal loopholes, standard macroeconomic framework has resulted in offering many open spaces which macroeconomists are invited to fill in with their own ideas and theories. We believe these open spaces have been intrinsic causes of many macroeconomic controversies such as the one between neoclassical and Keynesian schools of economics. These controversies, moreover, give us an impression that their macroeconomic models are mutually exclusive and cannot be integrated like oil and water.

On the contrary, as system dynamics researchers we believe that macroeconomy as a system could be modeled as an integrated whole so that controversies such as descried above are nothing but different behaviors caused by slightly different conditions of the same system structure. In this sense, its system dynamics model, if built completely, could synthesize these controversies as different macroeconomic system behaviors, rather than the behaviors of different economic system structures. This has been our main motivation for constructing a wholistic SD macroeconomic model in this book.

For the construction of synthetic model, a double entry accounting system of corporate balance sheet turns out to be very effective for describing many transactions among macroeconomic sectors. To some reader this approach seems to make our modeling unnecessarily complicated compared with the standard macroeconomic framework. We pose, however, that this is the simplest way to describe complicated macroeconomic behaviors per se.

Producers

Let us now describe some fundamental transactions which are missing in the standard textbook framework. We begin with producers. In the macroeconomic system, they face two important decisions: production for this year and investment for the futures. We have already assumed that production decision is made by the equation (7.19) by following a system dynamics approach of inventory management, while investment decision is assumed to be made by a standard macroeconomic investment function (7.21).

Based on these decisions, major transactions of producers are, as illustrated in Figure 7.10, summarized as follows.

- Producers are constantly in a state of cash flow deficits as analyzed in chapter 4. To make new investment, therefore, they have to borrow money from banks and pay interest to the banks.

- Out of the revenues producers deduct the amount of depreciation and pay wages to workers (consumers) and interests to the banks. The remaining revenues become profits before tax.

- They pay corporate tax to the government.
- The remaining profits are paid to the owners (that is, consumers) as dividends.

Consumers

Consumers have to make two decisions: how much to consume and how much to invest the remaining income between saving and government securities - a portfolio choice. Consumption decision is assumed to be made by a standard consumption function (7.3). (It could also be made dependent on their financial assets). As to the portfolio decision we simply assume that consumers first save the remaining income as deposits, out of which, then, they purchase government securities.

Transactions of consumers are illustrated in Figure 7.11, some of which are summarized as follows.

- Consumers receive wages and dividends.
- In addition, they receive interest from banks and the government that is derived from their financial assets consisting of bank deposits and government securities.
- Financial investment of government securities is made out of the account of deposits. (In this model, no corporate shares are assumed to be purchased).
- Out of the cash income as a whole, consumers pay income taxes, and the remaining amount becomes their disposal income.
- Out of their disposal income, they spend on consumption. The remaining amount is saved. Accordingly, no cash is assumed to be withheld by the consumers.

Government

Government faces decisions such as how much taxes to levy as revenues and how much to spend as expenditures. Tax revenues are assumed to be collected according to the standard formula in (7.20), while expenditures are determined either by growth-dependent amount (7.26) or revenue-dependent amount (7.27). In the model, expenditures are easily switched to either one. Revenue-dependent expenditure is set as default.

Transactions of the government are illustrated in Figure 7.12, some of which are summarized as follows.

- Government receives, as tax revenues, income taxes from consumers and corporate taxes from producers. It also levies excise tax on production.

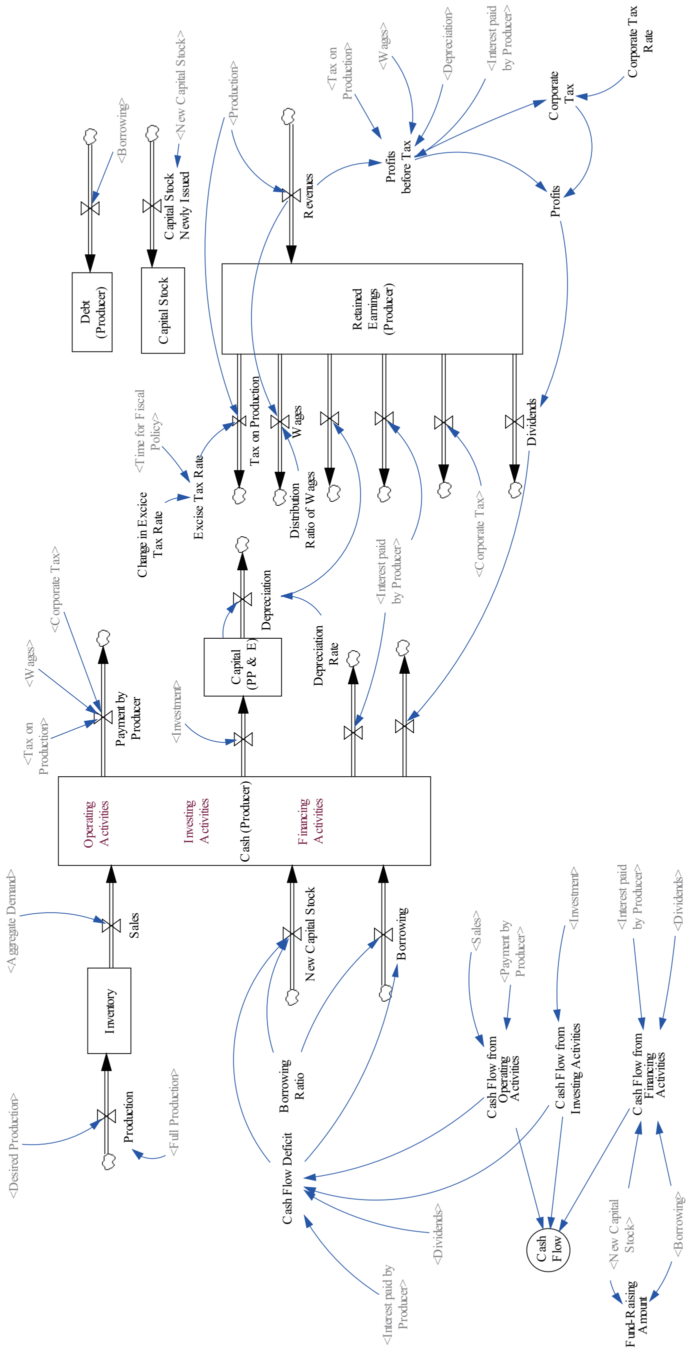

Figure 7.10: Transactions of Producers

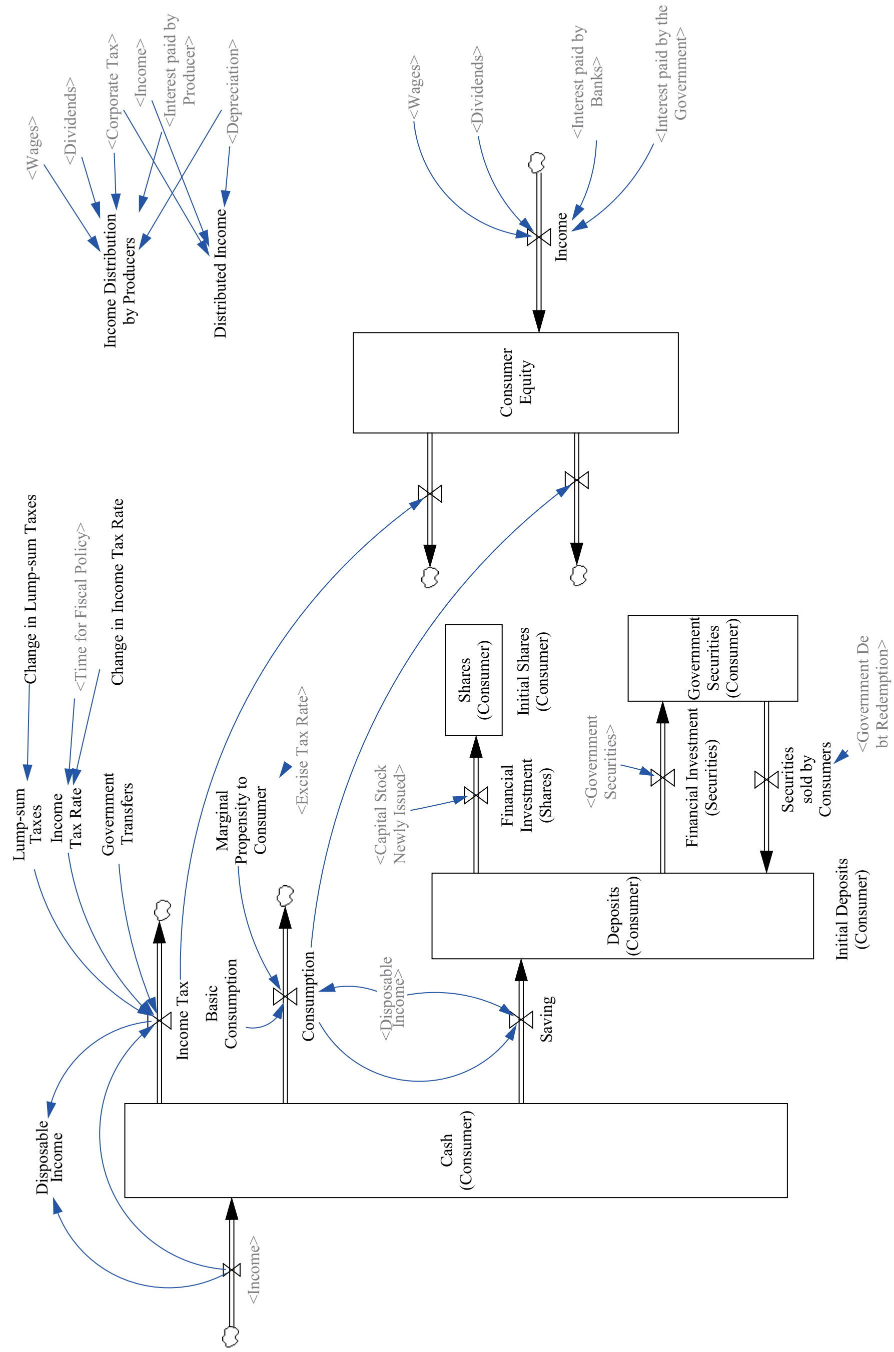

Figure 7.11: Transactions of Consumer

- Government spending consists of government expenditures and payments to the consumers such as debt redemption and interests against its securities.
- Government expenditures are assumed to be endogenously determined by either the growth-dependent expenditures or tax revenue-dependent expenditures.
- If spending exceeds tax revenues, government has to borrow cash from consumers by newly issuing government securities.

Banks

In our model, banks are assumed to play a very passive role; that is, they only make loans to producers by the amount asked by them. In other words, they don't purchase government securities and accordingly need to make no portfolio decisions between loans and securities. This assumption is dropped in the following chapters. Transactions of banks are illustrated in Figure 7.13, some of which are summarized as follows.

- Banks receive deposits from consumers, against which they pay interests.
- They make loans to producers and receive interests. Prime interest rate for loans is assumed to be the same as the interest rate for deposits. This assumption is dropped in the following chapters.
- Their retained earnings thus become interest receipts from producers less interest payment to consumers.

7.5 Behaviors of Aggregate Demand Equilibria

We now see how aggregate demand equilibrium of (Y^*, i^*) is attained in our SD model constructed above. This model is built by deleting the equation (7.10) and in this sense, as already discussed above, Y^* needs not be equal to a production level of full capacity, Y_{full}. Surely, the full production level is a maximum level of output in the economy beyond which no physical output is possible. To introduce this upper bound of production level, the equation (7.19) has to be revised as follows.

$$Y = \mathrm{Min}(Y_{full},\ Y^D) \tag{7.35}$$

Moreover, the full capacity output level in equation (7.14) is specified as follows:

$$Y_{full} = e^{\kappa t}\frac{1}{\theta}K \tag{7.36}$$

where κ is an annual increase rate of technological progress, and θ is a capital-output ratio. For simplicity, labor force is not considered here. The production process of GDP in our SD model is illustrated in Figure 7.14.

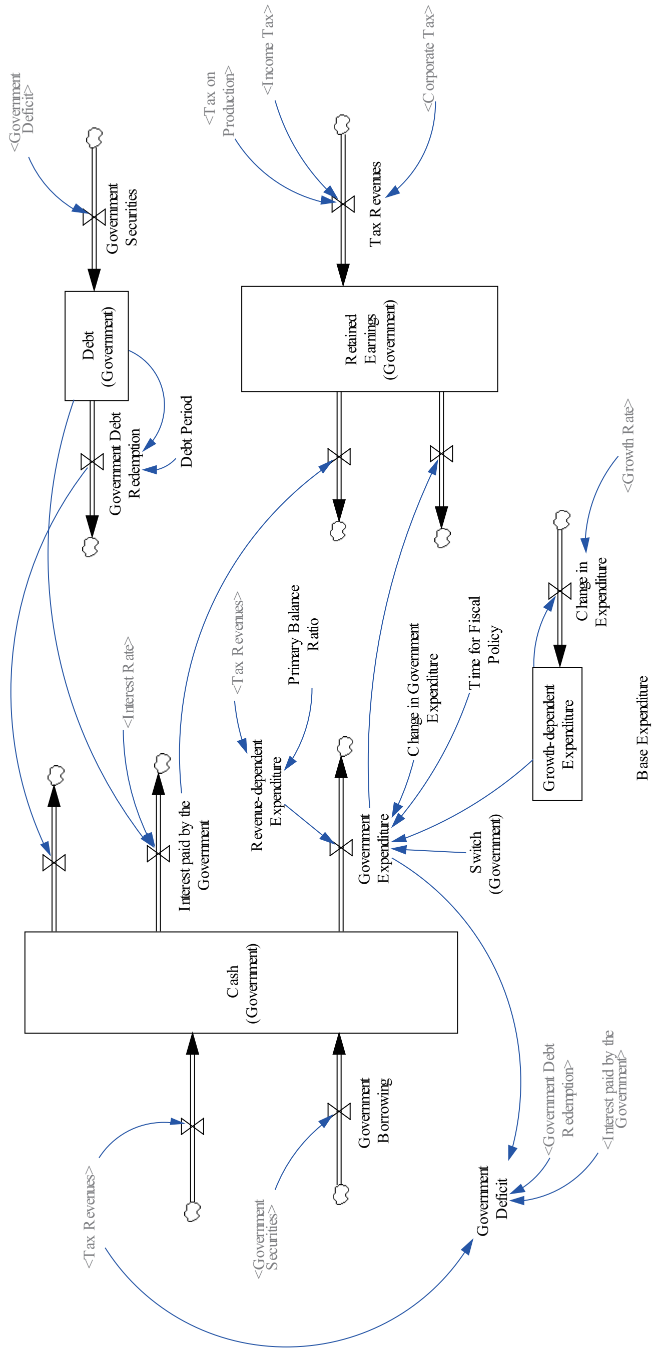

Figure 7.12: Transactions of Government

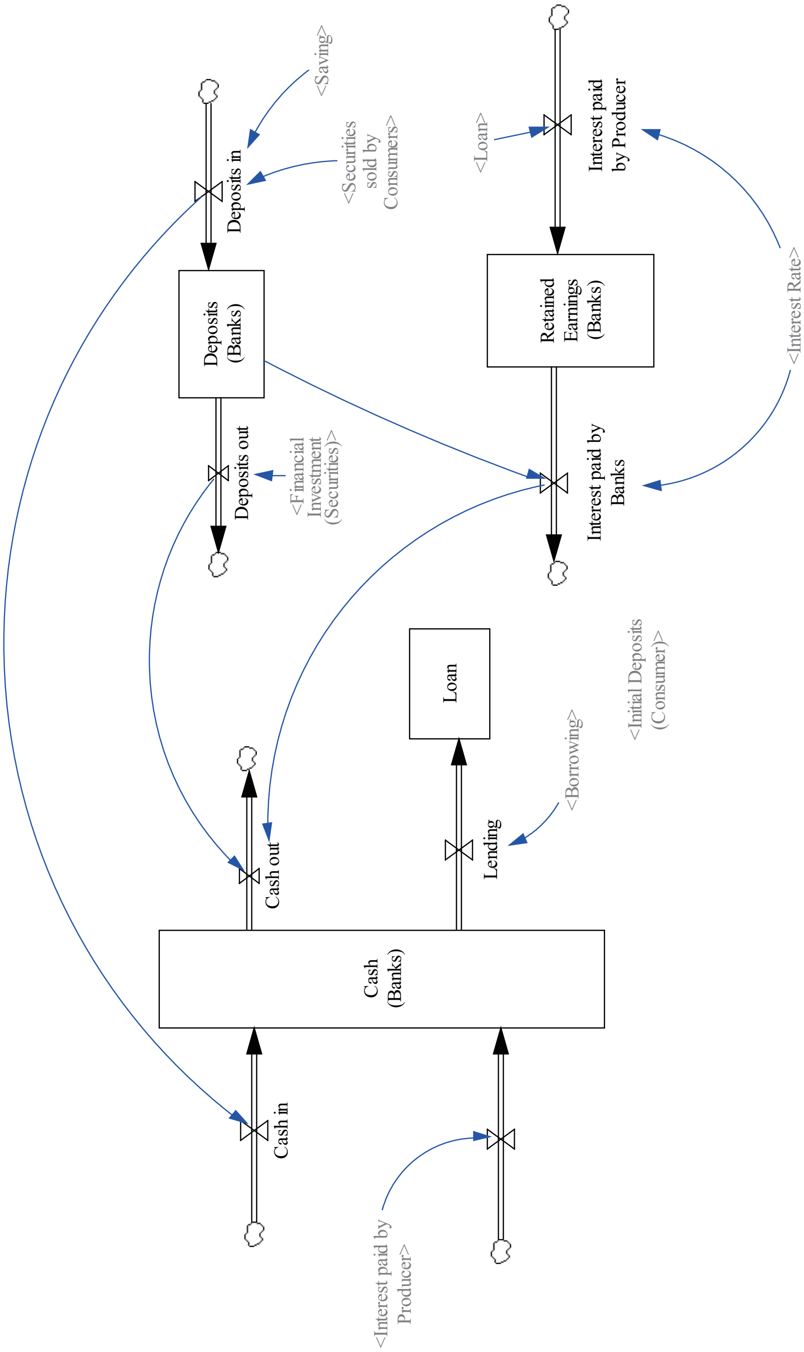

Figure 7.13: Transactions of Banks

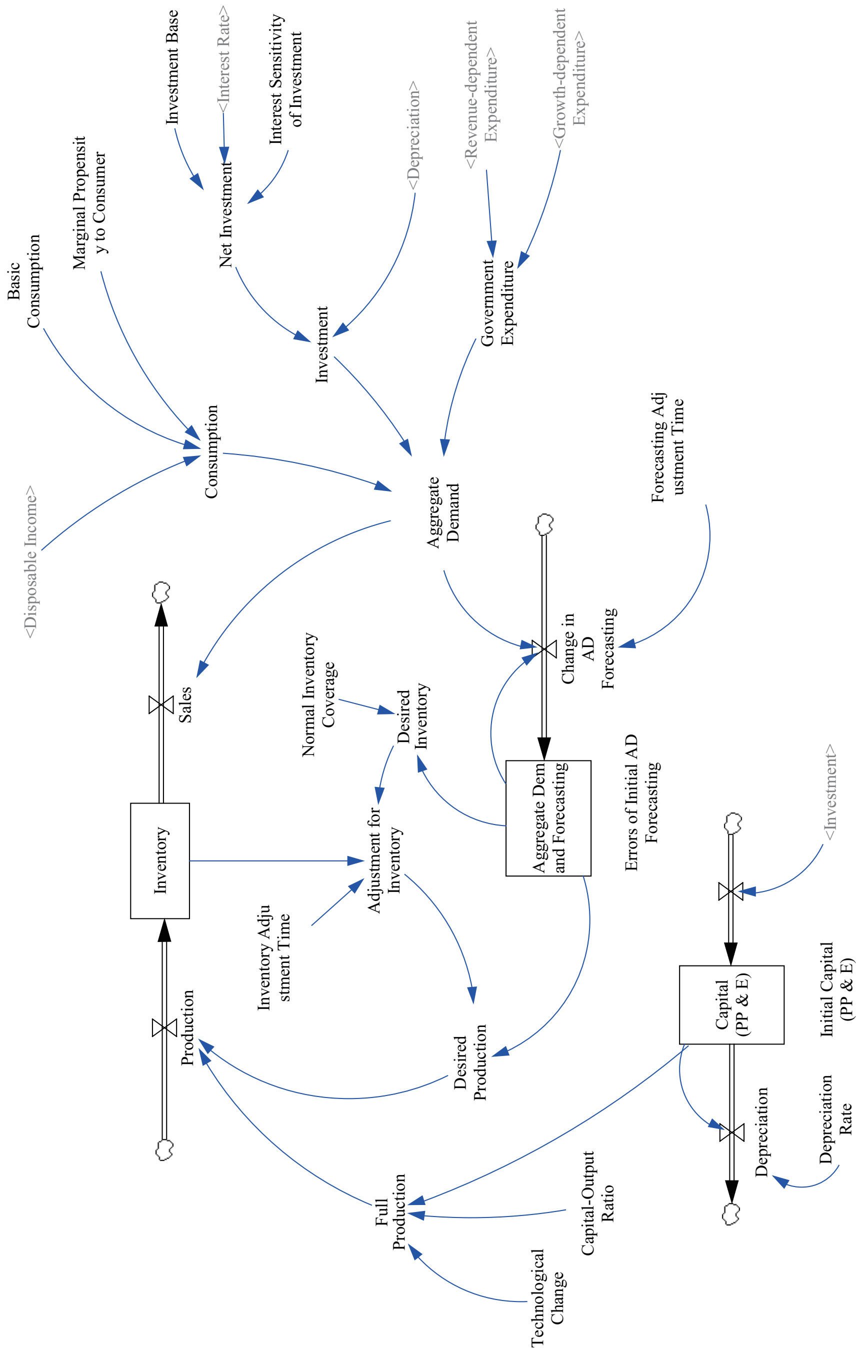

Figure 7.14: Full Capacity Production

Top diagram in Figure 7.15 shows mostly equilibrium growth path of production around full capacity level. Bottom diagram is loci of aggregate demand equilibrium (Y^*, i^*) such as an intersection between IS-LM curves as illustrated in Figure 7.8. Our model can capture these dynamic movements of the aggregate demand equilibria in contrast with comparative static ones in standard textbooks. This may be another contribution of SD macroeconomic modeling.

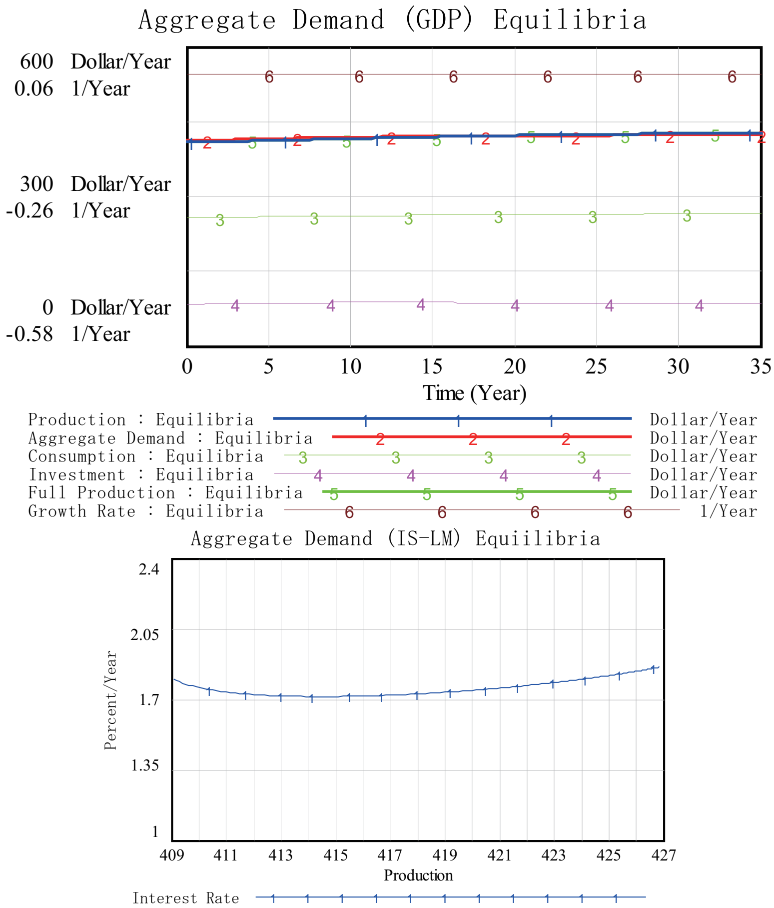

Figure 7.15: Aggregate Demand Mostly Equilibria

Let us now consider a disequilibrium case which is triggered by changing the amount of basic consumption from 101 to 90. Top diagram in Figure 7.16 shows that initial equilibrium amount of $Y = 409.09$ is thrown into disequilibrium until a discrepancy between full production and desired production is brought to an equilibrium about $Y^* = 422$ around the period 9 once again. Beyond this point, however, the economy is once again thrown into recession; that is to say, aggregate demand equilibrium of GDP is shown to be constantly lower than the full production level. Bottom diagram is loci of aggregate demand disequilibrium (Y^*, i^*) such as an intersection between IS-LM curves as illustrated in Figure 7.8.

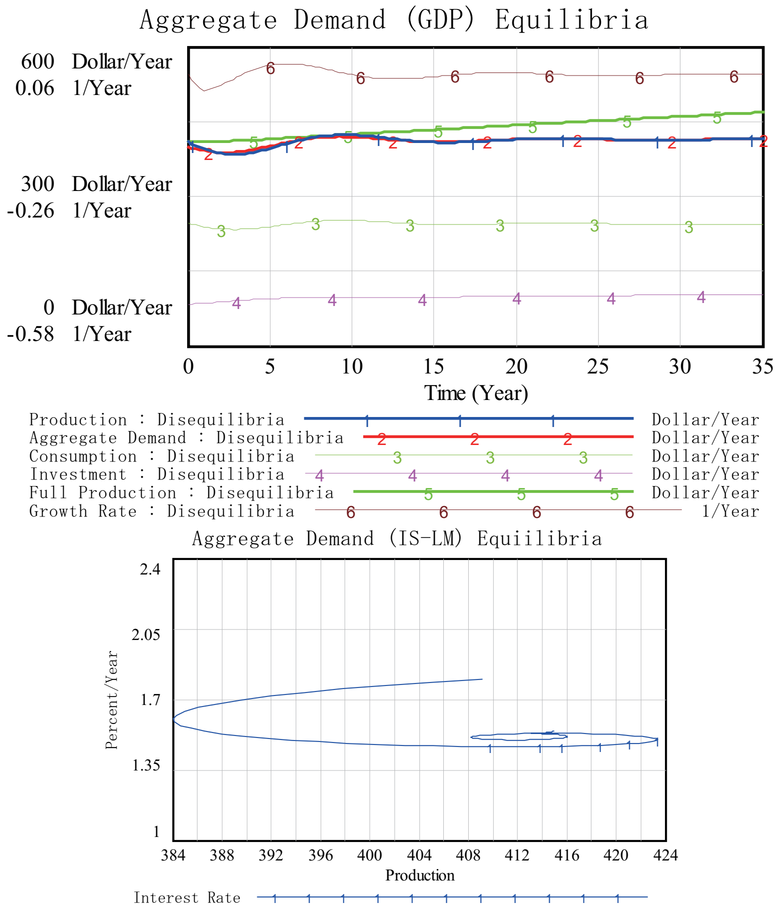

Figure 7.16: Aggregate Demand Disequilibria

Monetary Policy

How can we, then, attain a true Y_{full} equilibrium in which full capacity output is completely sold out? As already illustrated in Figure 7.3, it could be done by an increase in aggregate demand. The essence of the Keynesian theory is that aggregate demand can be stimulated by monetary and fiscal policies; that is, macroeconomy is manageable!

Let us examine monetary policy first by increasing the amount of money supply by 8 at period 12. According to textbook explanation, this increase in money supply shifts the LM curve to the right, and accordingly i^* is lowered while Y^* increases. In the top diagram of Figure 7.17 a full capacity equilibrium is shown to be attained again around $Y_{full} = Y^* = 440$ at period 17. Unfortunately, however, this equilibrium cannot be sustained, because capital continues to accumulate and accordingly production capacity also continues to increase, eventually exceeding aggregate demand.

Bottom diagram shows how interest rate begins to decline due to the increase in money supply. However, it eventually begins to increase as aggregate demand fails to sustain a full capacity equilibrium.

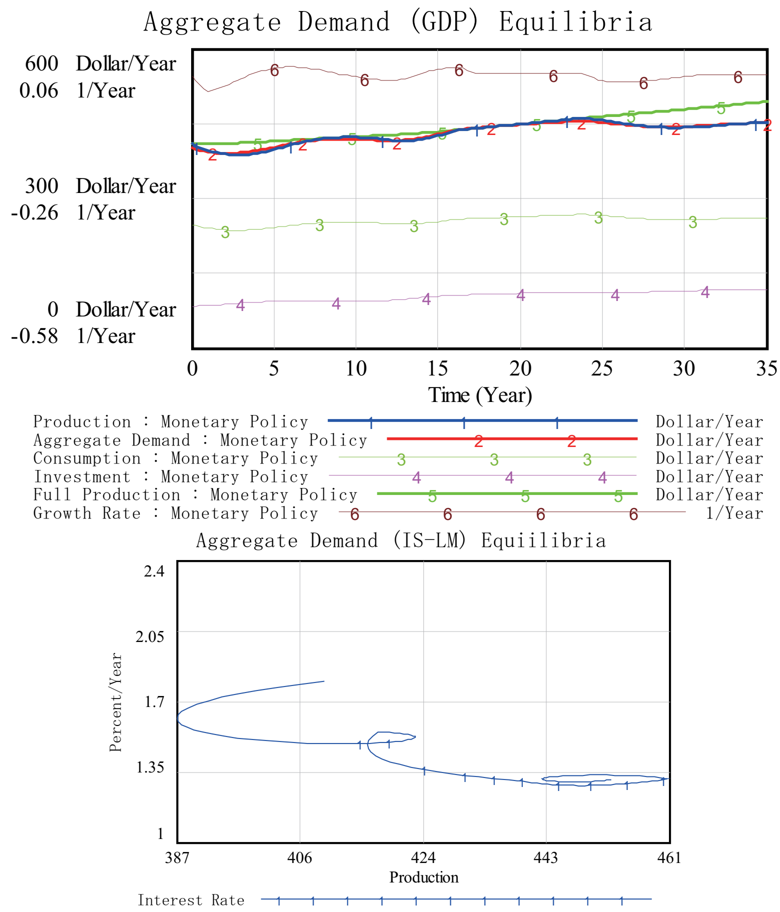

Figure 7.17: Effect of Monetary Policy

Fiscal Policy

Now we try to attain a Y_{full} equilibrium through fiscal policy in place of monetary policy. Specifically we try to increase government expenditures by 16 at period 14. According to textbook explanation, this increase shifts the IS curve to the right and stimulates the economy by increasing Y^*. However, as illustrated in the bottom diagram of Figure 7.18, it also pushes up i^*, and eventually discourages investment, which is a well known crowding-out effect of government expenditures.

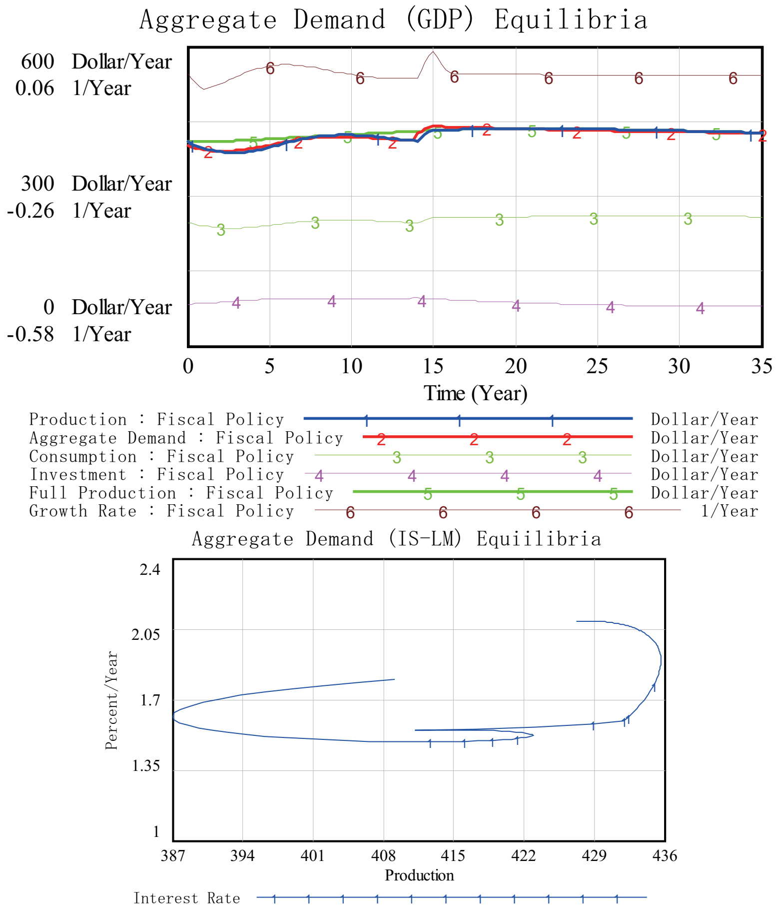

Figure 7.18: Effect of Fiscal Policy

To confirm this, let's take a look at the top diagram, in which a full capacity equilibrium is shown to be attained again around $Y_{full} = Y^* = 432$ around period 15. Beyond this point, however, aggregate demand and production continues to decline, while full capacity output also begin to decline over the period 21 due to a continued increase in interest rate and discouraged investment following it. This movement suggests the existence of crowding-out effect caused by fiscal policy. Figure 7.19 compares the behaviors of interest rate among disequilibria (line 1), monetary policy (line 2), and fiscal policy (line 3). Compared with the case of monetary policy which lowers interest rate to stimulate investment, fiscal policy puses up interest rate, discouraging the investment.

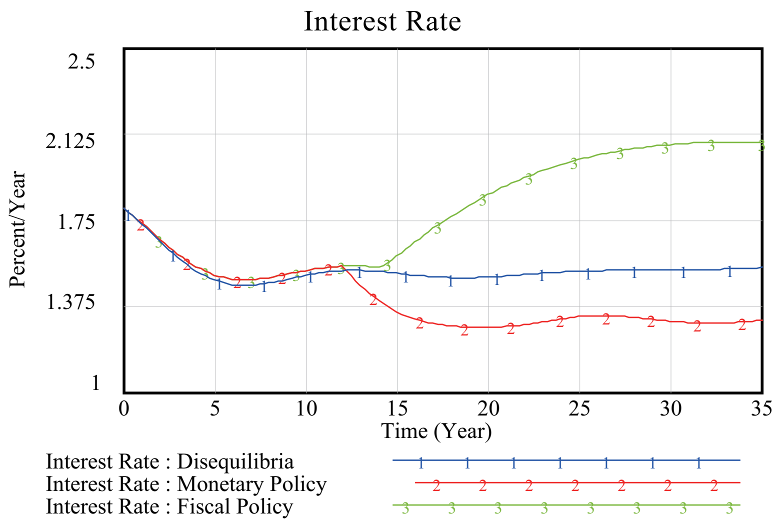

Figure 7.19: Behaviors of Interest Rate Compared

Government Debt

Figure 7.20 shows how government debt has accumulated by this fiscal policy from 0 at the period 14 to 417 at the period 35, which is close to the GDP of 427. In reality the increasing government debt will lower the price of government securities, and further increase the interest rate. This will not only cause a loss of government credibility, but also bring investment activities simultaneously to a complete standstill; in short, a total breakdown of national economy eventually.

In standard textbooks fiscal policy is usually introduced as a very effective policy to stimulate the economy, while the skyrocketing effect of government debt, the other side of the coin, is left out from the picture, giving an impression to the students that fiscal policy works well without any problem. Our system dynamics analysis is able to successfully capture the other side of the coin. Hence, this could be another contribution of our SD macroeconomic modeling.

In this Keynesian aggregate demand analysis, no feedback structure is built in to reverse the situation of hyper-inflation and a possible collapse of the economy, simply because price is assumed to be fixed, which will be dropped in the next section.

7.6 Price Flexibility

It is now clear that the Keynesian theory of aggregate demand equilibria is imperfect from a SD model-building point of view, because price level is assumed

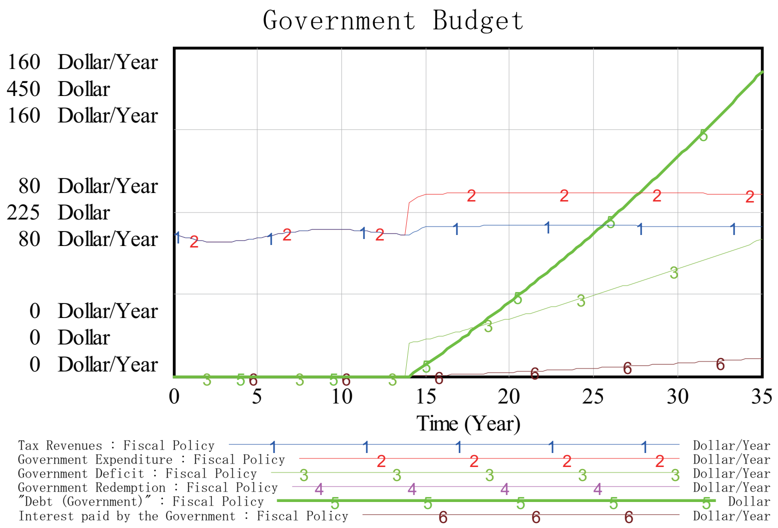

Figure 7.20: Skyrocketing Government Debt

to be sticky and there exists no built-in mechanism to restore a full capacity production equilibrium unless monetary and fiscal policies are carried out.

In fact, let us rewrite the aggregate demand equilibrium of GDP obtained in equation (7.25) as a function of price:

$$Y^*(P) = A + B\frac{M^s}{P}V, \tag{7.37}$$

where A and B are combined constant amounts. Then it becomes clear that this equation only provides a relation between Y^* and P. Hence, Y^* is called an aggregate demand function of price. It is now obvious that, unless price is flexible, there exists no mechanism to attain a true equilibrium such that

$$Y_{full} = Y^*(P) \tag{7.38}$$

It is shown in the previous section that, even though monetary and fiscal policies can attain a full capacity production equilibrium, central bank and government need to constantly fine-tune their policies to sustain such equilibrium. Can they really perform the task under a sticky price in the short run? If so, how short is a short run, practically speaking, to apply such policies?

From a dynamic point of view, it's very hard to specify how short is a short run. Is this moment in the short run or in the long run? It depends on when to specify an initial point. This moment could be in the short run to justify current policies. Or it could be already in the long run, and a long-run price adjustment mechanism, to be discussed below, may be under way. If so, current policy applications might worsen economic situations.

Accordingly, a better way of modeling a macroeconomic system has to allow price flexibility in the model from the beginning and let the price adjust disequilibria, including a fixed price as its special case. To do this formally, we have to bring a previously neglected equation (7.10). To avoid a redundancy of equation by doing so, we need to introduce another variable of price, and let it adjust discrepancies between full capacity output Y_{full} and desired production Y^D as in the equation of price adjustment mechanism (7.11). Such discrepancies are called GDP gap. Price, however, may also adjust directly to the discrepancies between inventory I_{inv} and its desired inventory I^*_{inv}, which are called inventory gap here. This is an adjustment process of attaining stability on a historical time already discussed in chapter 2.

Hence, such an adjustment equation could be described as

$$\frac{dP}{dt} = \Psi(Y^D - Y_{full}, I^*_{inv} - I_{inv}). \tag{7.39}$$

Let us specify the equation, as in the interest equation (7.33), as follows:

$$\frac{dP}{dt} = \frac{P^* - P}{Delay\ Time} \tag{7.40}$$

where the desired price P^* is obtained as

$$P^* = \frac{P}{\left((1-\omega)\frac{Y_{full}}{Y^D} + \omega\frac{I^*_{inv}}{I_{inv}}\right)^e} \tag{7.41}$$

where $\omega, 0 \leq \omega \leq 1$, is a weight between production and inventory ratios, and e is an elasticity.

This completes our SD macroeconomic modeling of Keynesian IS-LM model. Figure 7.21 illustrates adjustment processes of price and interest rate.

With the introduction of flexible price (which is attained by setting a ratio elasticity of effect on price = 1.0), behaviors of the model turns out to be surprisingly different from the previous model under a fixed price. First, aggregate demand equilibrium, Y^*, can no longer be attained as in the previous fixed price case. Instead, as top diagram of Figure 7.22 illustrates, they fluctuates alternatively, which we call aggregate demand alternations.

Second, this alternation moves along a full capacity output level, and occasionally approaches to a full capacity equilibrium such that $Y_{full} = Y^*$ as if butterflies moves around flowers and occasionally rest on them. This vividly contrast with the previous fixed price case in which aggregate demand equilibrium can be attained through monetary and fiscal policies, but it will eventually diverge from a full capacity output level. Therefore, under a flexible price, monetary and fiscal policies might not be effective to attain full capacity equilibrium.

Third, economic growth rates turn out to fluctuate periodically as illustrated in the left-hand diagram of Figure 7.23, in which the business cycle of growth rates, produced by the inner forces of the system structure itself, can be observed to have a period of about 15 years. This is an entirely unexpected behavior to us. Can we avoid this business cycle by practicing monetary and fiscal policies?

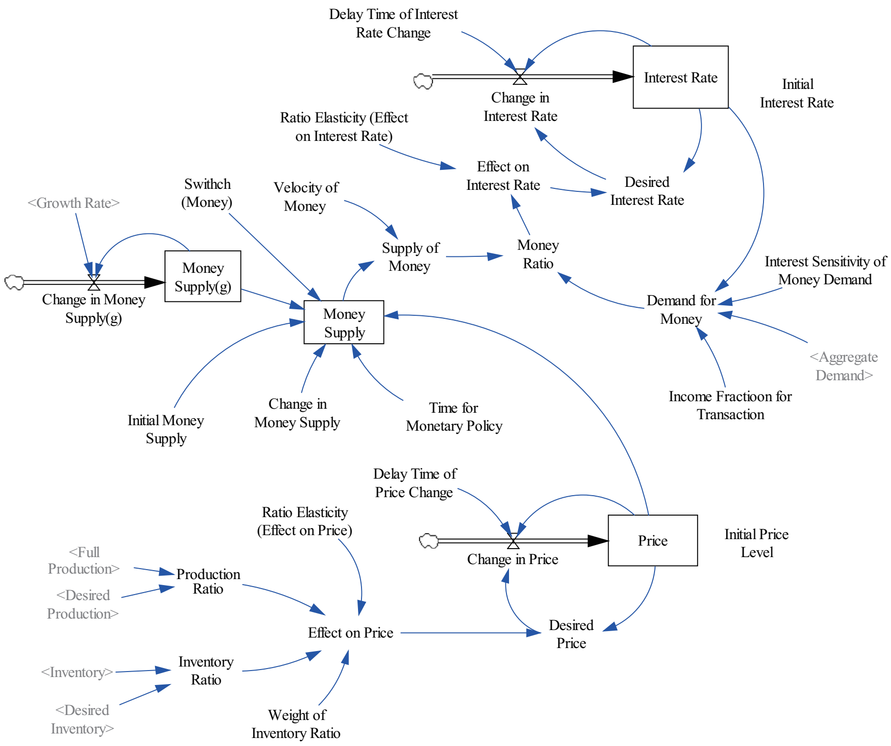

Figure 7.21: Price and Interest Rate Adjustment Processes

These will be open questions to be challenged later. Right-hand diagram shows cyclical movements of real money supply, demand and interest rate, which have similar fluctuation periods as business cycle of growth rate.

Figure 7.24 illustrates aggregate demand curve (line 1) and aggregate supply curves of production (lines 2) and full production (and 3). Aggregate demand curve is observed to be, roughly speaking, a downward-sloping, while aggregate supply curves to be horizontal.

Disequilbria under Price Flexibility

To create a disequilibrium situation, let us change the basic consumption from 101 to 90 in the same fashion as previous section. Under flexible price, no disequilibrium situation is successfully produced as the Figure 7.25 illustrates.

In other words, similar business cycles are observed, this time, at a larger scale. This can be confirmed with Figure 7.26

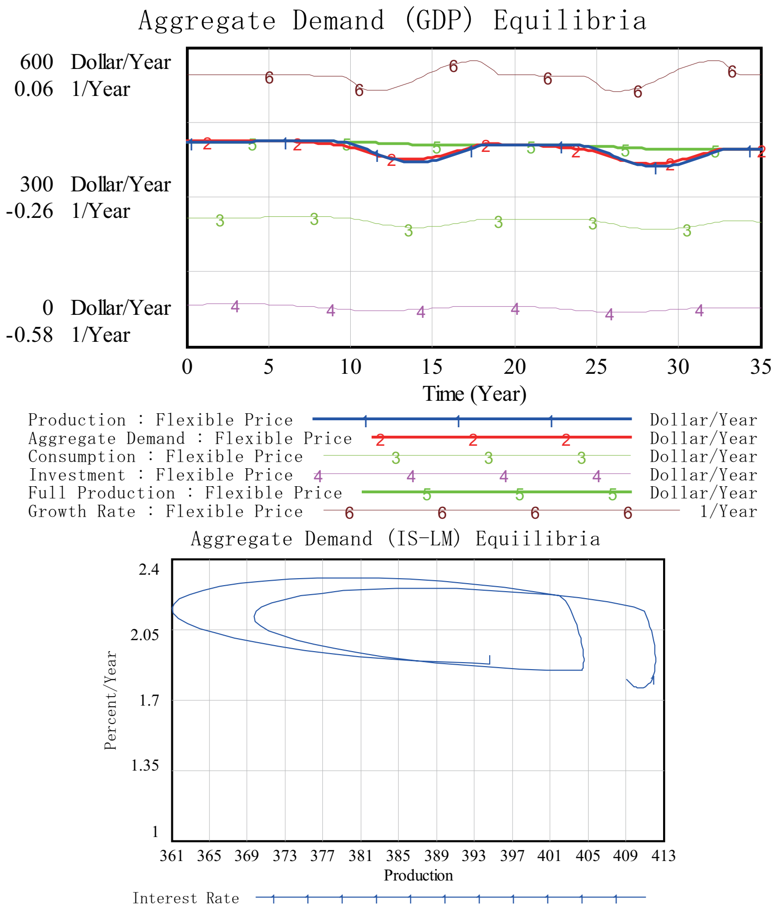

Figure 7.22: Flexible Price Equilibria

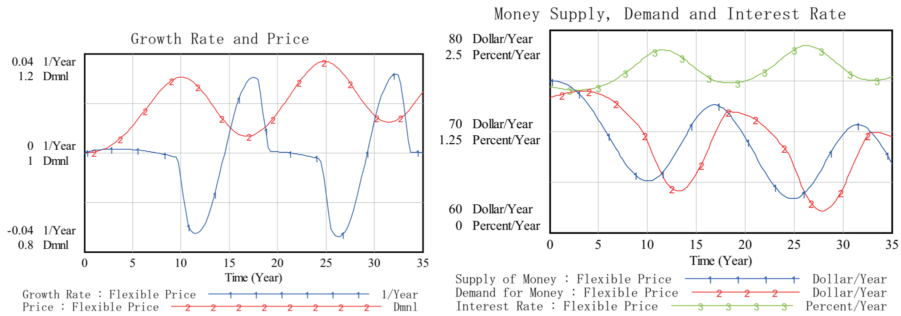

Figure 7.23: Growth, Price, Money Supply, Demand and Interest Rate

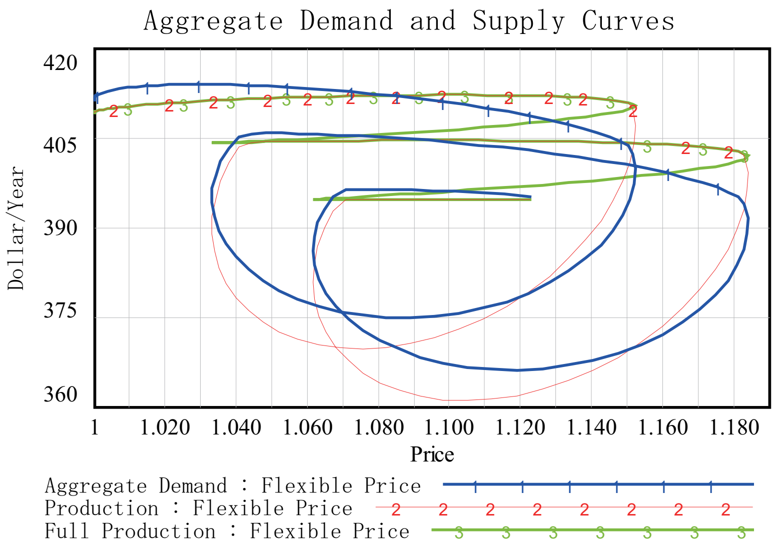

Figure 7.24: Aggregate Demand and Supply Curves

Growth-dependent Money Supply

It will be interesting to see how a change in money supply affects the behaviors of the above disequilibria under price flexibility, that is, price coefficient = 1.0. To do so, however, our macroeconomic model here must be first of all integrated with the money supply model developed in the previous chapters. Otherwise, it will be misleading to merely change money supply without examining its feedback relations within the system.

Even so, just for our curiosity, let us change money supply proportionally to an economic growth rate. Monetarist argue that money is neutral so that a change in money supply along with the economic growth does not affect true behaviors of its real part.

To observe the effect, let us introduce growth-dependent money supply in the same fashion as we introduced growth-dependent government expenditures in equation (7.26):

$$\frac{dM^s}{dt} = g(t)M^s. \tag{7.42}$$

In Figure 7.27, line 1 indicates a reference curve with constant money supply. Line 2 represents the behaviors with a growth-dependent money supply. Line 3 shows the behaviors with a growth-dependent money supply under disequilibria.

Specifically, left-hand diagram illustrates that real supply of money keeps fluctuating. Right-hand diagram shows that interest rate also continues to decrease as predicted by the theory.

Left-hand diagram of Figure 7.28 illustrates that production continues to grow. Right-hand diagram shows constant growth of money supply not only

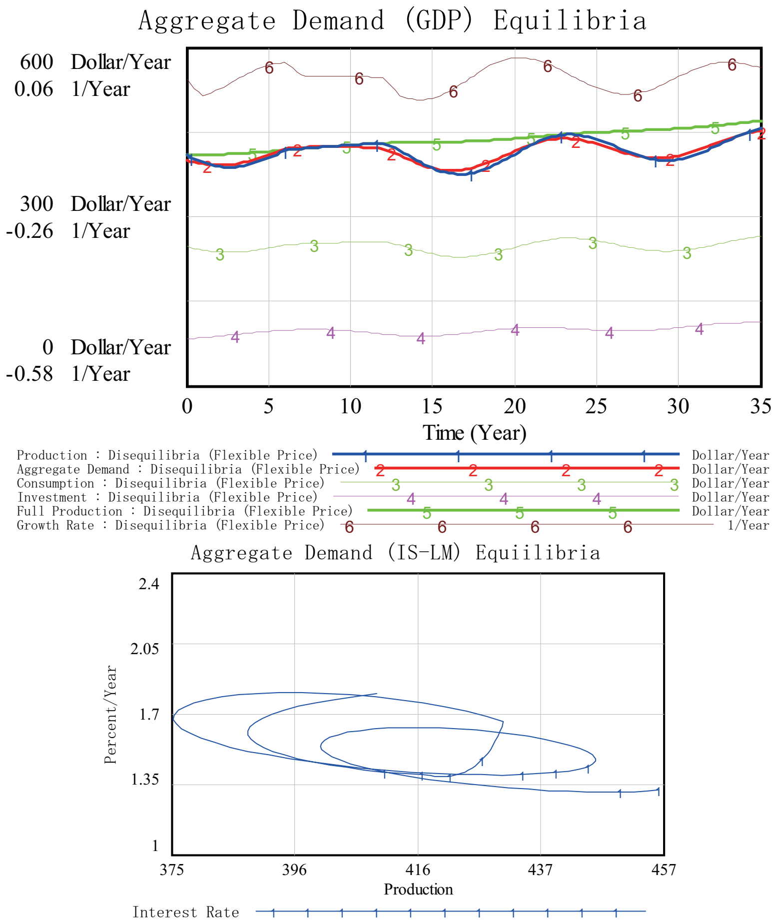

Figure 7.25: Flexible Price Disequilibria

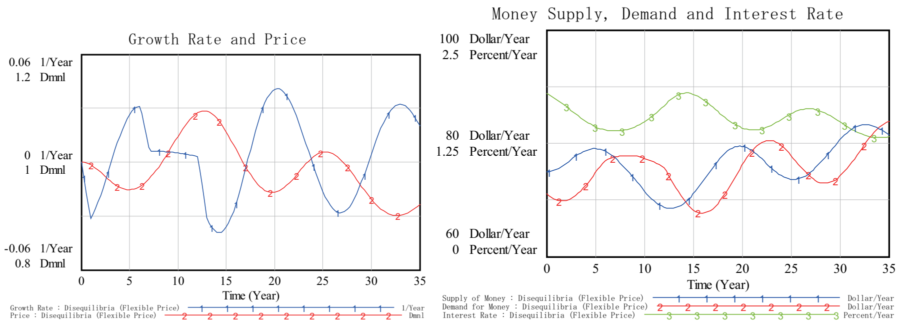

Figure 7.26: Growth, Price, Money Supply, Demand and Interest Rate

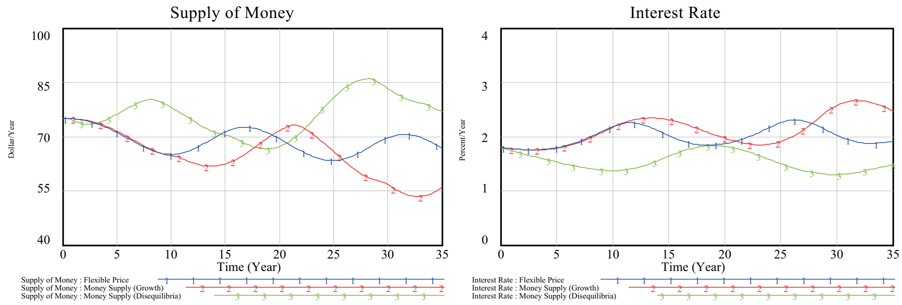

Figure 7.27: Growth-dependent Money Supply and its Effect on Interest Rate

stimulate an economic growth incessantly, but destabilize the economic behaviors, contrary to a monetarist belief that constant growth of money according to the economic growth stabilizes the economy.

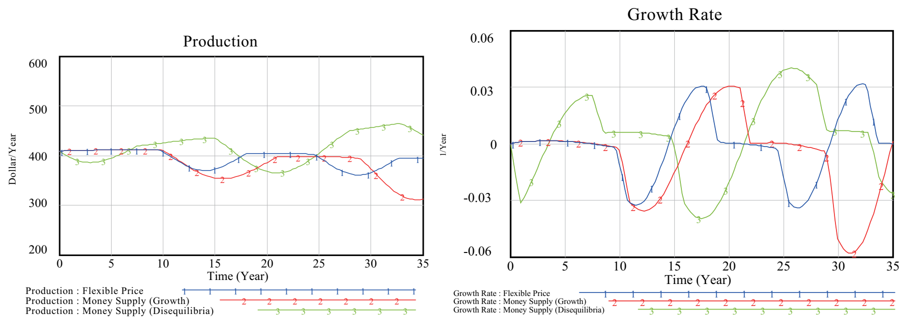

Figure 7.28: Growth-dependent Money Supply on GDP and Growth Rate

Figure 7.29 demonstrates that IS-LM curves extends to the right as the increasing money supply shifts LM curve to the right.

As pointed out above, however, true interpretations of these behaviors have to be postponed until the current macroeconomic model is integrated with the money supply models in the previous chapters.

7.7 A Comprehensive IS-LM Model

Keynesian IS-LM model has a serious limitation; that is, money supply is exogenously given. On the contrary our SD approach of IS-LM model can treat money supply endogenously, which is now presented in this section [Companion model: 4 GDP(IS-Money).vpm]. Money supply is defined in the previous chapters as follows:

$$\text{Money Supply} = \text{Currency in Circulation} + \text{Deposits} \tag{7.43}$$

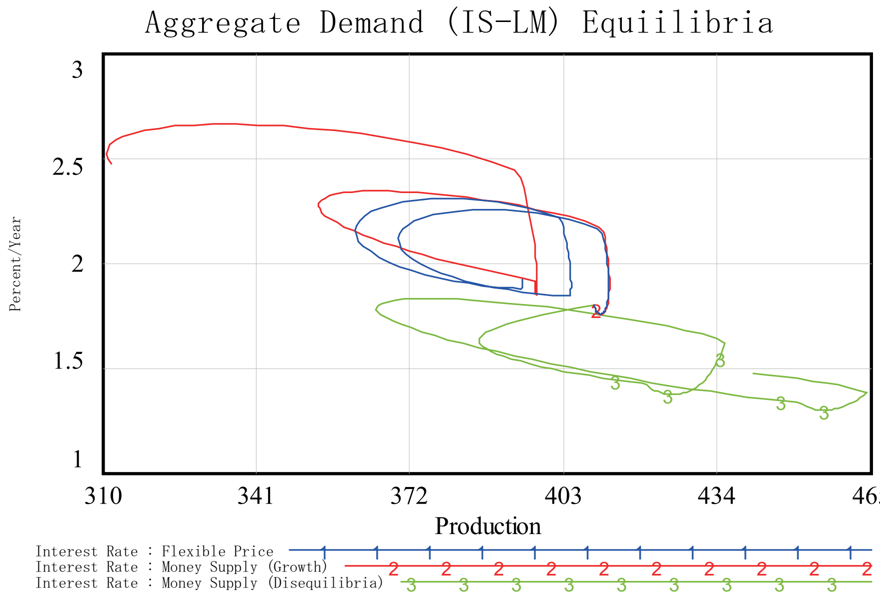

Figure 7.29: Growth-dependent Money Supply on IS-LM

Currency in circulation in our model consists of stocks of cash held by consumers, producers, government and banks. These cash stocks as well as deposits constitute money supply.

On the other hand, demand for money is also obtained in our model as outflows of the stocks of cash by consumers, producers, government and banks. In this way, supply of money and demand for money are endogenously determined in our comprehensive IS-LM model.

Based on these changes, desired interest rate defined in equation (7.34) also needs to be revised as

$$i^* = \frac{i}{\left(\frac{\text{Supply of Money}}{\text{Demand for Money}}\right)^e} \tag{7.44}$$

Figure 7.30 shows a revised processes of interest rate and price adjustment.

In this way, our IS-LM model now becomes more comprehensive. Yet, it has a serious theoretical flaws. First, money supply cannot be changed without the central bank, and secondly, real and monetary quantities are being mixed up. These will be fixed in the next chapter by integrating real and monetary sectors presented in this and previous two chapters.

Even so, it's worth a while to observe how our comprehensive SD model behaves in comparison with a traditional Keynesian IS-LM model presented above.

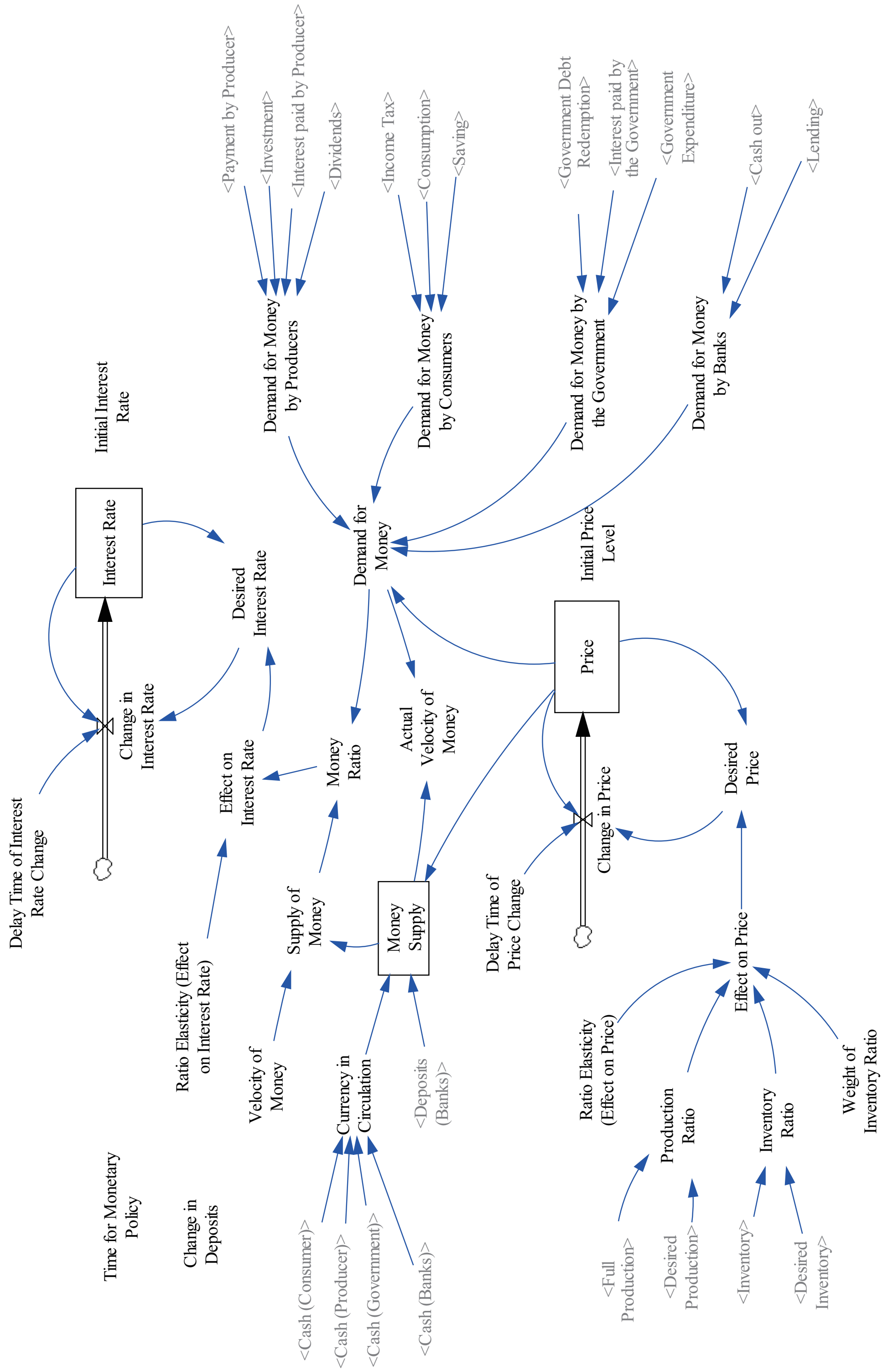

Figure 7.30: Interest Rate and Price Adjustment Process Revisited

Behaviors of the model

In the model aggregate demand equilibria are attained by setting a value of velocity of money to be 0.52, with all other model values remaining the same as before.

One of the Disequilibria can be triggered, as before, by reducing the amount of basic consumption from 101. Equilibria can be restored by introducing fiscal policy as before, with a skyrocketing government debt accumulated. This simulation is left to the reader.

There is no way, however, of introducing monetary policy in our model, simply because no central bank exists to create money supply within the system.

Price Flexibility

Let us now trigger disequilibria in a different way by introducing a technological progress of 0.3 % annually.

As Figure 7.31 indicates, aggregate demand and production fail to catch up with full production around the period of 14 due to the increase in its productivity. Under the circumstance, let us allow a price flexibility by setting a value of ratio elasticity of effect on price to be 1.5.

Figure 7.32 illustrates how a production gap can be filled with business cycles.

Figure 7.33 illustrates aggregate demand and supply curves.

7.8 Conclusion

In this chapter, we have successfully built a real part of macroeconomic system, based on our analytical tool of double entry accounting system. Our model comprises dynamic processes of determining GDP, interest rate and price level. It integrates both Keynesian and neoclassical frameworks, starting first with a standard Keynesian model, then expanding it as an aggregate demand equilibrium model of IS-LM curves, and finally introducing neoclassical long-run feature of price flexibility.

From the analysis of our SD macroeconomic modeling, some of the main features we have obtained are as follows.

- A standard Keynesian macroeconomic adjustment process overshoots an equilibrium GDP when SD inventory adjustment process is introduced.

- Under a balanced budget, a reduction in ramp-sum taxes does not stimulate the aggregate demand.

- Under a Keynesian sticky price, full capacity equilibrium cannot be sustained by monetary and fiscal policies unless they are constantly fine-tuned.

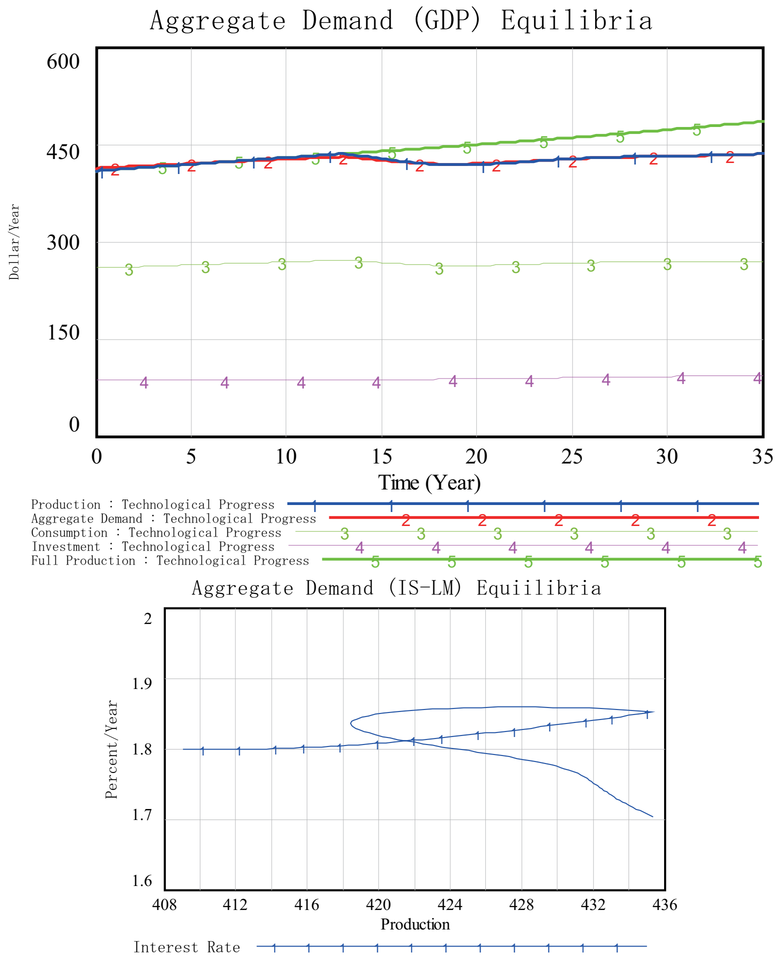

Figure 7.31: Disequilibria triggered by A Technological Progress

- Fiscal policy to attain full capacity equilibrium will skyrocket the government debt.
- Keynesian aggregate demand equilibria can be presented as loci of the intersections of IS-LM curves.
- Specifically, fiscal policy crowds out the investment opportunities by increasing interest rate. At the same time government debt continues to accumulate, which may eventually leads to an incredibility of government securities and a total collapse of the economy with worsening production capacity caused by the decrease in investment.
- Under a flexible price, aggregate demand equilibria can no longer be at-

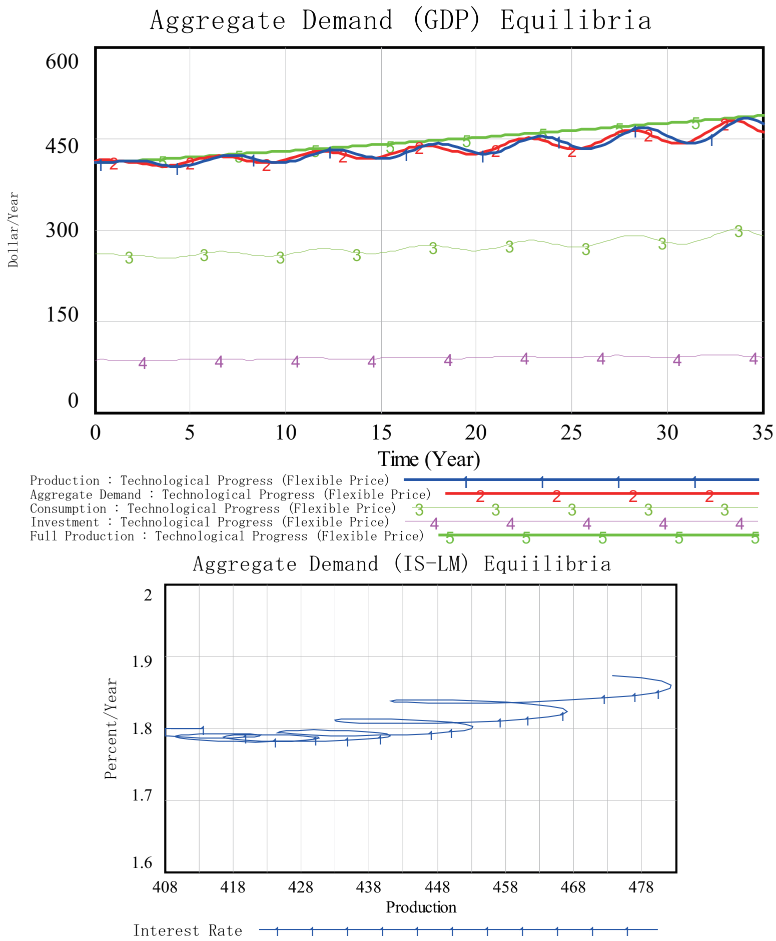

Figure 7.32: Business Cycles under Price Flexibility

tained, instead production and aggregate demand alternates. Moreover, they fluctuates around a full capacity output level.

- Under such circumstances, monetary and fiscal policies might be no longer effective as a tool to attain full capacity equilibrium.

- When money supply is fixed under a flexible price, price and interest rate continue to fluctuate.

- When money supply is changed proportionately to an economic growth rate, price and interest rate, as well as real money supply, begin to fluctuate larger than in the fixed money supply. Moreover, price and interest rate fluctuates oppositely to money supply.

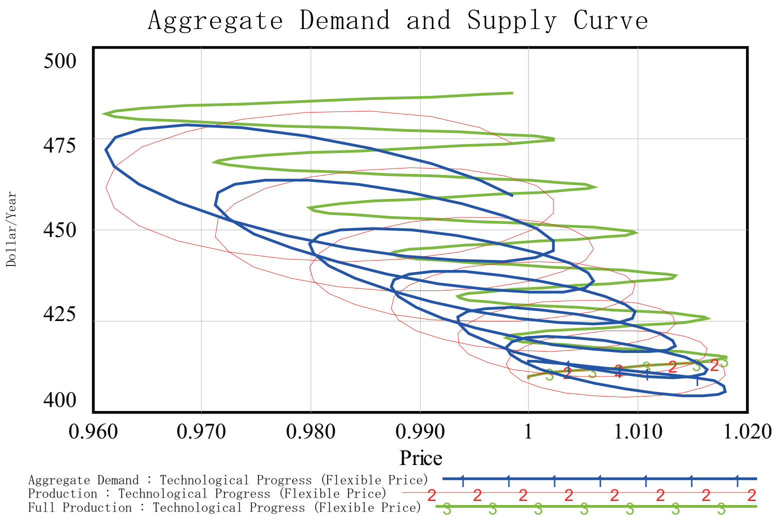

Figure 7.33: Aggregate Demand and Supply Curves

- Keynesian IS-LM model thus presented above has a serious limitation; that is, money supply is exogenously given.

It may be too early to confirm some of the above features until we will integrate the model presented in this chapter with the model of money supply in the previous two chapters, because an endogenous change in money supply may trigger an overall feedback reactions among all macroeconomic sectors, and the mechanistic change in money supply introduced here might be misleading.

Apparently, this leads to our next target toward the integration of the real and monetary economic models developed so far.

Appendix: SD Macroeconomic Model

Core part of our SD macroeconomic model is described as follows.

$$Y_{full} = \frac{1}{\theta}K \quad \text{(Full Capacity Output)} \tag{7.45}$$
$$Y = Min(Y_{full},\ Y^D) \quad \text{(Production Decision)} \tag{7.46}$$
$$Y^D = (I^*_{nv} - I_{nv}) + AD \quad \text{(Desired Production)} \tag{7.47}$$
$$\frac{dI_{nv}}{dt} = Y - AD \quad \text{(Inventory Adjustment)} \tag{7.48}$$
$$AD = C + I + G \quad \text{(Aggregate Demand)} \tag{7.49}$$
$$C = C_0 + cY_d \quad \text{(Consumption Decisions)} \tag{7.50}$$
$$Y_d = Y - T - \delta K \quad \text{(Disposable Income)} \tag{7.51}$$
$$T = T_0 + tY - T_r \quad \text{(Tax Revenues)} \tag{7.52}$$
$$I = \frac{I_0}{i} - \alpha i \quad \text{(Investment Decisions)} \tag{7.53}$$
$$\frac{dK}{dt} = I - \delta K \quad \text{(Net Capital Accumulation)} \tag{7.54}$$
$$\frac{dG}{dt} = gG \text{ (or } G = \beta T) \quad \text{(Government Expenditures)} \tag{7.55}$$
$$\frac{dP}{dt} = \Psi(Y^D - Y_{full}, I^*_{inv} - I_{inv}) \quad \text{(Price Adjustment)} \tag{7.56}$$
$$m^s = \frac{M^s}{P}V \quad \text{(Real Money Supply)} \tag{7.57}$$
$$m^d = aY - bi \quad \text{(Demand for Money)} \tag{7.58}$$
$$\frac{d\ i}{dt} = \Phi(m^d - m^s) \quad \text{(Interest Adjustment)} \tag{7.59}$$

This macroeconomic model consists of 15 equations with 15 unknown variables; that is, $Y_{full}, K, Y, Y^D, I_{nv}, AD, C, I, G, Y_d, T, I, P, MS, MD)$.

Chapter 8

Integration of Real and Monetary Economies

In the previous three chapters, monetary and real parts of macroeconomies are built separately. In this chapter[1], these three separate models are integrated to present a complete macroeconomic dynamic model consisting of real and monetary parts of macroeconomies. The integrated model is aimed to be generic, out of which diverse macroeconomic behaviors are shown to emerge. Specifically equilibrium growth path, business cycles and government debt issues are discussed in this chapter.

8.1 Macroeconomic System Overview

This chapter tries to integrate real and monetary parts of the macroeconomy that have been so far analyzed separately in the previous chapters [Companion model: Nominal GDP.vpm]. For this purpose, at least five sectors of the macroeconomy have to play macroeconomic activities simultaneously; that is, producers, consumers, banks, government and central bank. Figure 8.1 illustrates the overview of a macroeconomic system in this chapter, and shows how these macroeconomic sectors interact with one another and exchange goods and services for money. Foreign sector is still excluded from the current analysis.

The reader will be reminded that the integrated model to be developed in this chapter is a generic one by its nature, and does not intend to deal with some specific issues our macroeconomy is currently facing. Once such a generic macroeconomic model is build, we believe, any specific macroeconomic issue could be challenged by bringing real data in concern to this generic model without major structural changes in this integrated model.

[1] This chapter is partly based on the paper: Integration of Real and Monetary Sectors with Labor Market – SD Macroeconomic Modeling (3) – in “Proceedings of the 24th International Conference of the System Dynamics Society”, Nijmegen, The Netherlands, July 23 - 27, 2006. (ISBN 978-0-9745329-5-0)

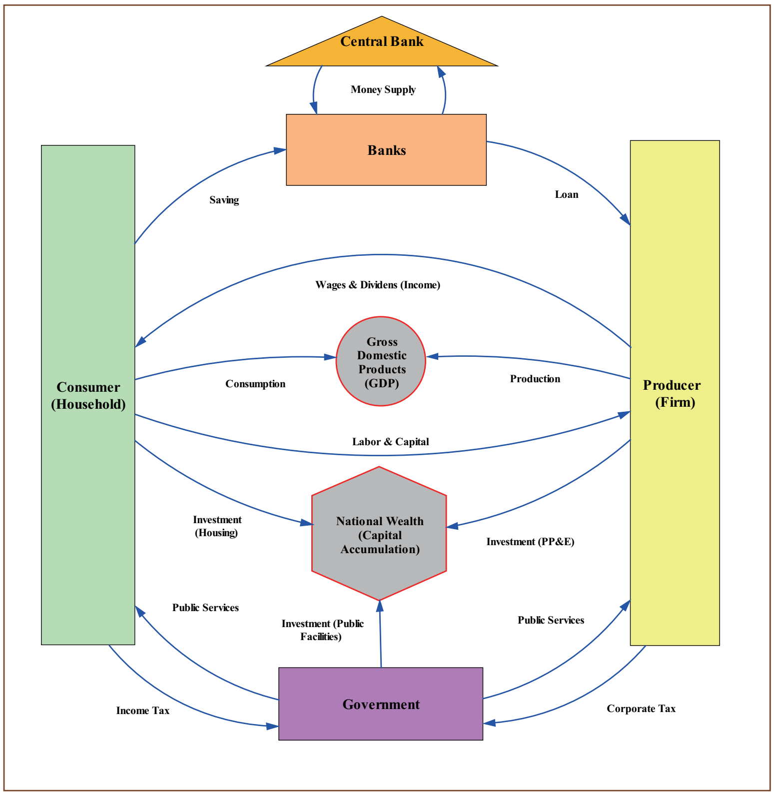

Figure 8.1: Macroeconomic System Overview

8.2 Changes for Integration

Let us now start to explain some major changes made to the previous models in order to integrate them.

Nominal and Real Units

In the previous models, all macroeconomic variables are assumed to have a dollar unit without specifically distinguishing nominal and real terms. In other words, GDP and other variables in the real sector of the previous model are implicitly interpreted as having real value of dollar.

In the integrated model of real and monetary sectors, variables in real production sector such as output and capital stock must have physical unit (which is specified as DollarReal in this chapter), while all market transactions among all sectors are made in terms of nominal unit of money; that is, Dollar. To convert a physical unit to a monetary unit for transactions between real and monetary sectors, price P is used that has a unit of Dollar/DollarReal.

Real and Nominal Interest Rates

With the introduction of real and nominal terms, interest rate introduced in the previous chapter has also to be reinterpreted as a real interest rate, meanwhile interest rate used for market transactions has to be nominal. The relationship between these two interest rates are shown by the following relation[2].

$$\text{Nominal interest rate} = \text{Real interest rate} + \text{Inflation rate} \tag{8.1}$$

Investment Order Placement and Delay

To reflect the fact that investment process takes time, a capital stock under construction is newly added to the capital accumulation process. That is, new investment is accumulated to the capital stock under construction, out of which capital stock (property, plant & equipment) is accumulated after a completion of capital under construction. This revision is illustrated in Figure 8.2.

Investment Function

The amount of desired investment is obtained as the difference between desired and actual capital stock plus depreciation such that

$$I(i) = \frac{K^*(i) - K}{\mathit{Time\ to\ Adjust\ Capital}} + \delta K \tag{8.2}$$

where δ is depreciation rate. The desired capital stock could be approximated by

$$K^* = \frac{\alpha(1-t)Y^*}{i+\delta} \tag{8.3}$$

where α is exponent on capital, and t is excise tax rate[3]. Furthermore, the desired output Y^* is represented by the variable: Aggregate Demand Forecasting (Long-run) as illustrated in Figure 8.2.

The new investment function obtained above replaces our previous investment function that is determined by the interest rate:

$$I(i) = \frac{I_0}{i} - \alpha i \tag{8.4}$$

where α is an interest sensitivity of investment. Surely, our model is open to any type of investment function which the reader considers to be more appropriate.

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[2]This formulation is the so-called Fisher effect; to be precise, nominal rate of interest is the sum of real interest, inflation rate and their cross product. See [58, pp. 320-323] for the detailed discussion on the real rate of interest in relation with the uniform rate of profit.

[3]For the derivation of this equation, see the section of production function in the next chapter.

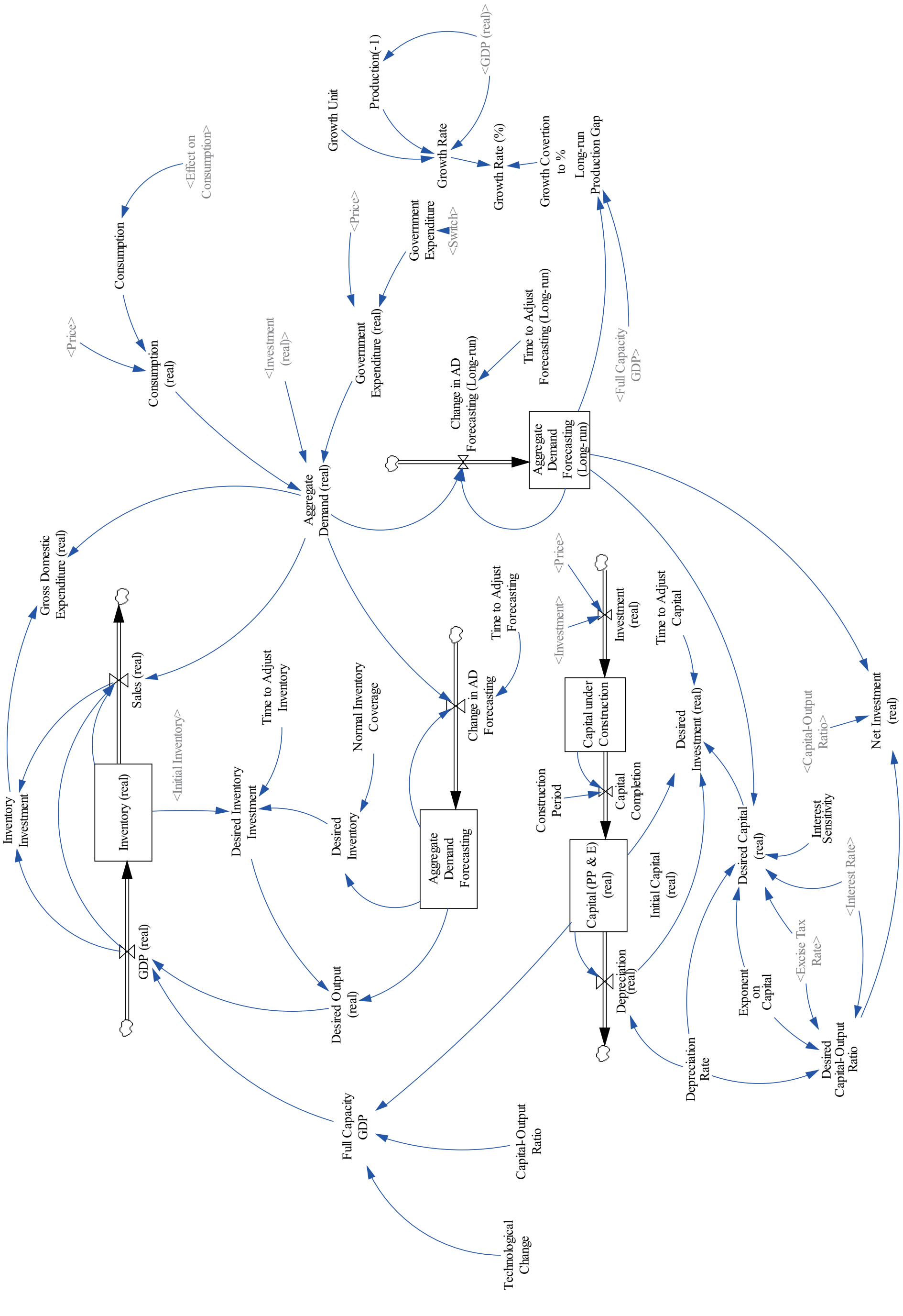

Figure 8.2: Real Production of GDP

Consumption Function

In the previous model, consumption is assumed to be determined by a constant marginal propensity to consume as expressed in equation (7.3). Now that nominal price is explicitly used in the integrated model, it's appropriate to consider that consumers respond to a price level. Specifically, marginal propensity to consume is now assumed to be dependent on a relative price elasticity of consumption such that

$$c(P) = \frac{c}{\left(\frac{P}{P_0}\right)^e} \tag{8.5}$$

where P_0 is an initial price level and e is a relative price elasticity of consumption. As a relative price level goes up, marginal propensity to consume gets smaller. In this way, consumption is affected by the relative size of prices and its elasticity.

Accordingly, the revised consumption function becomes

$$C(P) = C_0 + c(P)Y_d \tag{8.6}$$

The consumption function thus defined has a feature of a downward-sloping demand function, similar to a demand curve of microeconomic level of consumers.

Money Supply and Demand for Money

The integrated model here intends to be a complete system of macroeconomy, and money supply and demand for money have to be sought *within the system itself*. This revision is partly made in the previous chapter under the section of "A Comprehensive IS-LM Model".

Let us consider money supply first. In the previous model it is treated as exogenously fixed parameter, because there exists no mechanism to change money supply within the system. With the introduction of the central bank, money supply is now created within the system. It is here defined as follows[4]:

$$\text{Money Supply} = \text{Currency in Circulation} + \text{Deposits} \tag{8.7}$$

Currency in circulation may be represented by the sum of cash stocks held by consumers, producers, government and banks, while deposits are the amount of money consumers deposit with banks. For instance, whenever consumers purchase consumption goods from producers, the ownership of money changes hands from consumers to produces, and in the model this movement is represented as a decrease in consumers' stock of cash and a simultaneous increase in producers' stock of cash. In this way currency in circulation keeps moving among the cash stocks of consumers, producers, government and banks, decreasing one cash stock and increasing another cash stock simultaneously.

On the other hand, demand for money consists of three motives: transaction, precautionary and speculative motives, according to the standard textbooks

[4]In our simple model, it may not be needed to classify monetary aggregates further into M1, M2 and M3.

such as [30]. In our previous IS-LM model (equation (7.32)), real demand for money is formalized as consisting of transaction motives and speculative motives. Money demanded for market transactions in our integrated model is nothing but cash outflows by consumers, producers, government and banks. Consumers need cash to buy consumption goods, and producers need cash to make investment. And these needs for transaction have to be met out of their cash stocks.

As to a speculative motive, demand for cash is assumed to move back and forth freely between deposits and cash stocks of consumers to maintain a certain level of currency ratio (= Cash / Deposits).

$$\text{Cash Demand} = \frac{\text{Currency Ratio * Deposits - Cash}}{\text{Cashing Time}} \tag{8.8}$$

Currency ratio in turn is assumed to be determined by nominal interest rate. Specifically, whenever nominal interest rate drops, currency ratio tends to rise so that consumers increase their demand for cash. As an extreme case if nominal interest rate drops to the level of a so-called liquidity trap (almost close to zero per cent in late 1990s in Japan), currency ratio is assumed to become one so that no deposits are made. In this way, speculative demand for cash is made dependent on the nominal interest rate in the model.

Demand for money (nominal) thus interpreted has a unit of dollar/year, while money supply as a stock of cash has a unit of dollar. Therefore, money supply has to be multiplied by its velocity that has a unit 1/year, to secure unit equivalence in SD model as already formalized in the equation (7.22)[5].

Figure 8.3 illustrates our revised model of money supply and demand for money. It also shows adjustment processes of real interest rate and price level, which is already discussed in the section of a comprehensive IS-LM model in the previous chapter.

Discount Loans by the Central Bank

In this integrated model, banks are assumed to make loans to producers as much as desired so long as their vault cash is available. Thus, they are persistently in a state of shortage of cash as well as producers. In the case of producers, they could borrow enough fund from banks. From whom, then, should the banks borrow in case of cash shortage?

In a closed economic system, money or currency has to be created within the system. Under the current financial system, only the central bank is endowed with a power to create currency within the system, and make loans to the commercial banks as a last resort of currency provider to avoid bankruptcies of the whole economic system. This process of lending money by creating (or printing) currency is known as *money out of nothing*.

[5]This part of treatment for demand and supply of money corresponds to the Quantity Equation:

$$\text{Money (M) * Velocity (V) = Price (P) * Transaction (T),}$$

where V is called transaction velocity of money in [39, p. 82].

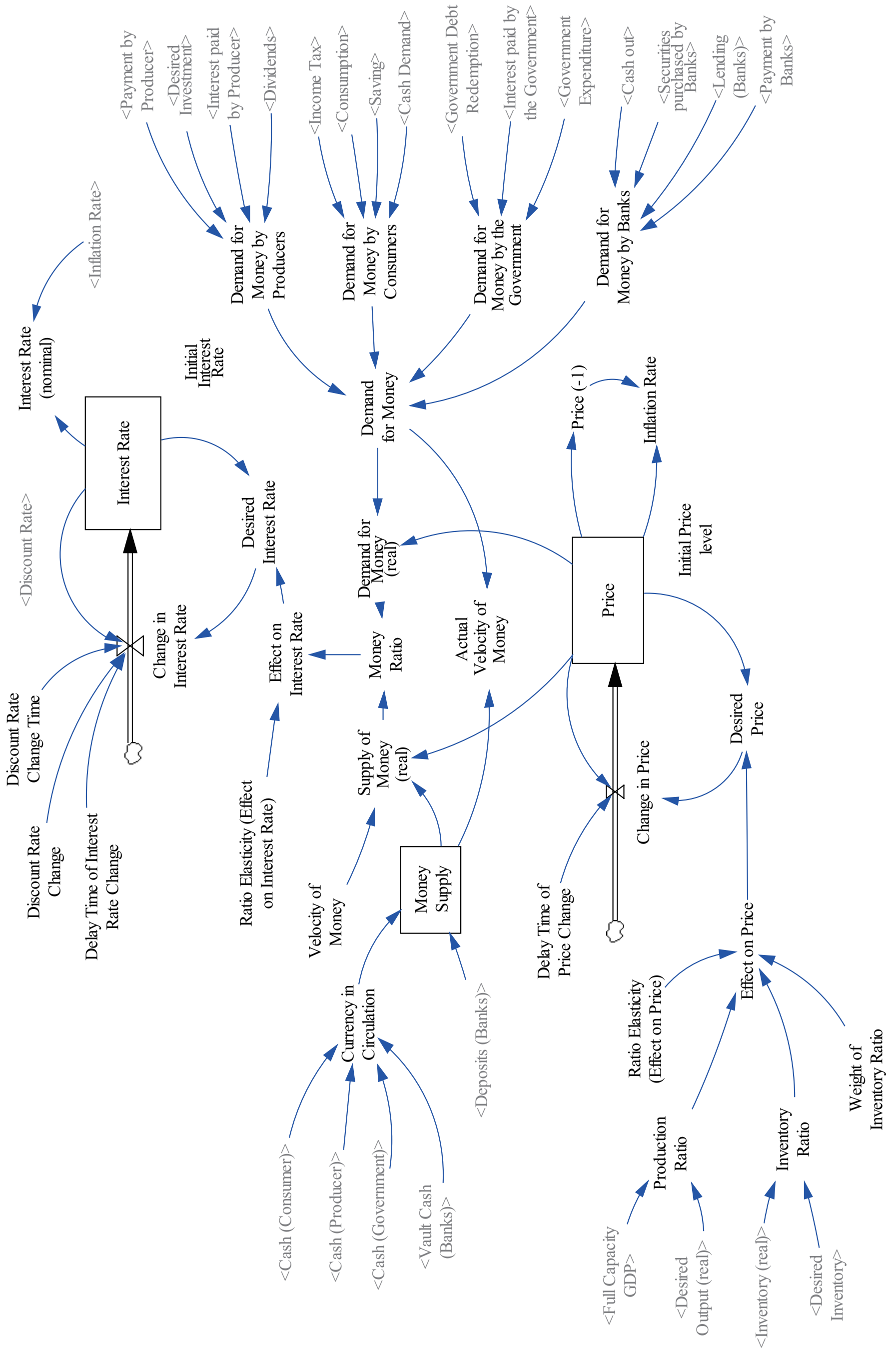

Figure 8.3: Interest Rate and Price Adjustments

Figure 8.4 indicates unconditional amount of annual discount loans and its growth rate by the central bank at the request of desired borrowing by banks. In other words, currency has to be incessantly created and put into circulation in order to sustain an economic growth under mostly equilibrium states. Roughly speaking, a growth rate of credit creation has to be in average equal to or slightly greater than the economic growth rate as suggested by the right hand diagram of Figure 8.4.

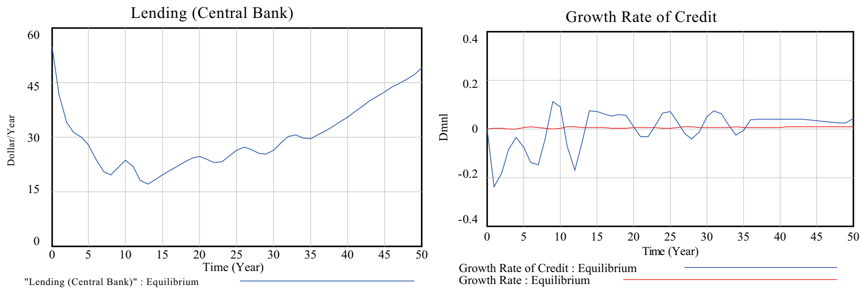

Figure 8.4: Lending by the Central Bank and its Growth Rate

In this way, the central bank begins to exert an enormous power over the economy through its credit control. What happens if the central bank fails to supply enough currency intentionally or unintentionally? An influential role of the central bank which caused economic bubbles and the following burst in Japan during 90's was completely analyzed by Warner in [54, 55]. Our macroeconomic model might provide an analytical foundation to support his new findings in the role of the central bank.

Four Types of Interest Rates

Due to the introduction of the central bank a fraction of bank deposits have to be reserved as required reserve with the central bank. This is called *a fractional reserve banking system*. Accordingly, the amount of loans banks can make to producers becomes less than that of deposits by consumers, and, if the same interest rate is applied as in the previous model, their interest income from loans becomes less than their interest payment against deposits. To avoid this negatively retained earnings of the banks, a higher interest rate has to be applied to the loans, which is called here *a prime rate*.

$$\text{Prime rate} \; > \; \text{Nominal interest rate} \tag{8.9}$$

The difference between these two interest rates is made large enough to avoid negatively retained earnings of banks. Moreover, it is assumed here that positive earnings, if any, will be completely distributed among bank workers as consumers.

With the introduction of credit loan by the central bank, another type of interest rate needs to be applied to the transaction, which is called *a discount* *rate*. The central bank is given a power to set its rate as a part of its monetary policies whenever making loans to commercial banks. It is set to be 0.8%, or 0.008 in our model.

Now the economy has four different types of interest rates; discount rate, real rate of interest, nominal rate of interest, and prime rate[6]. How are they related one another? It is assumed that the initial value of the real rate of interest (which is set to be 0.02 in our model) is increased by the amount of discount rate such that

$$\text{Initial value} = \text{initial interest rate} + \text{discount rate} \tag{8.10}$$

Nominal rate of interest and prime rate are assumed to be determined in our previous models as

$$\text{Interest rate (nominal)} = \text{real interest rate} + \text{inflation rate} \tag{8.11}$$

and

$$\text{Prime rate} = \text{interest rate (nominal)} + \text{prime rate premium}, \tag{8.12}$$

where prime rate premium is set to be 0.03 in our model to attain positive profits to the banks. Accordingly, discount rate affect all of the other three types of interest rate, giving a legitimacy of monetary policies to the central bank.

8.3 Transactions Among Five Sectors

Let us now describe some transactions by the central bank that is additionally brought to the model here. For the convenience to the reader, let us also repeat some of the transactions, with some revisions, by producers, consumers, government and banks that were already presented in the previous chapters.

Producers

Major transactions of producers are, as illustrated in Figure 8.5, summarized as follows.

- Out of the GDP revenues producers pay excise tax, deduct the amount of depreciation, and pay wages to workers (consumers) and interests to the banks. The remaining revenues become profits before tax.
- They pay corporate tax to the government out of the profits before tax.
- The remaining profits after tax are paid to the owners (that is, consumers) as dividends.

[6]To be precise, an overnight rate needs to be added, which is called a federal fund rate in the United States, or a call rate in Japan. It is the interest rate applied to the loans of reserved fund at the central bank by commercial banks. Current monetary policy is said to use this rate as a target rate so that it could influence all the other interest rates. In this model, it is represented by the discount rate for simplicity.

- Producers are thus constantly in a state of cash flow deficits. To continue new investment, therefore, they have to borrow money from banks and pay interest to the banks.

Consumers

Transactions of consumers are illustrated in Figure 8.6, some of which are summarized as follows.

- Consumers receive income as wages and dividends from producers.
- Financial assets of consumers consist of bank deposits and government securities, against which they receive financial income of interests from banks and government. (In this chapter, no corporate shares are assumed to be held by consumers).
- In addition to the income such as wages, interests, and dividends, consumers receive cash whenever previous securities are partly redeemed annually by the government.
- Out of these cash income as a whole, consumers pay income taxes, and the remaining income becomes their disposal income.
- Out of their disposal income, they spend on consumption. The remaining amount are either saved or spent to purchase government securities.

Government

Transactions of the government are illustrated in Figure 8.7, some of which are summarized as follows.

- Government receives, as tax revenues, income taxes from consumers and corporate taxes from producers as well as excise tax on production.
- Government spending consists of government expenditures and payments to the consumers for its partial debt redemption and interests against its securities.
- Government expenditures are assumed to be endogenously determined by either the growth-dependent expenditures or tax revenue-dependent expenditures.
- If spending exceeds tax revenues, government has to borrow cash from banks and consumers by newly issuing government securities.

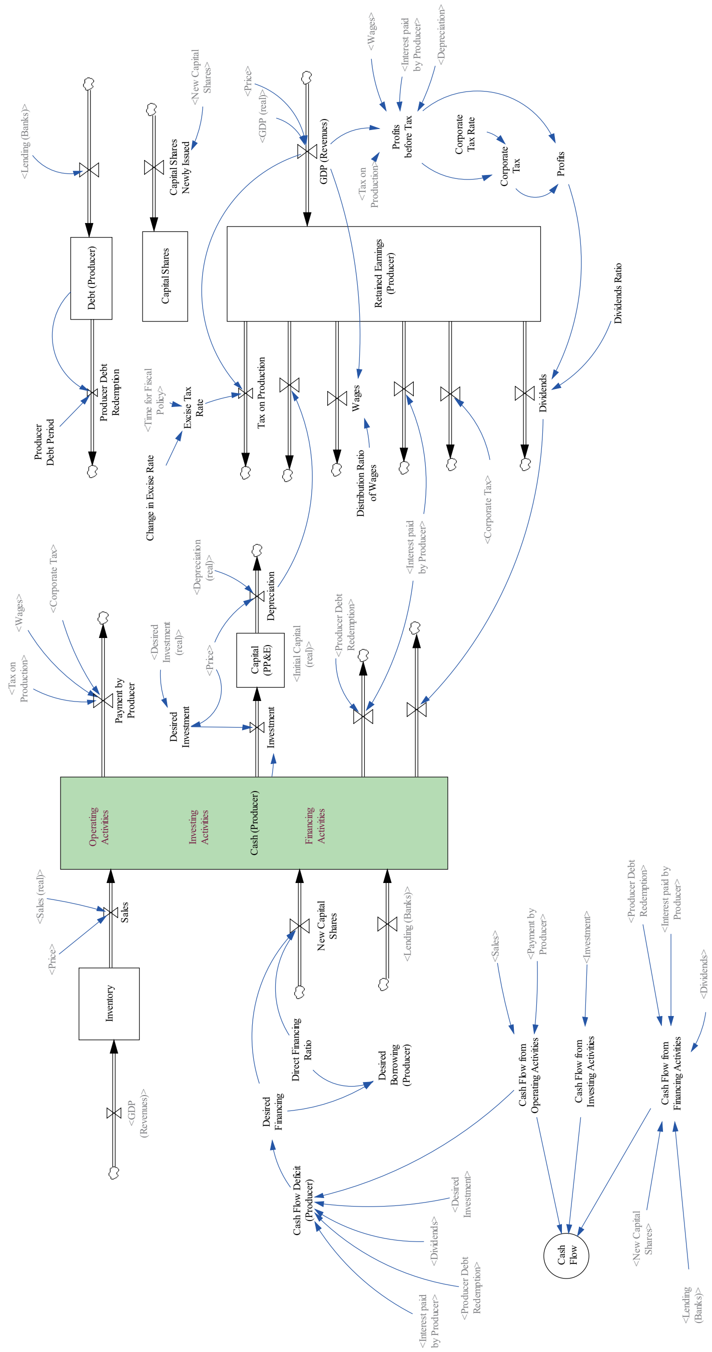

Figure 8.5: Transactions of Producers

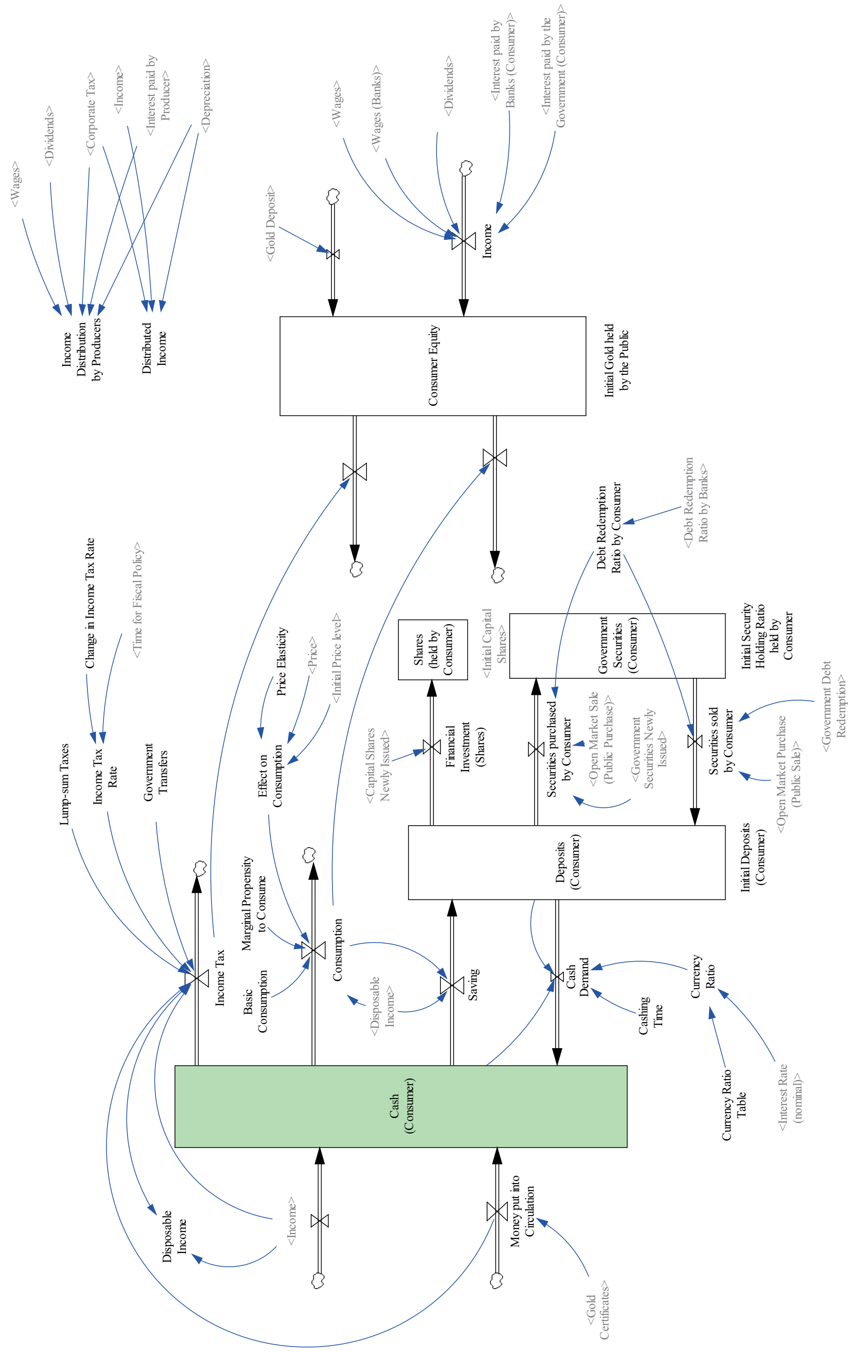

Figure 8.6: Transactions of Consumer

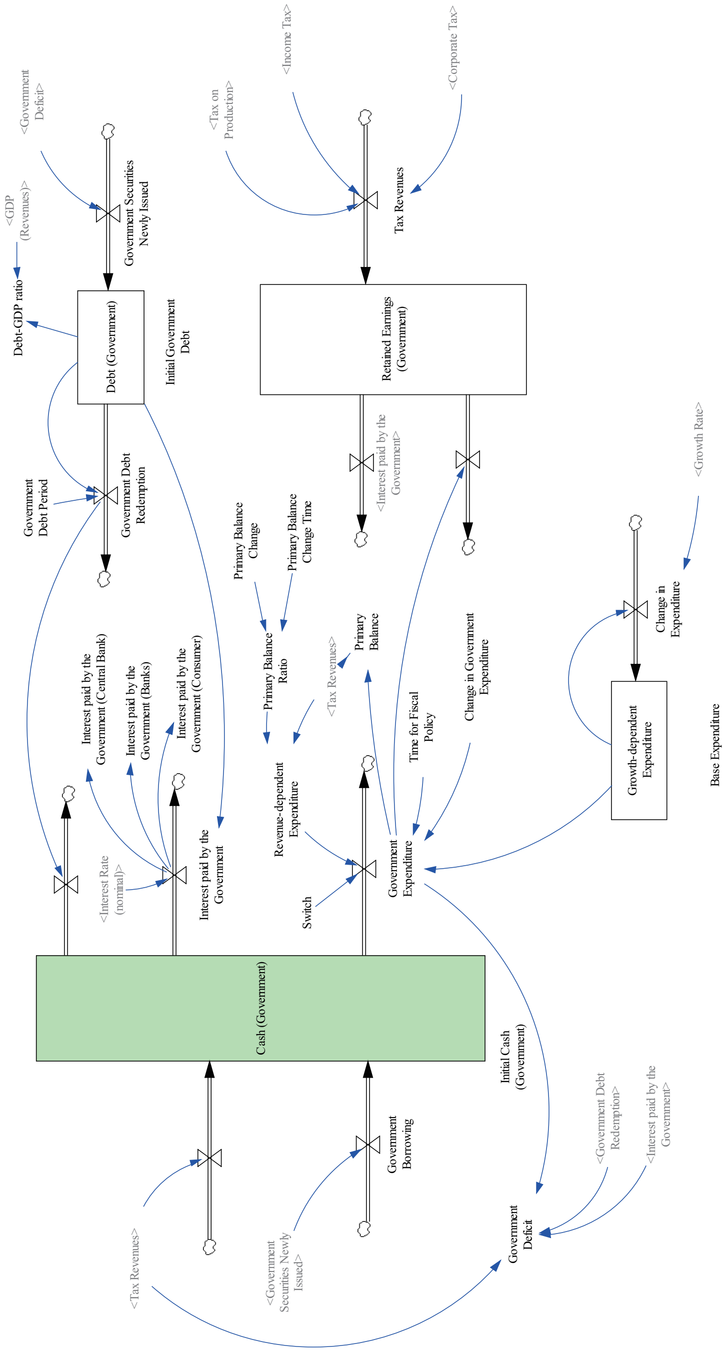

Figure 8.7: Transactions of Government

Banks

Transactions of banks are illustrated in Figure 8.8, some of which are summarized as follows.

- Banks receive deposits from consumers, against which they pay interests.
- They are obliged to deposit a fraction of the deposits as the required reserves with the central bank (which is called *a fractionalreserve banking system*).
- Out of the remaining deposits they purchase government securities, against which interests are paid from the government.
- Then, loans are made to producers and they receive interests for which a prime rate is applied.
- Their retained earnings thus become interest receipts from producers and government less interest payment to consumers. Positive earning will be distributed among bank workers as consumers.

Central Bank

In this integrated model, the central bank plays a very important role of providing a means of transactions and store of value; that is, currency. To make a story simple, its sources of assets against which currency is issued are confined to gold, discount loans and government securities. The central bank can control the amount of money supply through the amount of monetary base consisting of currency outstanding and reserves over which it has a direct power of control. This is done through monetary policies such as a manipulation of required reserve ratio and open market operations as well as direct lending control.

Transactions of the central bank are illustrated in Figure 8.9, some of which are summarized as follows.

- The central bank issues currency or money (historically gold certificates) against the gold deposited by the public.
- It can also issue currency by accepting government securities through open market operation, specifically by purchasing government securities from the public (consumers) and banks. Moreover, it can issue currency by making discount loans to commercial banks. (These activities are sometimes called *money out of nothing.*)
- It can similarly withdraw currency by selling government securities to the public (consumers) and banks, and through debt redemption by banks.
- Banks are required by law to reserve a certain fraction of deposits with the central bank. By controlling this required reserve ratio, the central

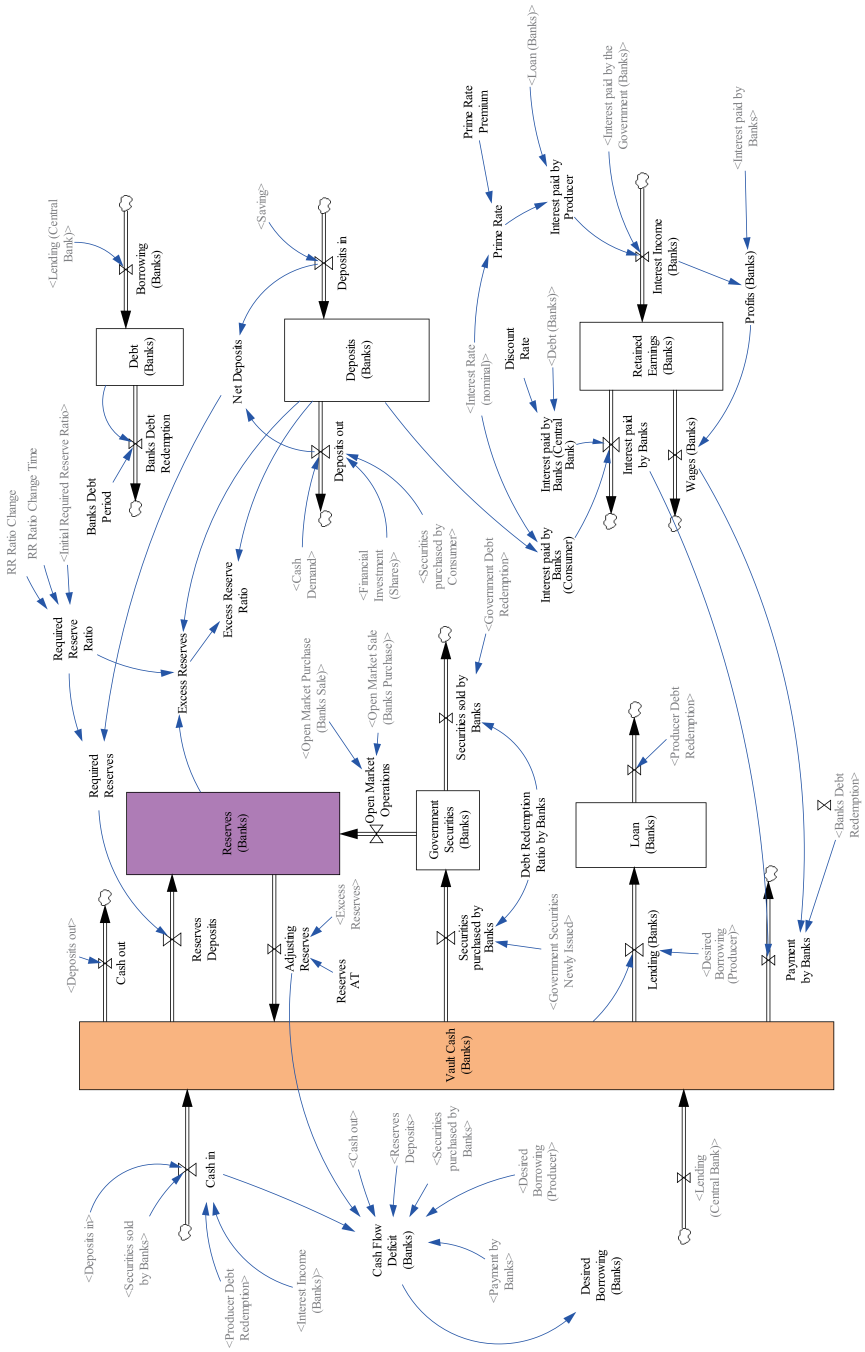

Figure 8.8: Transactions of Banks

bank can control the monetary base directly. The central bank can, thus, control the amount of money supply through monetary policies such as open market operations, reserve ratio and discount rate.

- Another powerful but hidden control method is through its direct influence over the amount of discount loans to banks (known as *window guidance* in Japan.)

8.4 Behaviors of the Integrated Model

Mostly Equilibria in the Real Sector

The integrated model is now complete. It is a generic model, out of which diverse macroeconomic behaviors will be produced. Let us start with an equilibrium growth path of the macroeconomy. In the previous IS-LM model, Keynesian aggregate demand equilibrium is defined as an equilibrium state where aggregate demand is equal to production. Moreover, it is also emphasized that the Keynesian aggregate demand equilibrium is not a full capacity equilibrium.

Let us call an equilibrium state *a full capacity aggregate demand equilibrium* if the following two output and demand levels are met:

$$\text{Full Capacity GDP} = \text{Desired Output} = \text{Aggregate Demand} \tag{8.13}$$

If the economy is not in the equilibrium state, then actual GDP is determined by

$$\text{GDP} = \text{MIN (Full Capacity GDP, Desired Output)} \tag{8.14}$$

In other words, if desired output is greater than full capacity GDP, then actual GDP is constrained by the production capacity, meanwhile in the opposite case, GDP is determined by the amount of desired output which producers wish to produce, leaving the capacity idle.

Does the equilibrium state, then, exist in the sense of full capacity GDP? By trial and error, mostly equilibrium states are acquired in the integrated model when a ratio elasticity of the effect on price e is 1, and a weight of inventory ratio ω is 0.1, as illustrated in Figure 8.10.

The reader may wonder why this is a state of mostly equilibria, because growth rates and inflation rates still fluctuate as shown in Figure 8.11. Specifically, growth rates fluctuates between 0.8% and - 0.2%, and inflation rates between 0.2% and - 0.2%. Our heart pulse rate, even a healthy person, fluctuates between 60 and 70 per minute. This is a normal state. In a similar fashion, it is reasonable to consider these fluctuations as normal equilibrium states.

In what follows, these equilibrium states are used as a benchmarking state of the comparison, and illustrated by line 2 or red lines in the Figures below.

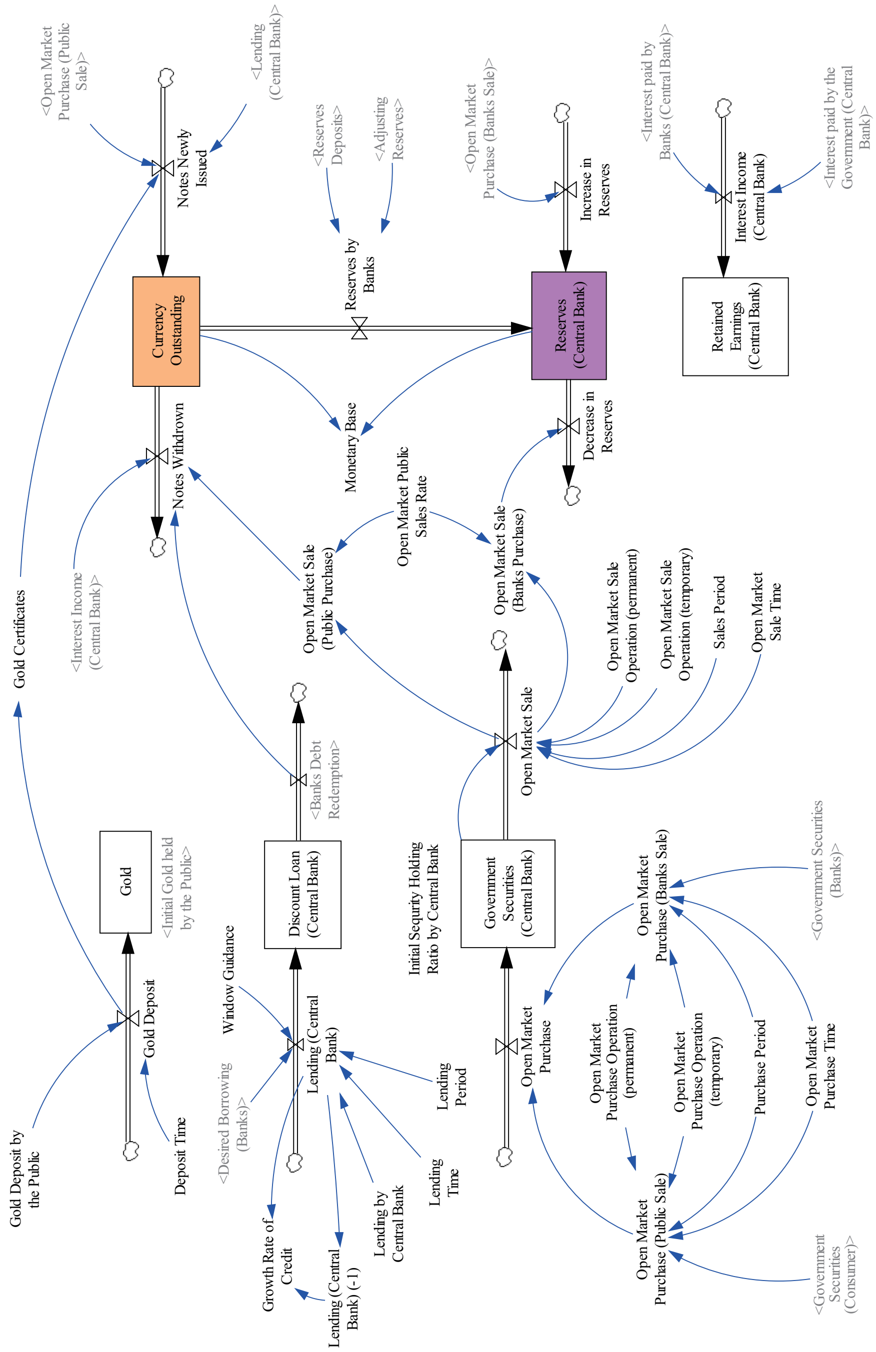

Figure 8.9: Transactions of Central Bank

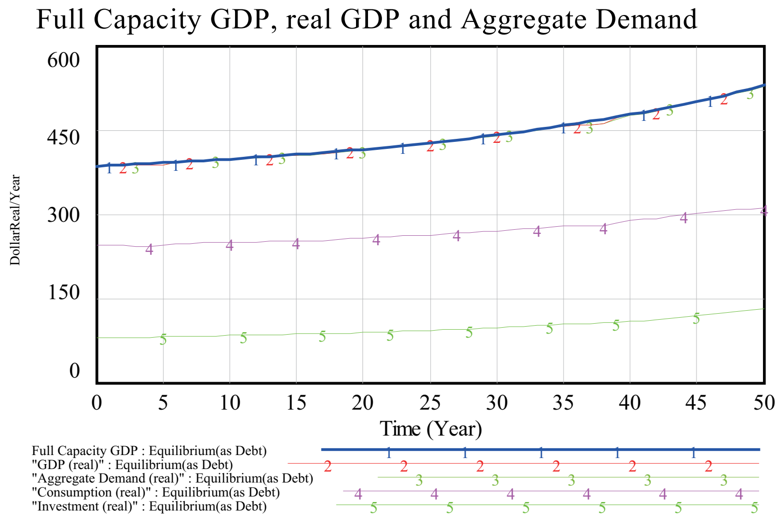

Figure 8.10: Mostly Equilibrium States

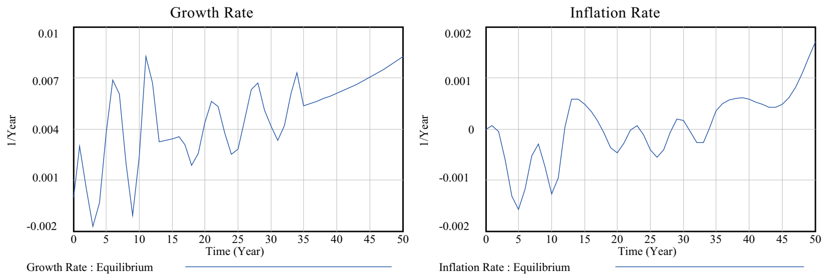

Figure 8.11: Growth and Inflation Rates of Mostly Equilibrium States

Fixprice Disequilibria

We are now in a position to make some analytical simulations for the model. First, let us show that without price flexibility it's hard to attain mostly equilibrium states. When price is fixed; that is, ratio elasticity of the effect on price is set to be zero, disequilibria begin to appear all over the period. Figure 8.12 illustrates how fixprice causes disequilibria everywhere. The economy seems to stagger; that is, economic growth rates become lower than the equilibrium ones over many periods.

Business Cycles by Inventory Coverage

From now on, let us assume the mostly equilibrium path. One of the interesting questions is to find out a macroeconomic structure that may produce economic fluctuations or business cycles. How can the above equilibrium growth path be

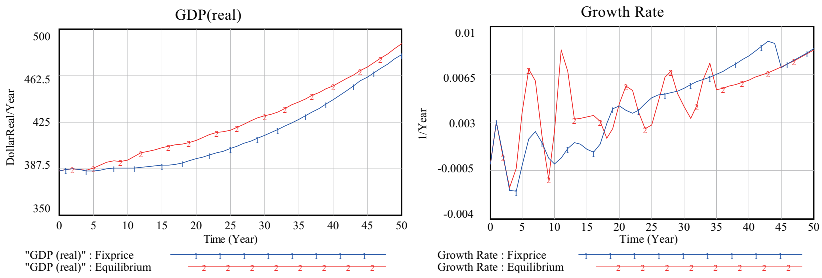

Figure 8.12: Fixprice and Mostly Equilibrium States

broken and business cycles will be triggered?

Our integrated model can successfully produce at least two ways of causing macroeconomic fluctuations. First, they can be caused by increasing normal inventory coverage period, Specifically, suppose the normal inventory coverage period increases from 0.25 or 3 months to 0.42 or about 5 months. The economy, then, begins to be troubled with short business cycles of about 9 years as Figure 8.13 portrays.

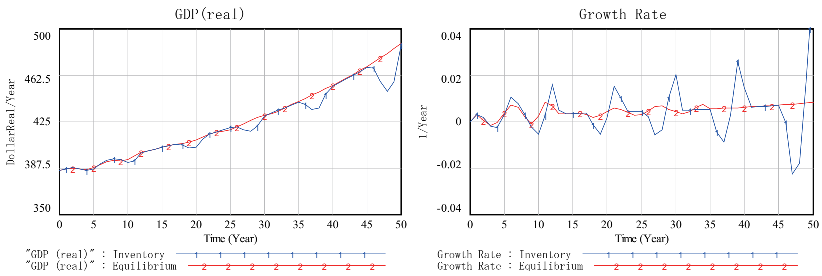

Figure 8.13: Business Cycles by Inventory Coverage

Business Cycles by Elastic Price Fluctuation

Secondly, the equilibrium growth path can also be broken and business cycle is triggered, though, in a totally different fashion. This time let us assume that a price response to the excess demand for products and inventory gap becomes more sensitive so that ratio elasticity now becomes elastic with a value of 1.3 from 1, and a weight of inventory ratio to production ratio becomes 0.6 from 0.1. Again, the economy is thrown into business cycle of between 5 and 8 years as depicted in Figure 8.14.

In this way, two similar business cycles are triggered, out of the same equilibrium growth path, by two different causes; one by an increase in inventory coverage period, and the other by the elasticity of price changes. The ability to produce these different behaviors of business cycles and economic fluctuations indicates a richness of our integrated generic model.

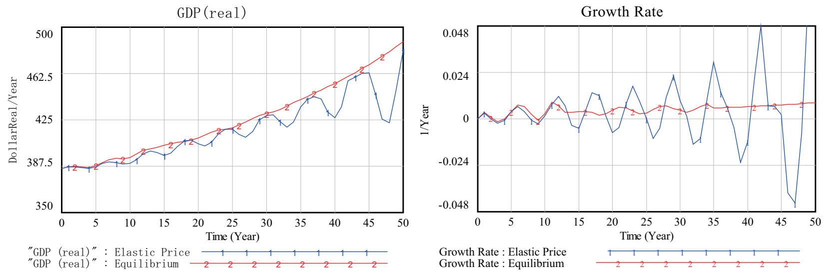

Figure 8.14: Business Cycles by Elastic Price Fluctuation

Economic Recession by Credit Crunch

With the introduction of discount credit loans to banks, the central bank seems to have acquired an almighty power to control credit. This hidden exerting power has been known in Japan as "window guidance".

To demonstrate how influential the power is, let us suppose that the central bank reduces the amount of credit loans by 10%; that is, window guidance value is reduced to 0.9 from 1. In other words, banks can borrow only 90% of the desired amount of borrowing from the central bank.

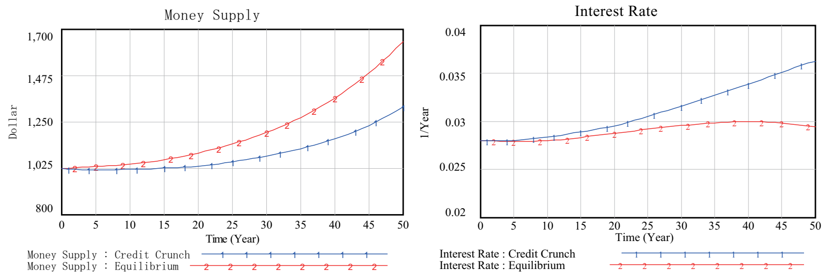

Figure 8.15: Money Supply and Interest Rate by Credit Crunch

Figure 8.15 illustrates how money supply shrinks and, accordingly, interest rate increases by the credit crunch caused by the central bank. Figure 8.16 illustrates the economy is now deeply triggered into recession in the sense that the GDP under credit crunch is always below the equilibrium GDP, and its economic growth rates seem to be lower in average than those of equilibrium with dwindling short-period business cycles. It is unexpected to see that the economic recession is provoked by the credit crunch rather than the business cycles as shown above. Economic recessions caused by the credit crunch can be said to be worse than the recessions caused by other business cycles.

As discussed above, growing economy needs new currencies to be incessantly put into circulation. If the central bank, instead of the government, is historically endowed with this important role, savvy control of credits by the central bank becomes crucial for the stability and growth of macroeconomy as demon-

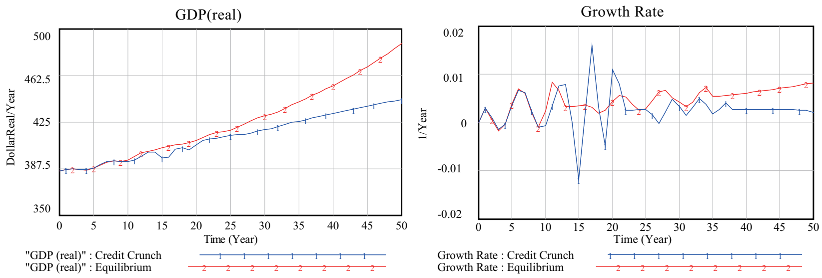

Figure 8.16: Economic Recession by Credit Crunch

strated here by the integrated model.

Monetary and Fiscal Policies for Equilibrium

So far, we have examined several states of disequilibria caused by fixprice, business cycles by inventory coverage and elastic price fluctuation, and credit crunch. Can we restore equilibrium, then ? According to the Keynesian theory, the answer is affirmative if monetary and fiscal policies are appropriately applied.

Let us consider the case of fixprice disequilibria and apply monetary policy, first. Suppose the central bank increases the purchase of government securities by 12% for 5 years starting at the year 6 (see the top left diagram of Figure 8.17). Then, money supply continues to grow gradually, and interest rate eventually starts to decrease (see top right and bottom left diagrams.). These changes in the monetary sector will eventually restore full capacity aggregate demand equilibrium (GDP=420.08) at the year 24 through the year 38 for 15 years (see the bottom right diagram). It takes 18 years for this open market operation to take its effect. Moreover, it is interesting to observe that during this period of sustained equilibrium due to the increased money supply, desired output (line 2 of the bottom right diagram) is higher than the real GDP. This does not cause inflation here due to the assumption of fixprice. Once this assumption is dropped, surely inflation arises. In this sense, monetary policy of open market purchase can be said to be inflationary by its nature.

Second scenario of restoring the equilibrium is to apply fiscal policy. In our model quite a few tools are available for fiscal policy such as changes in income tax rate, lump-sum taxes, excise rate, corporate tax rates and government expenditures. We employ here income tax rate. The reader can try other policy tools by running the model.

Facing the fixprice disequilibria, the government now decides to introduce an increase in income tax rate by 3%; that is, from the original 10% to 13 %, at the year 6 (see top left hand diagram in Figure 8.18). Under the assumption of balanced budget, or a unitary primary balanced ratio, an increase in income tax also becomes the same amount of increase in government expenditure (see top right diagram). This causes the increase in interest rate, which crowds out investment. Even so, aggregate demand is spontaneously stimulated to restore

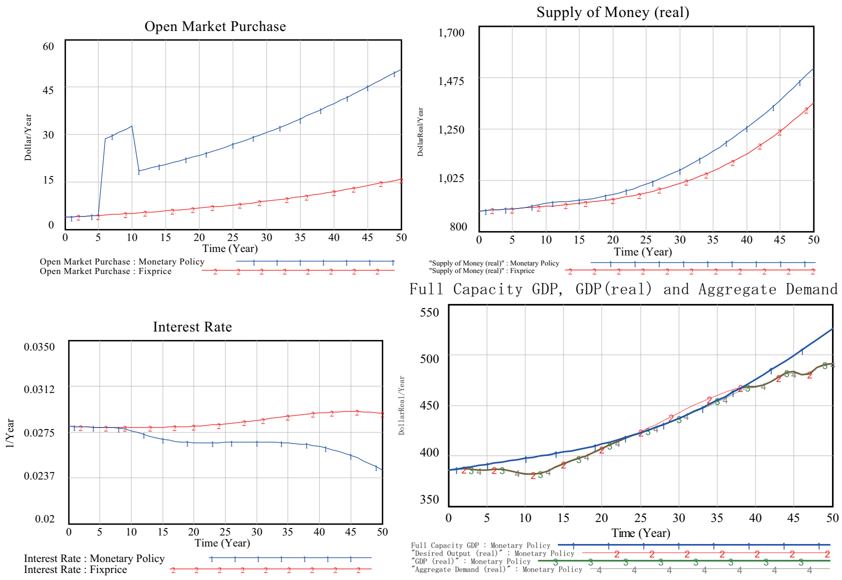

Figure 8.17: Monetary Policy: Open Market Purchase and GDP

the equilibrium (GDP = 393.89) at the year 7 through the year 26 for 20 years (see the bottom right diagram). This equilibrium is attained by a slightly higher desired output (line 2), which,however, does not trigger inflation due to the assumption of fixprice recession. Compared with the monetary policy, the effect of fiscal policy appears quickly from the next year.

In this way, our integrated generic model can provide effective scenarios of sustaining full capacity aggregate demand equilibrium growth path through monetary and fiscal policies.

Government Debt

So long as the equilibrium path in the real sector is attained by fiscal policy, no macroeconomic problem seems to exist. Yet behind the full capacity aggregate demand growth path attained in Figure 8.18, government debt continues to accumulate as the left diagram of Figure 8.19 illustrates. Primary balance ratio is initially set to one and balanced budget is assumed in our model; that is, government expenditure is set to be equal to tax revenues, as lines 1 and 2 overlap in the diagram. Why, then, does the government continue to suffer from the current deficit?

In the model government deficit is defined as

$$\text{Deficit} = \text{Tax Revenues - Expenditure - Debt Redemption - Interest} \quad (8.15)$$

Therefore, even if balanced budget is maintained, the government still has to keep paying its debt redemption and interest. This is why it has to keep borrow-

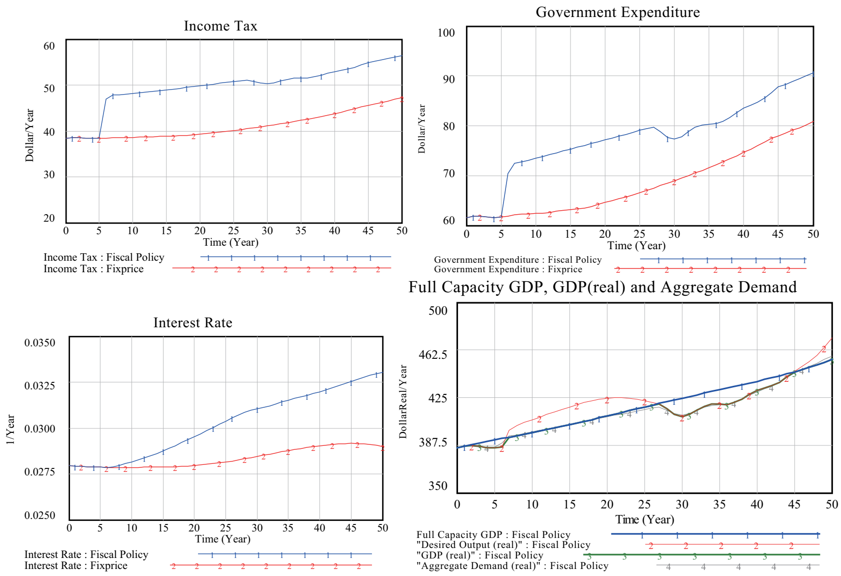

Figure 8.18: Fiscal Policy: Change in Income Tax Rate and Interest Rate

ing and accumulating its debt. Initial GDP in the model is obtained to be 386, while government debt is initially set to be 200. Hence, the initial debt-GDP ratio is around 0.52 year (similar to the current ratios among EU countries). The ratio continues to increase to 1.98 years at the year 50 in the model as illustrated in the right diagram of Figure 8.19. This implies the government debt becomes 1.98 years as high as the annual level of GDP (which is comparable to the Japanese debt ratio of 1.97 in 2010).

Can such a high debt be sustained? Absolutely no. Eventually this runaway accumulation of government debt may cause nominal interest rate to increase, because the government may be forced to pay higher and higher interest rate in order to keep borrowing, which may in turn launch a hyper inflation[7].

On the other hand, a higher interest rate may trigger a sudden drop of government security price, deteriorating the value of financial assets of banks, producers and consumers. The devaluation of financial assets may force some banks and producers to go bankrupt eventually. In this way, under such a hyper inflationary circumstance, financial crisis becomes inevitable and government is destined to collapse. This is one of the hotly debated scenarios about the consequences of the abnormally accumulated debt in Japan, whose current debt-GDP ratio is about 1.97 years; the highest among OECD countries!

Let us now consider how to avoid such a financial crisis and collapse. At the year 6 when fiscal policy is introduced to restore a full capacity aggregate demand equilibrium in the model, the economy seems to have gotten back to

[7]This feedback loop from the accumulating debt to the higher interest rate is not yet fully incorporated in the model.

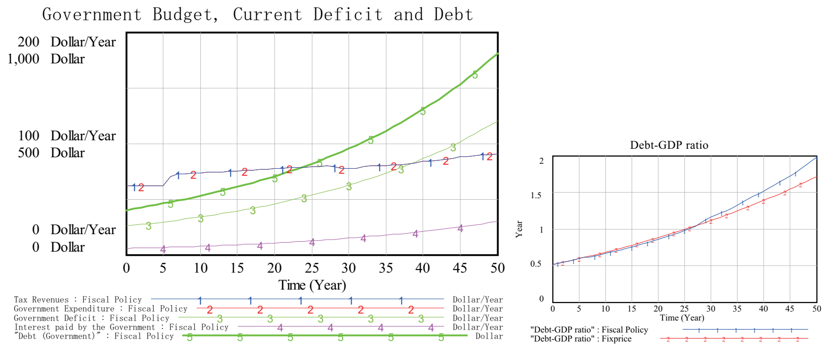

Figure 8.19: Accumulation of Government Debt

the right track of sustained growth path. And most macroeconomic textbooks emphasize this positive side of fiscal policy. A negative side of fiscal policy is the accumulation of debt for financing the government expenditure. Yet most macroeconomic textbooks neglect or less emphasize this negative side, partly because their macroeconomic framework cannot handle this negative side effect properly as our integrated model does here. In our example the debt-GDP ratio is 0.61 years at the introduction year of fiscal policy.

At the face of financial crisis as discussed above, suppose that the government is forced to reduce its debt-GDP ratio to around 0.6 by the year 50, To attain this goal, a primary balance ratio has to be reduced to 0.9 in our economy. In other words, the government has to make a strong commitment to repay its debt annually by the amount of 10 percent of its tax revenues. Let us assume that this reduction is put into action around the same time when fiscal policy is introduced; that is, the year 6. Under such a radical financial reform, as illustrated in Figure 8.20, debt-GDP ratio will be reduced to around 0.64 (line 1 in the right diagram) and the accumulation of debt will be eventually curved (left diagram).

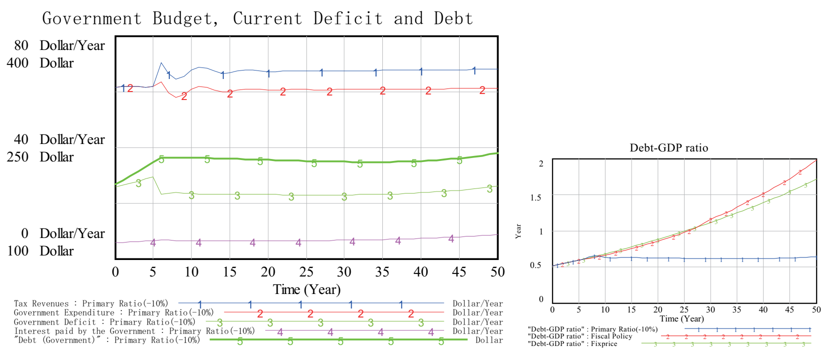

Figure 8.20: Government Debt Deduction

Even so, this radical financial reform becomes very costly to the government and its people as well. At the year of the implementation of 10 % reduction of a primary balance ratio, growth rate is forced to drop to minus 4.1 %, and the economy fails to sustain a full capacity aggregate demand equilibrium as illustrated in the left diagram of Figure 8.21. In other words, the reduction of the primary balance ratio by 10% nullifies the attained full capacity aggregate demand equilibrium by fiscal policy. The right diagram compares three states of GDP; line 3 is when price is fixed, line 2 is when fiscal policy is applied, and line 1 is when primary balance ratio is reduced by 10 %.

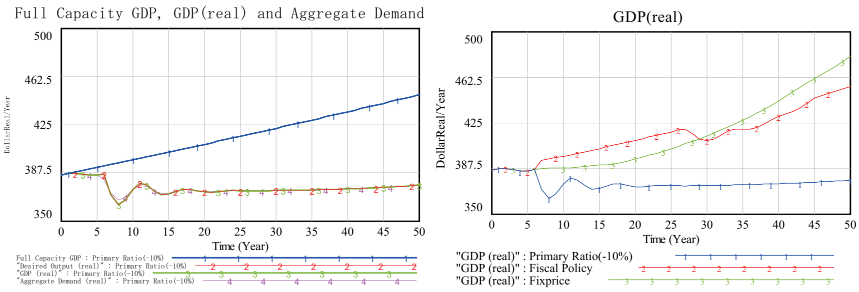

Figure 8.21: Effect of Government Debt Deduction

Price Flexibility

Is there a way to reduce government debt without sacrificing equilibrium? Monetary and fiscal policies above are applied to the disequilibria caused by fixprice. Let us make price flexible again by setting price elasticity to be 0.7 under the above fiscal situation of primary balance deduction. Left diagram of Figure 8.22 shows that equilibrium is attained at the peaks of business cycle, while right diagram shows that government debt is reduced by 10 % deduction of primary balance ratio.

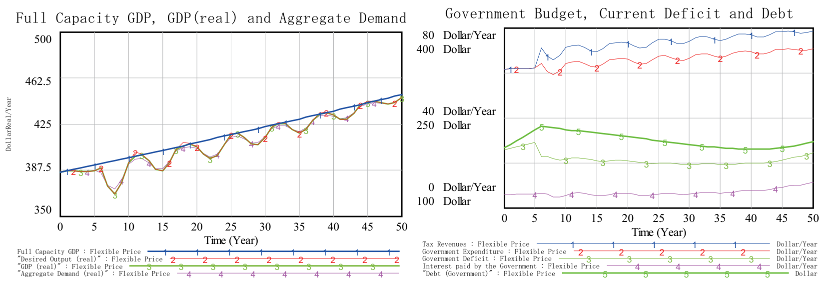

Figure 8.22: Price Flexibility, Business Cycles and Debt Reduction

In this way, the reduction of the government debt by diminishing a primary balance ratio is shown to be possible without causing a sustained recession by introducing flexible price. Yet, a financial reform of this radical type seems to allude to the only soft-landing solution path for a country with a serious debt problem such as Japan, so long as our SD simulation suggests, if a sudden collapse of the government and macroeconomy is absolutely to be avoided. Its success depends on how people can endure getting *worse before better*.

Figure 8.23 compares growth paths of the economy under five different situations such as almost equilibrium, fixprice, fiscal policy, fiscal policy with debt deduction, and fiscal policy with flexible price. Compared with the almost equilibrium path (line 1), debt-reducing path with flexible price causes a business cycle. Yet, compared with another debt-deduction path (line 4), this seems a better path.

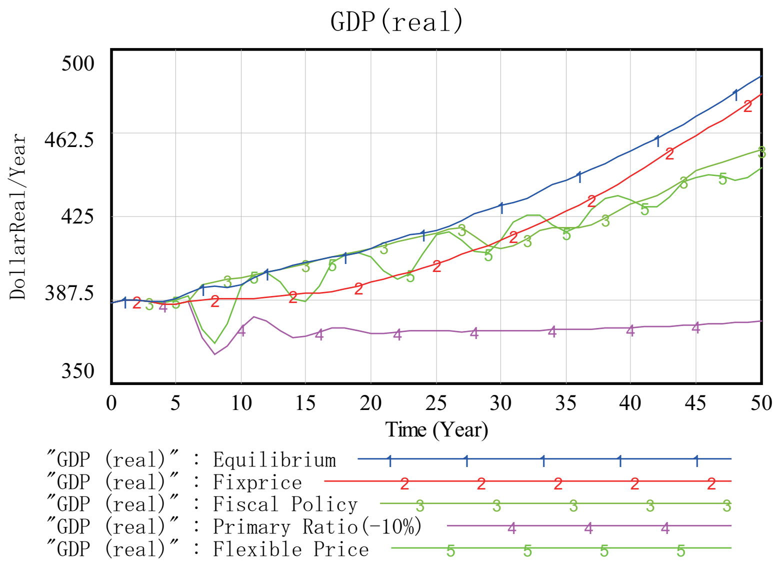

Figure 8.23: Comparison of GDP paths

Remember in our model labor market is not introduced. In other words, economic equilibrium and government debt reduction are shown to be possible under market price flexibility. This gives us an important clue for the working of market economy without labor market (or capitalist economy). Our society needs an economic production system for its survival. Market economy needs not be a capitalist market economy with labor market. Market economy without labor market here is shown to be an efficient system, which I called *MuRatopian Economy*([58]). Now is time to analyze a capitalist market economy with labor market in the next chapter.

8.5 Conclusion

The integration of the real and monetary sectors in macroeconomy is attained from the previously developed macroeconomic models. For this integration, five macroeconomic sectors are brought to the model such as producers, consumers, government, banks and the central bank. Moreover, several major changes are made to the previous models, among which distinction between nominal and real outputs and interest rates are the most crucial one.

The integrated macroeconomic model could be generic in the sense that diverse macroeconomic behaviors will be produced within the same model structure. To show such a capability, some macroeconomic behaviors are discussed. First, the existence of mostly equilibria is shown. And disequilibria are triggered by fixprice and business cycles by two different causes; inventory coverage period and price sensitivity.

Then, Keynesian monetary and fiscal policies are applied to the disequilibria caused by fixprice. Finally accumulating government debt issue is explored. As shown by these, the integrated generic model presented here, we believe, will provide a foundation for the analysis of diverse macroeconomic behaviors.

To make the model furthermore complete, however, at least the following fine-tuning revision has to be incorporated to the model: a feedback loop of the accumulating government debt to the interest rate.

Chapter 9

A Macroeconomic System

In the previous chapter, monetary and real parts of macroeconomies are integrated to present a macroeconomic dynamic model. In this chapter[1], population dynamics and labor market is to be brought to make the integrated model complete. This complete model is aimed to be generic, out of which diverse macroeconomic behaviors are shown to emerge.

9.1 Macroeconomic System Overview

This chapter tries to bring population and labor market to the stage to make the integrated model complete [Companion model: MacroSystem.vpm]. For this purpose, at least five sectors of the macroeconomy have to play macroeconomic activities simultaneously as in the previous integrated model; that is, producers, consumers, banks, government and central bank. Figure 9.1 illustrates the overview of a macroeconomic system in this chapter, and shows how these macroeconomic sectors interact with one another and exchange goods and services for money. Foreign sector is still excluded from the current analysis.

The reader will be reminded that the complete macroeconomic model to be developed below is a generic one by its nature, and does not intend to deal with some specific issues our macroeconomy is currently facing. Once such a generic macroeconomic model is build, we believe, any specific macroeconomic issue could be challenged by bringing real data in concern to this generic model without major structural changes in this integrated model.

9.2 Production Function

In the previous model, full capacity output level is specified as follows:

[1]This chapter is based on the paper: Integration of Real and Monetary Sectors with Labor Market – SD Macroeconomic Modeling (3) – in "Proceedings of the 24th International Conference of the System Dynamics Society", Nijmegen, The Netherlands, July 23 - 27, 2006. (ISBN 978-0-9745329-5-0)

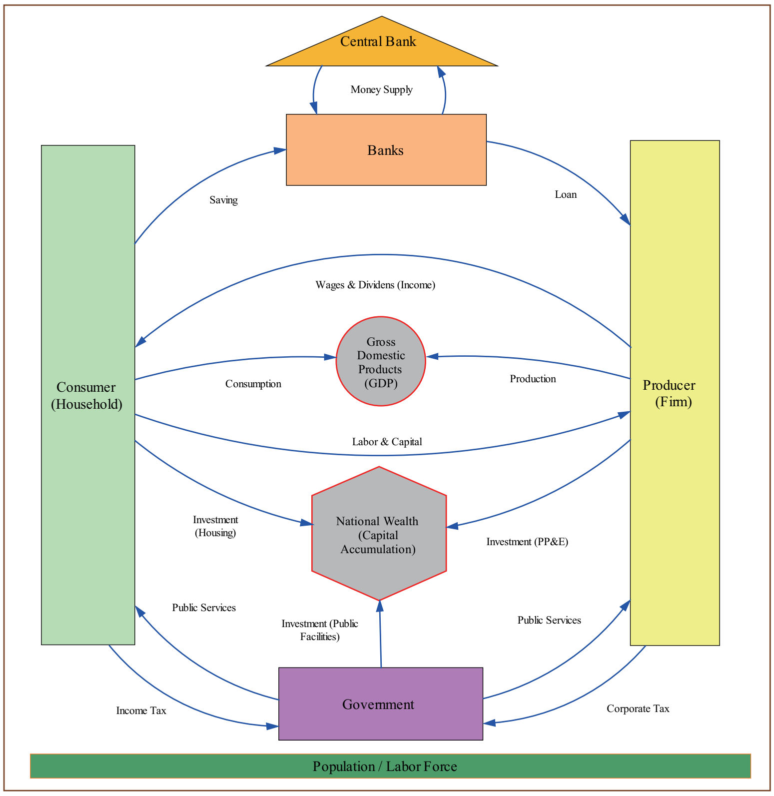

Figure 9.1: Macroeconomic System Overview

$$Y_{full} = e^{\kappa t} \frac{1}{\theta} K \tag{9.1}$$

where κ is an annual increase rate of technological progress, and θ is a capital-output ratio. In order to fully consider the role of employed labor L, it needs to be replaced with Cobb-Douglas production function:

$$Y_{full} = F(K, L, A) = AK^{\alpha} L^{\beta} \tag{9.2}$$

where A is a factor of technological change, and α and β are exponents on capital and labor, respectively. GDP thus produced is redefined as full capacity GDP.

With the introduction of the employed labor and totally available labor force, it also becomes possible to define potential output or GDP as

$$Y_{potential} = F(K, LF, A) = AK^{\alpha} LF^{\beta} \tag{9.3}$$

where LF is the total amount of labor force which is defined as the sum of the employed and unemployed labor.

Accordingly, the desired price P^* defined in equation (7.41) needs be slightly revised to reflect the gap between potential GDP and desired output Y^D as

$$P^* = \frac{P}{\left((1-\omega)\frac{Y_{potential}}{Y^D} + \omega\frac{I^*_{inv}}{I_{inv}}\right)^e} \tag{9.4}$$

where $\omega, 0 \leq \omega \leq 1$, is a weight between production and inventory ratios, and e is an elasticity.

Let us assume that productivity due to technological progress grows exponentially such that

$$A = \bar{A}e^{\kappa t} \tag{9.5}$$

where κ is an annual increase rate of technological progress, which may be possible to be endogenously determined within the system. Following the method by Nathan Forrester [26], let us normalize this production function with the initial potential GDP at $t = 0$:

$$\bar{Y}_{potential} = F(\bar{K}, \bar{LF}, \bar{A}) = \bar{A}\bar{K}^{\alpha}\bar{LF}^{\beta} \quad \text{(Initial Potential Output)} \tag{9.6}$$

Then, we have

$$Y_{full} = e^{\kappa t}\bar{Y}_{potential}\left(\frac{K}{\bar{K}}\right)^{\alpha}\left(\frac{L}{\bar{LF}}\right)^{\beta} \tag{9.7}$$

Now let us define profits after tax. In our integrated model, three types of taxes are levied: tax on production (excise tax), corporate tax and income tax. The former two taxes are paid by producers (Figure 9.5), while income tax, consisting of lump-sum tax and a proportional part of income tax, is paid by consumers (Figure 9.6). With these into consideration, profits after tax Π are now defined as

$$\Pi = ((1-t)PY_{full} - (i+\delta)P_K K - wL)(1-t_c) \tag{9.8}$$

where t is an excise tax rate, t_c is a corporate profit tax rate, i is a real interest rate, δ is a depreciation rate, and w is a nominal wage rate.

One remark may be appropriate for the definition of capital cost $iP_K K$. The amount of capital against which interests are paid are the amount of debt outstanding by producers (which is the same as the outstanding loan by banks) in the integrated model, yet at an abstract theoretical level it is regarded the same as the book value of capital from which depreciation is deducted. Our model based on double accounting system enables to handle this distinction. Specifically, capital cost (= interest paid by producers) are calculated in the model as

$$iP_K K \approx \text{Prime Rate x Loan (or Debt by producers)} \tag{9.9}$$

First order condition for profit maximization with respect to capital stock is calculated by partially differentiating profits with respect to capital as

$$\begin{aligned}
\left(\frac{1}{1-t_c}\right)\frac{\partial \Pi}{\partial K} &= \alpha(1-t)Pe^{\kappa t}\left(\frac{\bar{Y}_{full}}{\bar{K}}\right)\left(\frac{K}{\bar{K}}\right)^{\alpha-1}\left(\frac{L}{\bar{L}}\right)^{\beta} - (i+\delta)P_K \\
&= \frac{\alpha(1-t)Pe^{\kappa t}\left(\frac{\bar{Y}_{full}}{\bar{K}}\right)\left(\frac{K}{\bar{K}}\right)^{\alpha}\left(\frac{L}{\bar{L}}\right)^{\beta}}{\frac{K}{\bar{K}}} - (i+\delta)P_K \\
&= \frac{\alpha(1-t)PY_{full}}{K} - (i+\delta)P_K \\
&= 0
\end{aligned} \tag{9.10}$$

The demand function for capital is thus obtained as

$$K = \frac{\alpha(1-t)PY_{full}}{(i+\delta)P_K} \tag{9.11}$$

At a macroeconomic level of one commodity, price of output P is treated with the same as the price of capital stock P_K. Hence, a desired level of capital stock K^* could be approximately calculated by desired output Y^* as

$$K^*(i) = \frac{\alpha(1-t)Y^*}{i+\delta} \tag{9.12}$$

In our model desired output Y^* is represented by the variable: Aggregate Demand Forecasting (Long-run) as illustrated in Figure 9.2 (see also [26]).

The amount of desired investment is now obtained as the difference between desired and actual capital stock such that

$$I(i) = \frac{K^*(i) - K}{\text{Time to Adjust Capital}} + \delta K \tag{9.13}$$

Furthermore, let us define desired capital-output ratio as follows:

$$\theta^*(i) = \frac{K^*}{Y^*} = \frac{\alpha(1-t)}{i+\delta} \tag{9.14}$$

Then, the above investment function can be rewritten as[2]

$$I(i) = \frac{(\theta^*(i) - \theta)Y^*}{\text{Time to Adjust Capital}} + \delta K \tag{9.15}$$

The new investment function obtained above replaces our previous investment function in equation (7.21) that is determined by the interest rate:

$$I(i) = \frac{I_0}{i} - ai \tag{9.16}$$

where a is an interest sensitivity of investment.

[2]To be precise, this reformulation cannot be used as an alternative investment function without minor behavioral changes. Hence equation (9.13) is used in this model.

First order condition for profit maximization with respect to labor is calculated as follows:

$$
\begin{aligned}
\left(\frac{1}{1-t_c}\right)\frac{\partial \Pi}{\partial L} &= \beta(1-t)Pe^{\kappa t}\left(\frac{\bar{Y}_{full}}{\bar{L}}\right)\left(\frac{K}{\bar{K}}\right)^{\alpha}\left(\frac{L}{\bar{L}}\right)^{\beta-1} - w \\
&= \frac{\beta(1-t)Pe^{\kappa t}\left(\frac{\bar{Y}_{full}}{\bar{L}}\right)\left(\frac{K}{\bar{K}}\right)^{\alpha}\left(\frac{L}{\bar{L}}\right)^{\beta}}{\frac{L}{\bar{L}}} - w \\
&= \frac{\beta(1-t)PY_{full}}{L} - w \\
&= 0 \qquad (9.17)
\end{aligned}
$$

Demand for labor is thus obtained as

$$L^d = \frac{\beta(1-t)PY_{full}}{w}. \qquad (9.18)$$

Specifically, it is a decreasing function of real wage rate such that $R = w/P$.

From this demand function for labor, desired level of labor L^* could be approximately obtained by desired output Y^* and expected wage rate w^e as

$$L^*(Y^*, w^e) = \frac{\beta(1-t)PY^*}{w^e} \qquad (9.19)$$

The expected wage rate is assumed to be determined as

$$w^e = w(1 + \text{inflation rate}) \qquad (9.20)$$

The determination of the wage rate will be discussed in the following section.

Net employment decision is now made according to the difference between desired and actual amount of labor such that

$$E(Y^*, w^e) = \frac{L^*(Y^*, w^e) - L}{\text{Time to Adjust Labor}} \qquad (9.21)$$

Net employment thus defined has a downward-sloping shape such that

$$\frac{\partial E}{\partial w^e} = -\frac{\beta(1-t)Y^*}{\text{Time to Adjust Labor}}\frac{1}{(w^e)^2} < 0. \qquad (9.22)$$

The amount of wages to be paid by producers is determined by

$$W = wL \qquad (9.23)$$

as illustrated in Figure 9.5.

With the above first-order conditions, profits after tax are now rewritten as

$$
\begin{aligned}
\Pi &= ((1-t)PY_{full} - (i+\delta)P_K K - wL)(1-t_c) \\
&= ((1-t)PY_{full} - \alpha(1-t)PY_{full} - \beta(1-t)PY_{full})(1-t_c) \\
&= (1-t)(1-t_c)(1-\alpha-\beta)PY_{full} \qquad (9.24)
\end{aligned}
$$

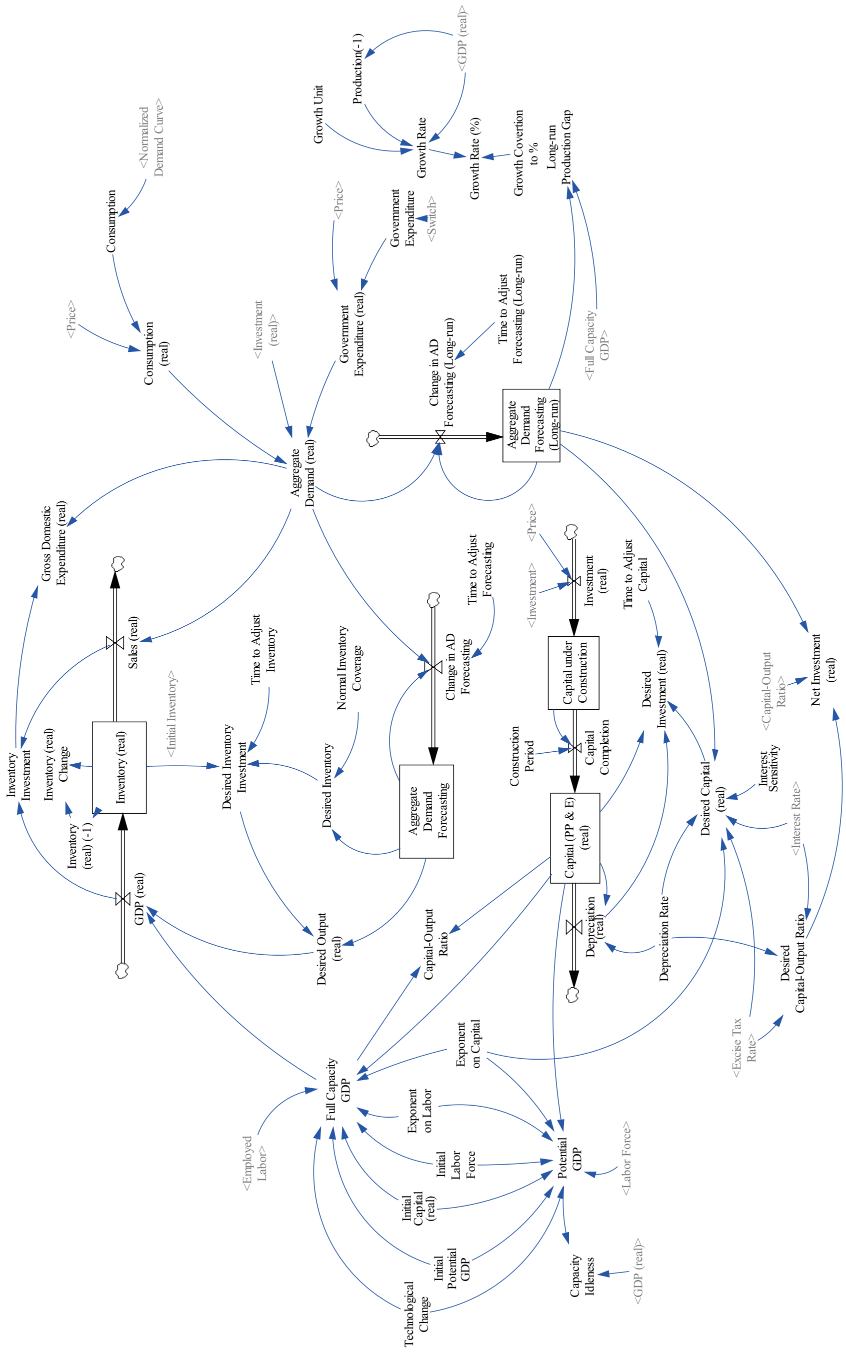

Figure 9.2: Real Production of GDP

Hence, if constant returns to scale is assumed as production technology, as often done; that is, $\alpha + \beta = 1$, then no profits after tax are made available, out of which dividends have to be paid to shareholders. Accordingly, a diminishing returns to scale is assumed in our model; that is, $\alpha = 0.4$ and $\beta = 0.5$ so that $\alpha + \beta < 1$.

9.3 Population and Labor Market

Labor Market and Wage Rate Adjustment

So far labor demand is assumed to be fully met as the equation (9.21) indicates. To determine the real wage rate in the labor market, it is necessary to introduce the availability of labor supply, and the population dynamics of the economy by which labor supply is constrained.

Population dynamics is modeled according to the World3 model[3]. It consists of five cohorts of age groups, and two population cohorts between age 15 to 44 and 45 to 64 are considered to be a productive population cohort.

In this macroeconomic model, the productive population cohort is further broken down into five categorically-different subgroups: high school, college education, voluntary employed, employed labor, and unemployed labor, as illustrated by Figure 9.3. Employed and unemployed labor constitutes a total labor force, by which potential GDP is calculated together with the amount of capital.

Nominal wage rate is now determined in the labor market as follows:

$$\frac{dw}{dt} = \phi(L^* - L^s) \tag{9.25}$$

where L^* denotes demand for desired labor, while L^s indicates supply of labor forces. Labor demand (and net employment) is in return determined by a real wage rate in equation (9.18).

Let us further specify the wage rate equation, as in the interest rate and price equations, as follows:

$$\frac{dw}{dt} = \frac{w^* - w}{\text{DelayTime}} \tag{9.26}$$

where the desired wage rate w^* is obtained as

$$w^* = \frac{w}{\left(\frac{L^s}{L^*}\right)^e} \tag{9.27}$$

where e is a labor ratio elasticity.

These features are reflected in Figure 9.4.

[3]Vensim version of the World3 model is provided in the vendor's's sample models by Ventana Systems, Inc.

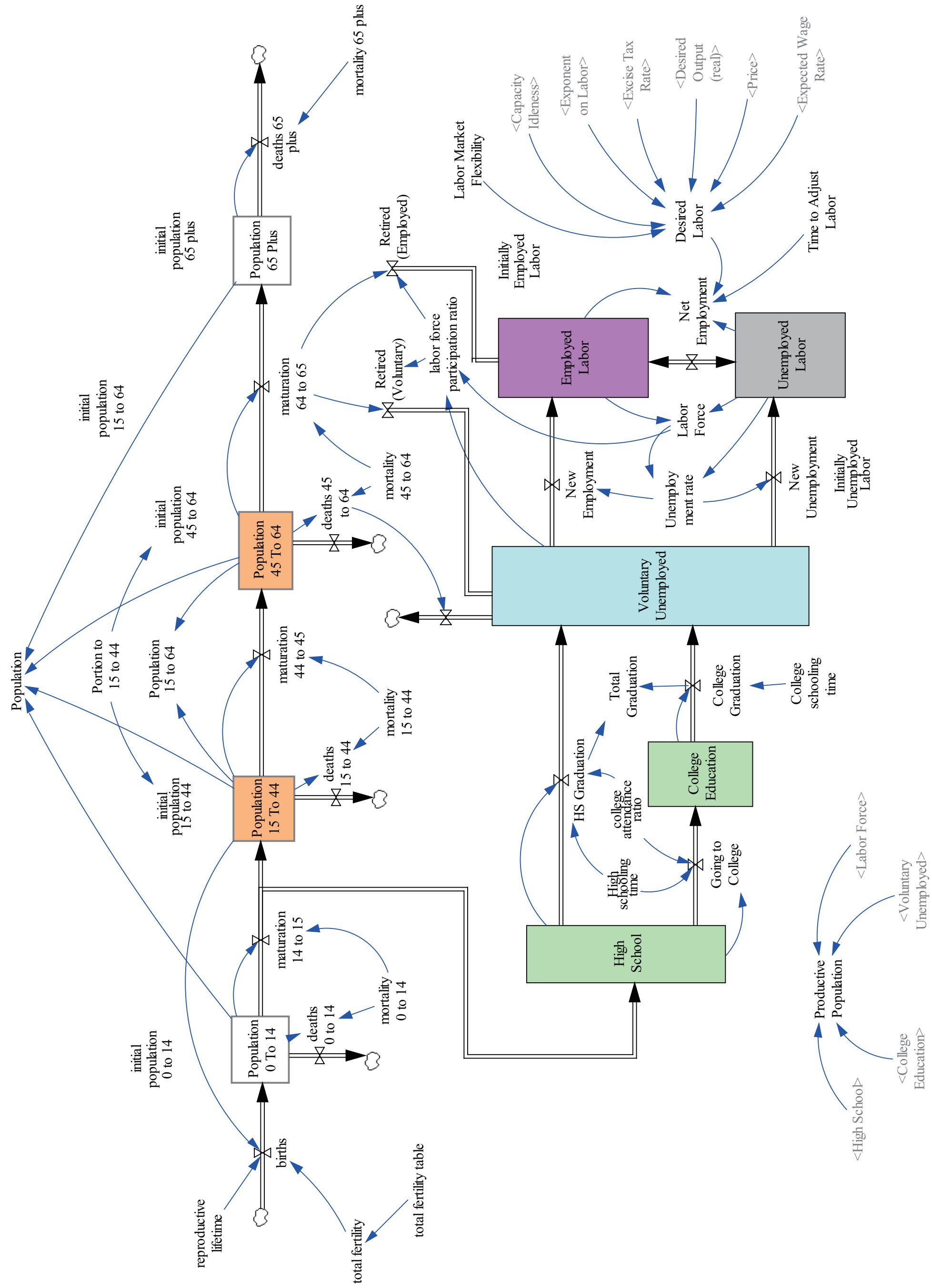

Figure 9.3: Population and Labor Market

Price Adjustment by Cost-push Force

In the model, price is assumed to be adjusted by the demand-pull forces generated by discrepancies between desired aggregate demand and potential GDP, and between inventory gap such that

$$\frac{dP}{dt} = \Psi(Y^D - Y_{potential}, I^*_{inv} - I_{inv}). \tag{9.28}$$

With the introduction of wage determination in equation (9.26), it now becomes possible to add cost-push forces to the price adjustment process. Theses forces are represented by a change in the nominal wage rate such that

$$w_g = \frac{d\ log(w)}{dt} \tag{9.29}$$

The price adjustment process is now influenced by demand-pull and cost-push forces as well such that

$$\frac{dP}{dt} = \Psi_1(Y^D - Y_{potential}, I^*_{inv} - I_{inv}) + \Psi_2(w_g) \tag{9.30}$$

Figure 9.4 illustrates our revised model for adjustment processes of price and wage rate as well as interest rate.

9.4 Transactions Among Five Sectors

Let us now describe some major transactions by producers, consumers, government, banks and central bank, most of which are already described in the previous chapter. For the convenience to the reader, they are repeated here.

Producers

Major transactions of producers are, as illustrated in Figure 9.5, summarized as follows.

- Out of the GDP revenues producers pay excise tax, deduct the amount of depreciation, and pay wages to workers (consumers) and interests to the banks. The remaining revenues become profits before tax.
- They pay corporate tax to the government out of the profits before tax.
- The remaining profits after tax are paid to the owners (that is, consumers) as dividends.
- Producers are thus constantly in a state of cash flow deficits. To continue new investment, therefore, they have to borrow money from banks and pay interest to the banks.

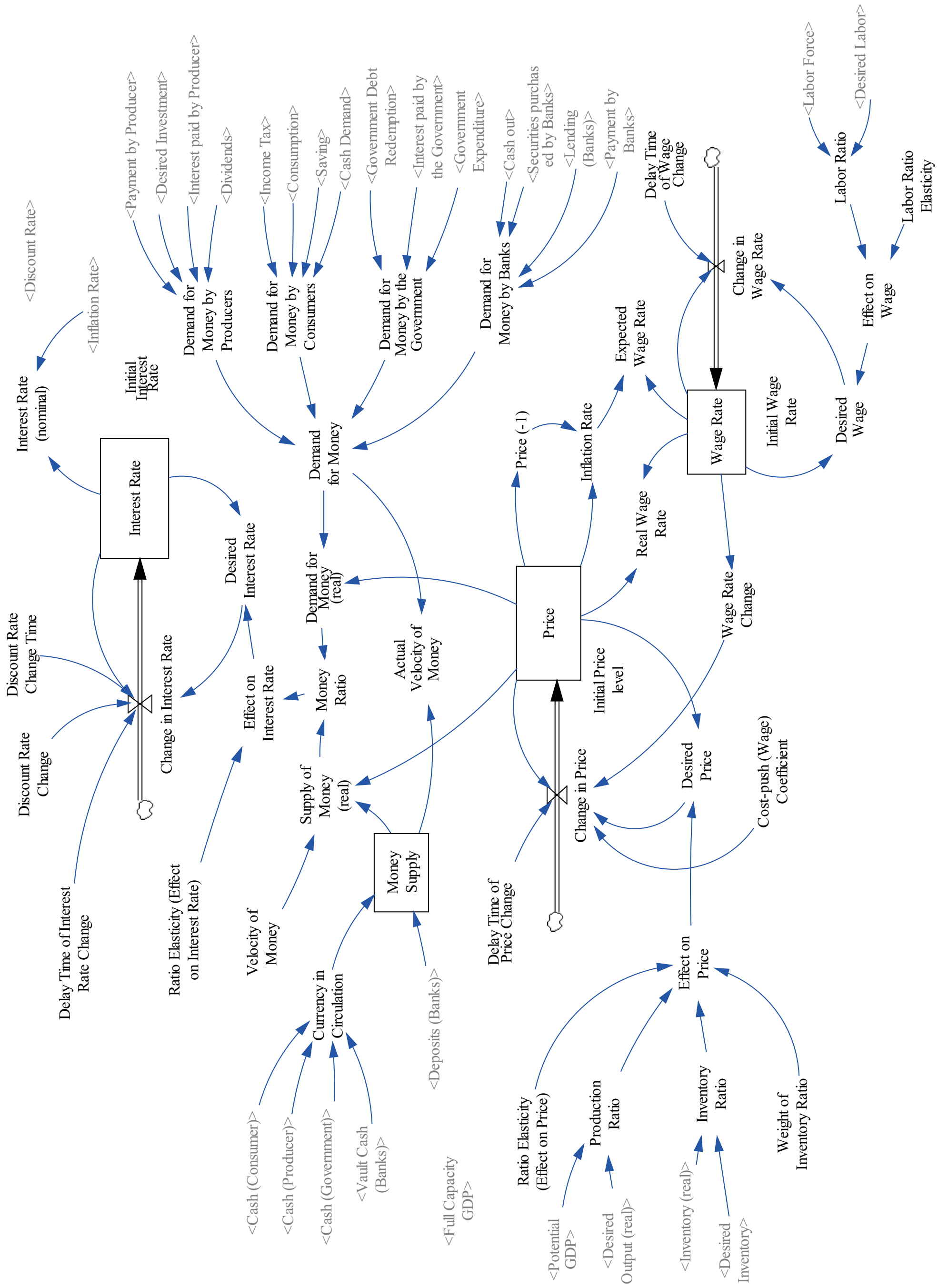

Figure 9.4: Interest Rate, Price and Wage Rate

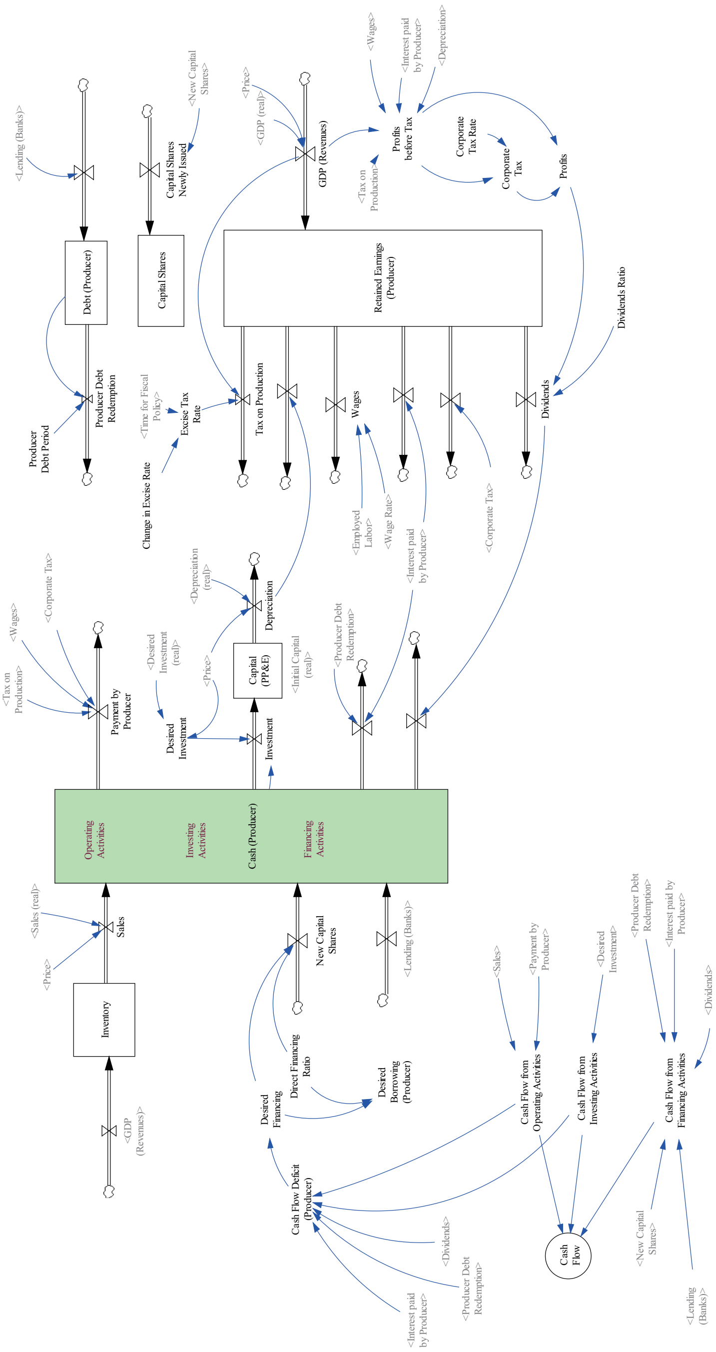

Figure 9.5: Transactions of Producers

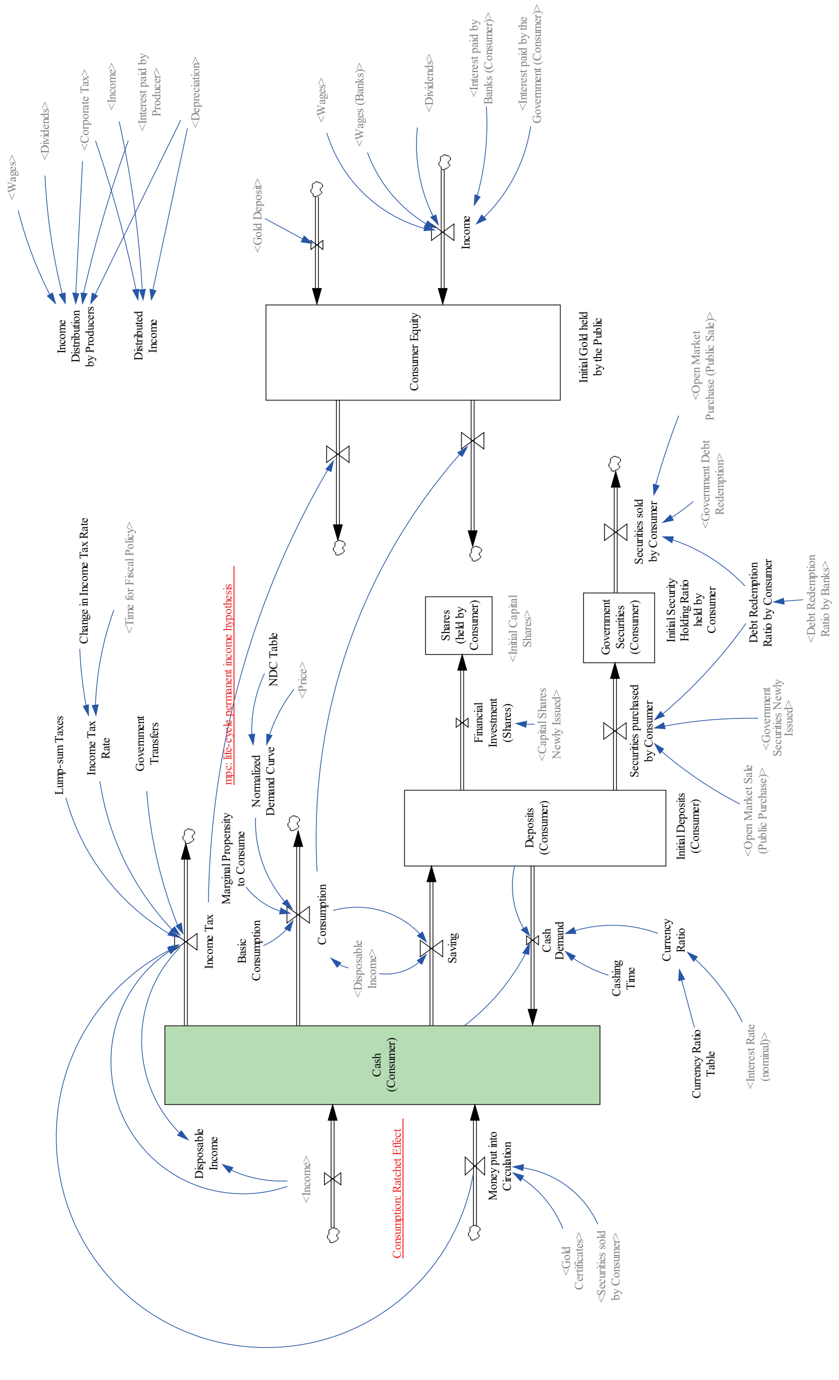

Figure 9.6: Transactions of Consumer

Consumers

Transactions of consumers are illustrated in Figure 9.6, some of which are summarized as follows.

- Consumers receive wages and dividends from producers.
- Financial assets of consumers consist of bank deposits and government securities, against which they receive financial income of interests from banks and government. (No corporate shares are assumed to be held by consumers).
- In addition to the income such as wages, interests, and dividends, consumers receive cash whenever previous securities are partly redeemed annually by the government.
- Out of these cash income as a whole, consumers pay income taxes, and the remaining income becomes their disposal income.
- Out of their disposal income, they spend on consumption that is determined by their marginal propensity to consume and price elasticity. The remaining amount are either spent to purchase government securities or saved.

Government

Transactions of the government are illustrated in Figure 9.7, some of which are summarized as follows.

- Government receives, as tax revenues, income taxes from consumers and corporate taxes from producers as well as excise tax on production.
- Government spending consists of government expenditures and payments to the consumers for its partial debt redemption and interests against its securities.
- Government expenditures are assumed to be endogenously determined by either the growth-dependent expenditures or tax revenue-dependent expenditures.
- If spending exceeds tax revenues, government has to borrow cash from banks and consumers by newly issuing government securities.

Banks

Transactions of banks are illustrated in Figure 9.8, some of which are summarized as follows.

- Banks receive deposits from consumers, against which they pay interests.

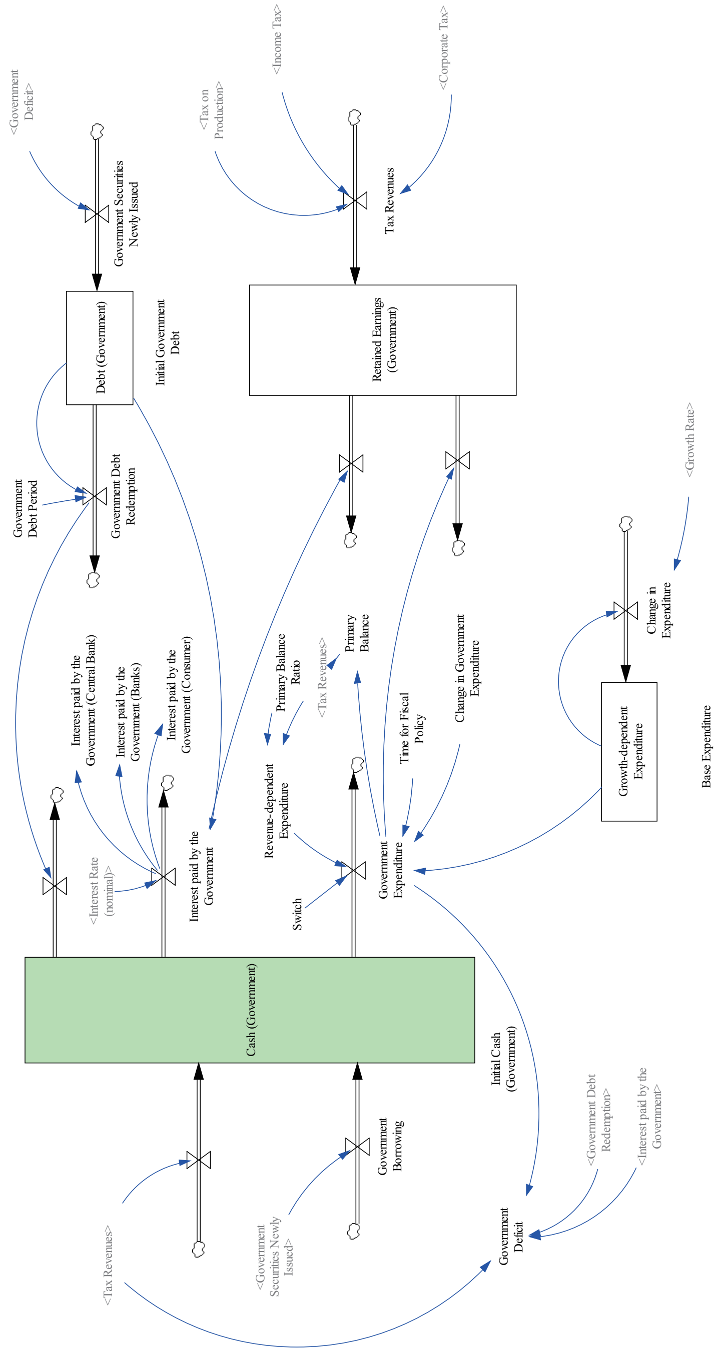

Figure 9.7: Transactions of Government

- They are obliged to deposit a portion of the deposits as the required reserves with the central bank (which is called *a fractional banking system*).
- Out of the remaining deposits they purchase government securities, against which interests are paid from the government.
- Then, loans are made to producers and they receive interests for which a prime rate is applied.
- Their retained earnings thus become interest receipts from producers and government less interest payment to consumers. Positive earning will be distributed among bank workers as consumers.

Central Bank

In this complete macroeconomic model, the central bank plays a very important role of providing a means of transactions and store of value; that is, money or currency. To make a story simple, its sources of assets against which currency is issued are confined to gold, loan and government securities. In short, money is mostly issued as debt by the government and commercial banks. The central bank can control the amount of money supply through the amount of monetary base consisting of currency outstanding and reserves over which it has a direct power of control. This is done through monetary policies such as a manipulation of required reserve ratio and open market operations as well as direct lending control.

Transactions of the central bank are illustrated in Figure 9.9, some of which are summarized as follows.

- The central bank issues currency or money (historically gold certificates) against the gold deposited by the public, though this practice is currently insignificant and only reflects its historical origin of modern banking system.
- It can now mainly issue currency by accepting government securities through open market operation, specifically by purchasing government securities from the public (consumers) and banks. Moreover, it can issue currency by making discount loans to commercial banks. (These activities are sometimes called *money out of nothing*.)
- It can similarly withdraw currency by selling government securities to the public (consumers) and banks, and through debt redemption by banks.
- Banks are required by law to reserve a certain amount of deposits with the central bank. By controlling this required reserve ratio, the central bank can also control the monetary base or currency in circulation directly. The central bank can, thus, control the amount of money supply through monetary policies such as open market operations, reserve ratio and discount rate.

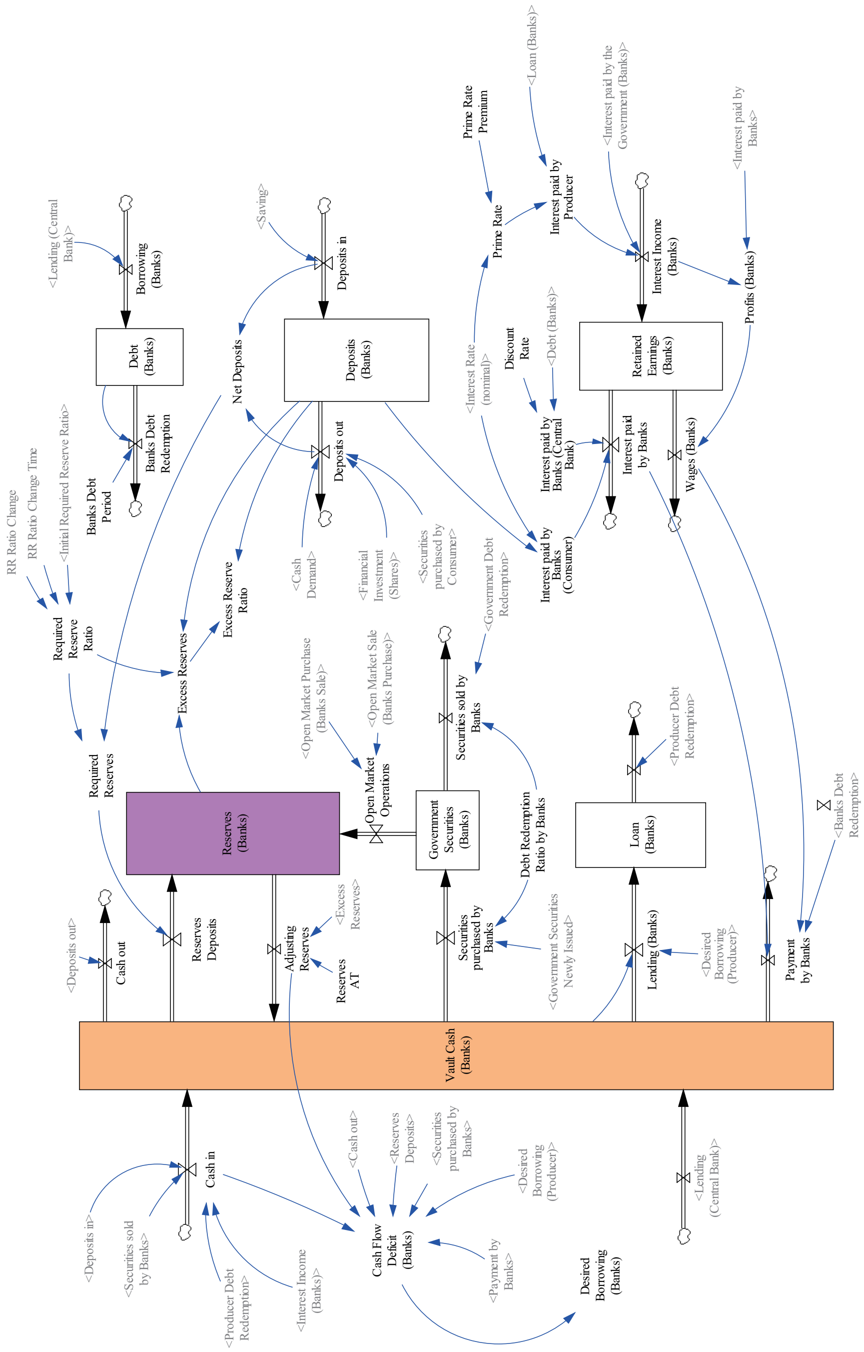

Figure 9.8: Transactions of Banks

- Another powerful but hidden control method is through its direct influence over the amount of credit loans to banks (known as *window guidance* in Japan.)

9.5 Behaviors of the Complete Macroeconomic Model

Mostly Equilibria in the Real Sector

The integrated model is now complete. It is a generic model, out of which diverse macroeconomic behaviors will be produced. Let us start with an equilibrium growth path of the macroeconomy. As in the previous model, let us call an equilibrium state *a full capacity aggregate demand equilibrium* if the following three output and demand levels are met:

$$\text{Full Capacity GDP} = \text{Desired Output} = \text{Aggregate Demand} \tag{9.31}$$

If the economy is not in the equilibrium state, then actual GDP is determined by

$$\text{GDP} = \text{MIN (Full Capacity GDP, Desired Output)} \tag{9.32}$$

In other words, if desired output is greater than full capacity GDP, then actual GDP is constrained by the production capacity, meanwhile in the opposite case, GDP is determined by the amount of desired output which producers wish to produce, leaving the capacity idle, and workers being laid off.

Even though, full capacity GDP is attained, full employment may not be realized unless

$$\text{Potential GDP} = \text{Full Capacity GDP} \tag{9.33}$$

Does the equilibrium state, then, exist in the sense of full capacity GDP and full employment? To answer these questions, let us define GDP gap as the difference between potential GDP and actual GDP, and its ratio to the potential GDP as

$$\text{GDP Gap Ratio} = \frac{Y_{potential} - \text{GDP}}{Y_{potential}} \tag{9.34}$$

By trial and error, mostly equilibrium states are acquired in the complete macroeconomic model whenever price is flexibly adjusted by setting its coefficient to be 1, together with all other adjusted parameters, as illustrated in Figure 9.10.

Labor market is newly introduced in this model. Therefore, our analyses in what follows are focused on the behaviors of GDP gap and unemployment. Figure 9.11 illustrates detailed behaviors of the GDP gap ratio and unemployment rate at the almost equilibrium states. Both figures tend to converge less than 1% after the year 5, and can be well regarded as the states of almost equilibria.

In what follows, these equilibrium states are used as benchmarking states of the comparison, and illustrated by line 2 or red lines in Figures.

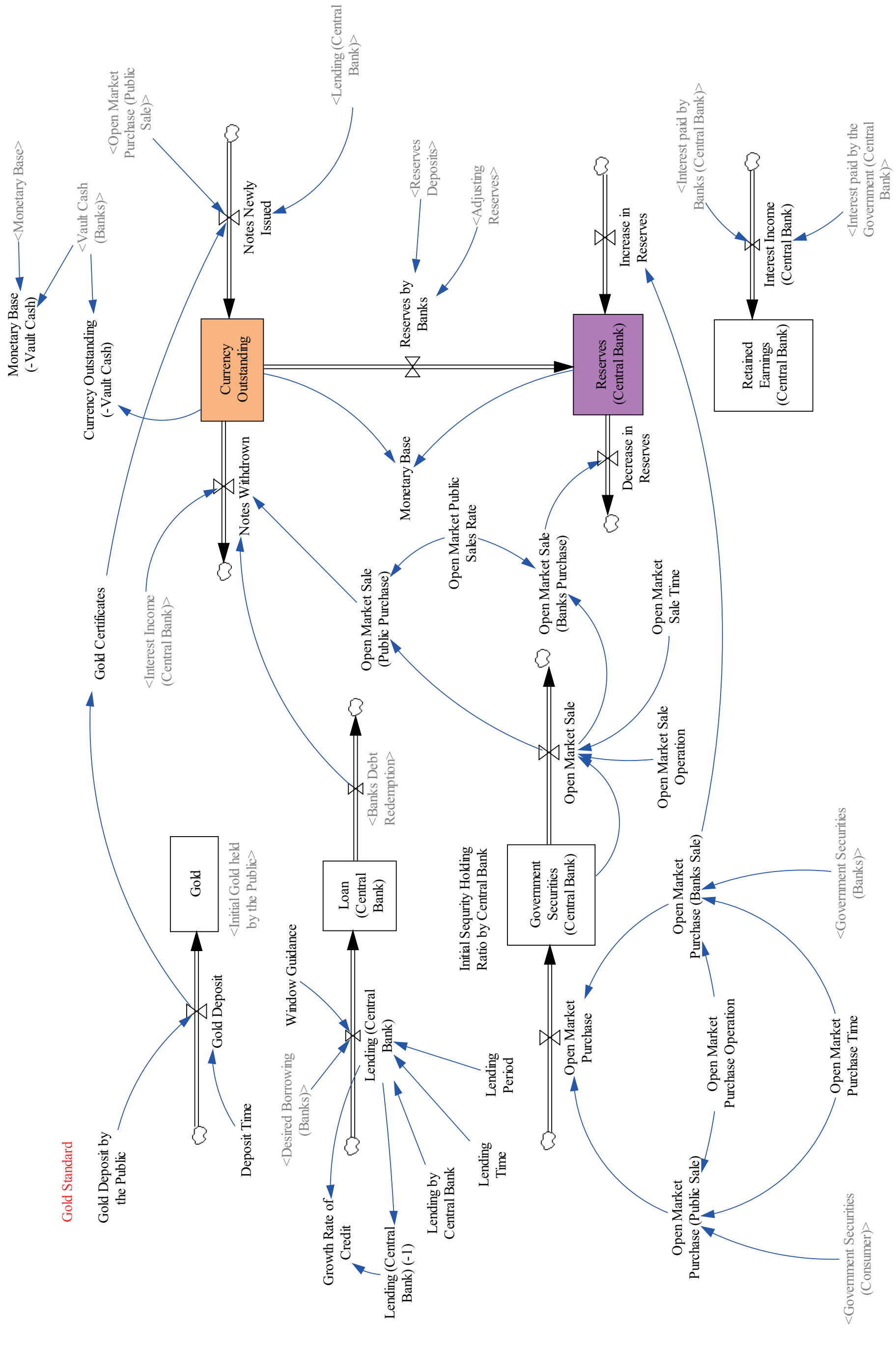

Figure 9.9: Transactions of Central Bank

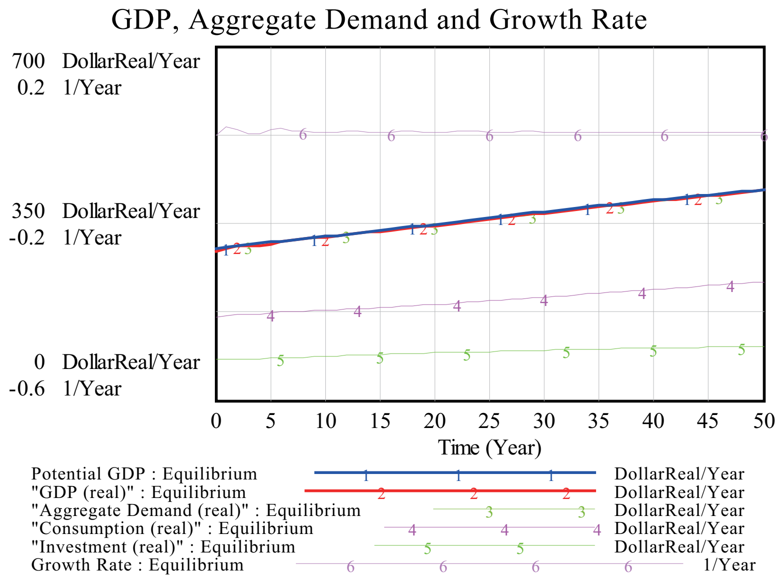

Figure 9.10: Mostly Equilibrium States

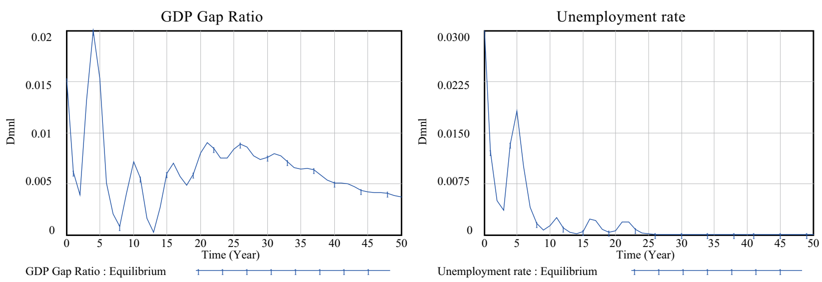

Figure 9.11: GDP Gap Ratio and Unemployment Rate of the Equilibrium States

Fixprice Disequilibria

We are now in a position to make some analytical simulations for the model. First, let us show that without price flexibility it's hard to attain mostly equilibrium states. When price is fixed; that is, price coefficient is set to be zero, disequilibria begin to appear all over the period. Left-hand diagram of Figure 9.12 illustrates how fixprice causes to expand GDP gap to 17% at the year 50. Right-hand diagram shows the unemployment rate fluctuates with its peak of 3.8% at the year 7. In this way under fixprice the economy seems to stagger.

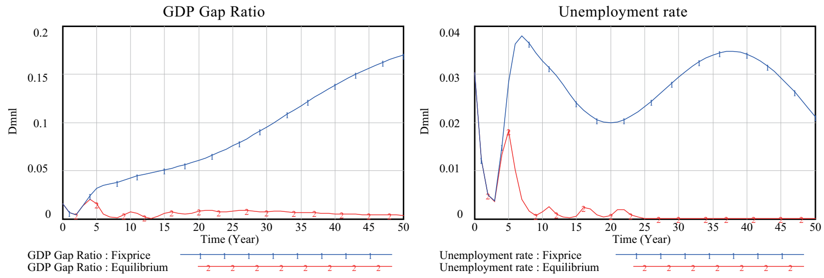

Figure 9.12: Fixprice and Mostly Equilibrium States

Business Cycles by Inventory Coverage

From now on, let us start our analyses with the mostly equilibrium path. One of the interesting questions is to find out a macroeconomic structure that may produce economic fluctuations or business cycles. How can the mostly equilibrium growth path be broken and business cycles will be triggered?

Our complete macroeconomic model can successfully produce at least two different ways of causing macroeconomic fluctuations as in the previous chapter; that is, changes in inventory coverage and price fluctuation. Firstly, they can be caused by increasing normal inventory coverage period, Specifically, suppose the normal inventory coverage now increases to 0.5 or 6 months instead of the initial value of 0.1 or 1.2 month. The economy, then, begins to be troubled with short period's business cycles of 6 or 7 years as Figure 9.13 portrays.

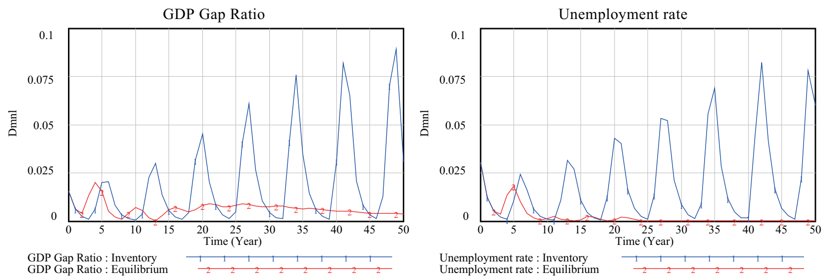

Figure 9.13: Business Cycles caused by Inventory Coverage

Business Cycles by Elastic Price Fluctuation

Secondly, the equilibrium growth path can also be broken and business cycle is triggered, in a totally different fashion, by price fluctuation. Price can be fluctuated by changes in its elasticity and cost-push factor such as changes in wage rate. Let us consider the former first by assuming that a price response to the excess demand for products becomes more sensitive so that output ratio elasticity now becomes elastic with a value of 1.6 from 1, and a weight of inventory ratio to the effect on price becomes 0.6 from 0.1. Again, the economy is thrown into business cycle as depicted in Figure 9.14.

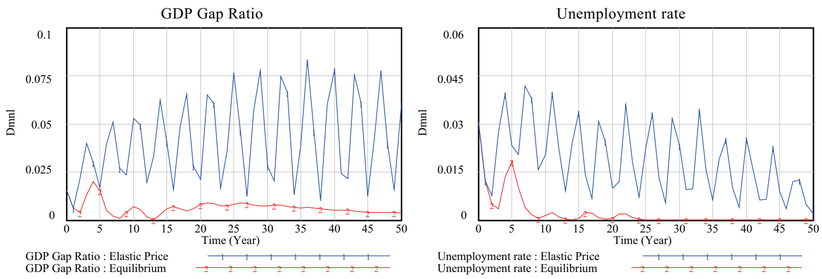

Figure 9.14: Business Cycles caused by Elastic Price Fluctuation

Business Cycles by Cost-push Price

Since price adjustment process is revised in equation (9.30), there exists another way to affect price fluctuation, this time, by the cost-push changes in nominal wage rate. Specifically, cost-push(wage) coefficient is now set to be 0.18 from 0. Again, the economy is thrown into business cycle as depicted in Figure 9.15.

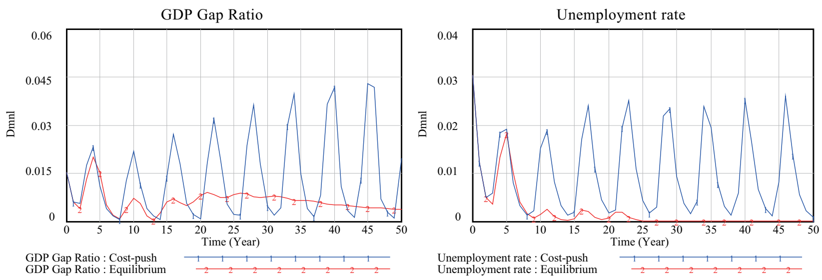

Figure 9.15: Business Cycles caused by Cost-push Price

It would be worth examining this case of business cycle furthermore. Figure 9.16 illustrates the fluctuations of price, nominal wage rate and unemployment rate triggered by cost(wage))-push fluctuations. Price and wage rate fluctuate in the same direction, while GDP gap and unemployment rate fluctuate counter-cyclically against price and wage fluctuations. In other words, when price and wage rates increase, GDP gap and unemployment rate decrease, and vise versa. Moreover, it is observed that GDP gap cycle is always followed by unemployment rate cycle.

In this way, two similar business cycles are triggered, out of the same almost equilibrium growth path, by two different causes; one by an increase in inventory coverage period, and the other by the price and wage changes. The ability to

Figure 9.16: Wage, Inflation and Unemployment Rates and GDP Gap

produce these different behaviors of business cycles and economic fluctuations indicates a richness of our macroeconomic generic model.

Economic Recession by Credit Crunch

With the introduction of discount credit loans to banks, the central bank seems to have acquired an almighty power to control credit. The power has been overlooked in standard textbooks. This hidden exerting power has been known in Japan as "window guidance".

To demonstrate how influential the power is, let us suppose that the central bank reduces the amount of discount credit loans by 30%; that is, window guidance value is reduced to 0.7 from 1. In other words, banks can borrow only 70% of the desired amount of borrowing from the central bank.

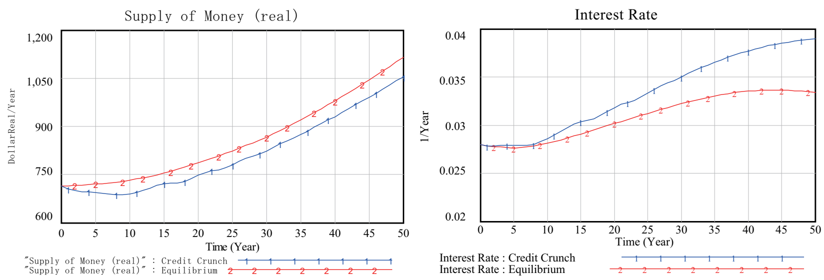

Figure 9.17: Money Supply and Interest Rate caused by Credit Crunch

Figure 9.17 illustrates how money supply shrinks and, accordingly, interest rate increases by the credit crunch caused by the central bank. Figure 9.18

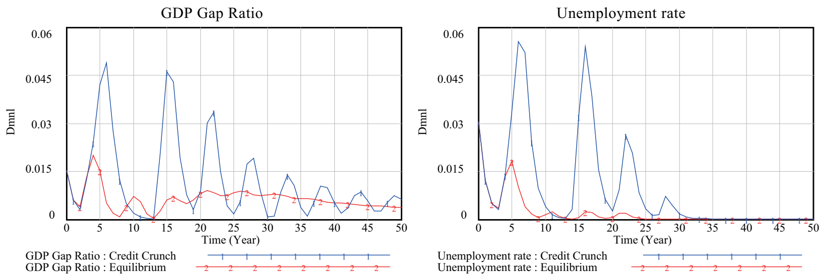

Figure 9.18: Economic Recessions caused by Credit Crunch

demonstrates that the economy is now deeply triggered into recession in the sense that GDP gaps under credit crunch appear as another business cycles, followed by similar unemployment rate cycles. It is a surprise to observe that economic recessions are provoked by the intentional credit crunch of the central bank in addition to the business cycles as shown above.

As discussed above, growing economy needs new currency to be incessantly put into circulation. If the central bank, instead of the government, is historically endowed with this important role, savvy control of credits by the central bank becomes crucial for the stability and growth of macroeconomy as demonstrated here.

Monetary and Fiscal Policies for Equilibrium

So far, we have examined several states of disequilibria caused by fixprice and business cycles. Can we restore equilibrium, then ? According to the Keynesian theory, the answer is affirmative if fiscal and monetary policies are appropriately applied.

To answer the question, let us start with the case of fixprice disequilibria and consider monetary policy, first, for the restoration of equilibria. Suppose the central bank increases the purchase of government securities by 30% for 5 years starting at the year 6 (see the top left diagram of Figure 9.19). Then, money supply continues to grow gradually, and interest rate eventually starts to decrease (see top right diagram.). These changes in the monetary sector will eventually restore full capacity GDP and almost full employment equilibria from the year 17 through the year 36 for 20 years. (see the bottom two diagrams).

Second scenario of restoring the equilibrium is to apply fiscal policy. In our model quite a few tools are available for fiscal policy such as changes in income tax rate, lump-sum taxes, excise rate, corporate tax rates and government expenditures. We employ here income tax rate. The reader can try other policy tools by running the model.

Facing the fixprice disequilibria, the government now decides to introduce an increase in income tax rate by 5%; that is, from the original 10% to 15 %,

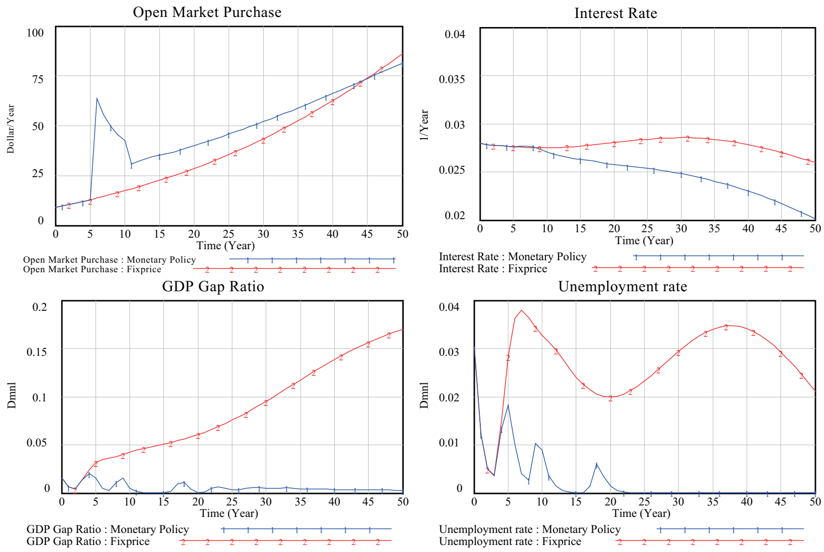

Figure 9.19: Monetary Policy of Open Market Purchase and GDP

at the year 15. Under the assumption of balanced budget, or a unitary primary balanced ratio, an increase in income tax also becomes the same amount of increase in government expenditure (see top left hand diagram in Figure 9.20).

This causes the increase in interest rate, which crowds out investment. Even so, aggregate demand is spontaneously stimulated to restore the full capacity GDP and almost full employment equilibrium at the year 16 through 28 for 13 years (see the bottom diagrams). Compared with the monetary policy, the effect of fiscal policy appears quickly from the next year.

In this way, our complete macroeconomic model can provide effective scenarios of sustaining full capacity and full employment equilibrium growth path through monetary and fiscal policies so long as they are timely applied.

Government Debt

So long as the equilibrium path in the real sector is attained by fiscal policy, no macroeconomic problem seems to exist. Yet behind the full capacity aggregate demand growth path attained in Figure 9.20, government debt continues to accumulate as the left diagram of Figure 9.21 illustrates. Primary balance ratio is initially set to one and balanced budget is assumed in our model; that is, government expenditure is set to be equal to tax revenues, as lines 1 and 2 overlap in the diagram. Why, then, does the government continue to suffer from the current deficit?

In the model government deficit is defined as

$$\text{Deficit} = \text{Tax Revenues - Expenditure - Debt Redemption - Interest} \quad (9.35)$$

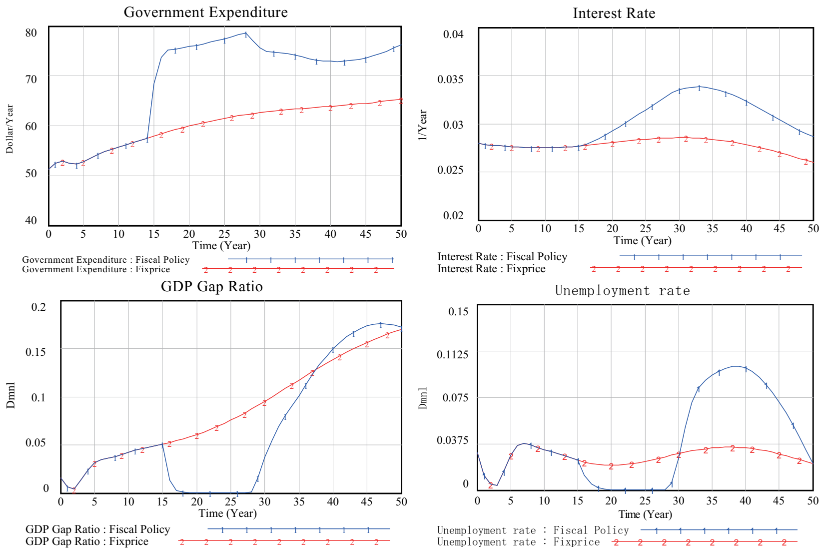

Figure 9.20: Fiscal Policy of a Change in Income Tax Rate and GDP

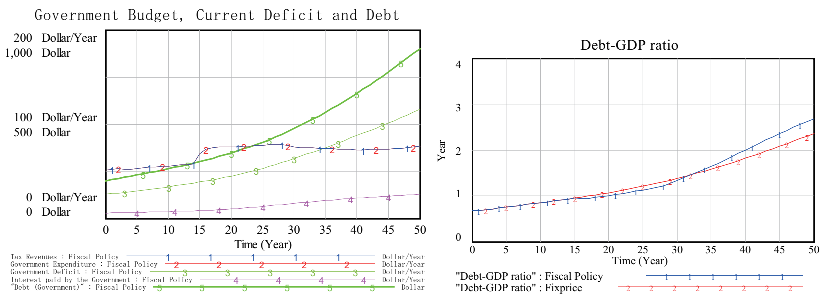

Figure 9.21: Accumulation of Government Debt

Therefore, even if balanced budget is maintained, the government still has to keep paying its debt redemption and interest. This is why it has to keep borrowing and accumulating its debt, a negative side of exponential growth in the current financial system. Initial GDP in the model is obtained to be 295, while government debt is initially set to be 200. Hence, the initial debt-GDP ratio is around 0.68 year (a little bit higher than the current ratios among EU countries). The ratio continues to increase to 2.7 years at the year 50 in the model as illustrated in the right diagram of Figure 9.21. This implies the government debt becomes 2.7 times as high as the annual level of GDP.

Can such a high debt be sustained? Absolutely no. Eventually this runaway accumulation of government debt may cause nominal interest rate to increase, because the government may be forced to pay higher and higher interest rate in order to keep borrowing, which may in turn launch a hyper inflation[4].

On the other hand, a higher interest rate may trigger a sudden drop of government security price, deteriorating the value of financial assets of banks, producers and consumers. The devaluation of financial assets may force some banks and producers to go bankrupt eventually. In this way, under such a hyper inflationary circumstance, financial crisis becomes inevitable and government is destined to collapse. This is one of the hotly debated scenarios about the consequences of the abnormally accumulated debt in Japan, whose debt-GDP ratio in 2010 is about 1.97 years; the highest among OECD countries!

Let us now consider how to avoid such a financial crisis and collapse. At the year 15 when fiscal policy is introduced to restore a full capacity aggregate demand equilibrium in the model, the economy seems to have gotten back to a right track of sustained growth path. And most macroeconomic textbooks emphasize this positive side of fiscal policy. A negative side of fiscal policy is the accumulation of debt for financing the government expenditure. Yet most macroeconomic textbooks neglect or less emphasize this negative side, partly because their macroeconomic framework cannot handle this negative side effect properly as our complete macroeconomic model does here. In our example the debt-GDP ratio is 0.68 years at the introduction year of fiscal policy.

At the face of financial crisis as discussed above, suppose that the government is forced to reduce its debt-GDP ratio to around one by the year 50, To attain this goal, a primary balance ratio has to be reduced to 0.87 in our economy. In other words, the government has to make a strong commitment to repay its debt annually by the amount of 13 percent of its tax revenues. Let us assume that this reduction is put into action around the same time when fiscal policy is introduced; that is, the year 15. Under such a radical financial reform, as illustrated in Figure 9.22, debt-GDP ratio will be reduced to around one (right diagram) and the accumulation of debt will be eventually curved (left diagram).

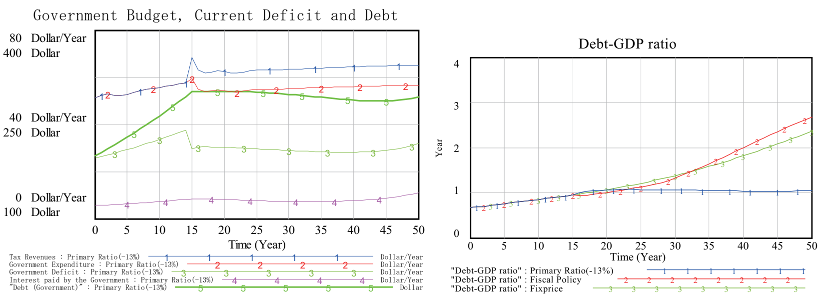

Figure 9.22: Government Debt Deduction

Even so, this radical financial reform becomes very costly to the government and its people as well. At the year of the implementation of 13 % reduction

[4]This feedback loop from the accumulating debt to the higher interest rate is not yet fully incorporated in the model.

of a primary balance ratio, growth rate is forced to drop to minus 4.86%, and the economy fails to sustain a full capacity and full employment equilibrium as illustrated by line 1 in Figure 9.23. In fact, GDP gap ratio continues to rise to 30% and unemployment rate peaks to 15.5% at the year 22. In other words, the reduction of the primary balance ratio by 13% totally nullifies the attained full capacity and full employment equilibrium by fiscal policy. The same diagrams compare three states of GDP; line 3 is when price is fixed, line 2 is when fiscal policy is applied, and line 1 is when primary balance ratio is reduced by 13%.

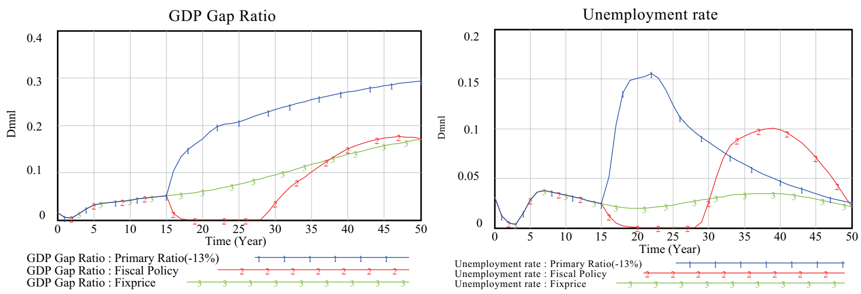

Figure 9.23: Effect of Government Debt Deduction

Price Flexibility

Is there a way to reduce government debt without sacrificing equilibrium? The monetary and fiscal policies introduced above are applied to the disequilibria caused by fixprice. Let us make price flexible again by setting price elasticity to be 1. Left diagram (line 1) of Figure 9.24 shows that GDP gap ratio is again reduced to be below 0.1, while right diagram (line 1) shows that unemployment rate gradually gets reduced to zero.

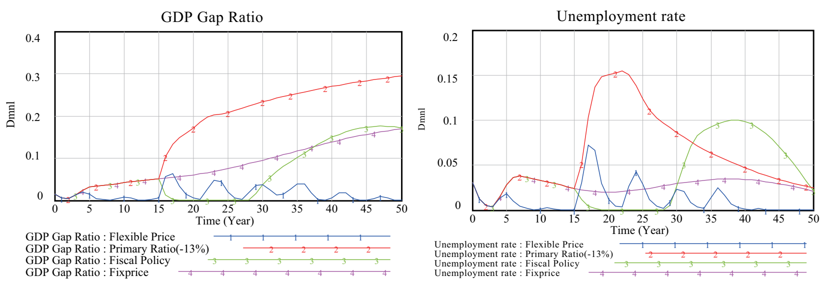

Figure 9.24: GDP Gap and Unemployment Rate under Price Flexibility

In this way, the reduction of the government debt by diminishing a primary balance ratio is shown to be possible without causing a sustained recession by introducing flexible price. A financial reform of this radical type seems to allude to a soft-landing solution path for a country with a serious debt problem such as Japan, so long as our SD simulation suggests, if a sudden collapse of the government and macroeconomy is absolutely to be avoided. Its success depends on how people can endure getting *worse before better*.

Figure 9.25 compares growth paths of the economy under several different situations among almost equilibrium (line 1), fixprice (line 2), fiscal policy (line 3), primary rate reduction (line 4) and flexible price (line 5). Compared with the almost equilibrium path (line 1), debt-reducing path (line 5) causes a business cycle. Yet, compared with another debt-deduction path (line 4), this seems a better path.

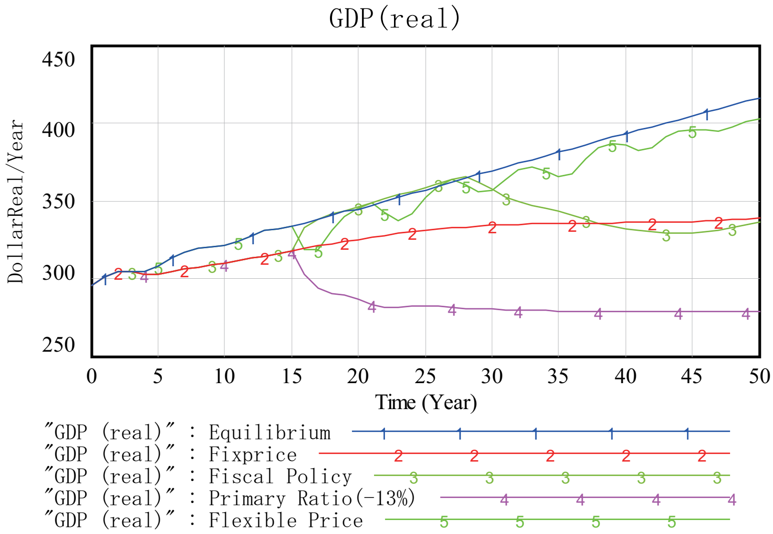

Figure 9.25: Comparison of GDP paths

9.6 Conclusion

Labor market is newly brought to the integrated model of the real and monetary sectors that is analyzed in the previous chapter. Accordingly, several changes are made in this chapter to make the model a complete macroeconomic model. First, production function is revised as Cobb-Douglas production function. Second, population dynamics is added and labor market is newly introduced. This also enables the price adjustment process by cost-push forces.

Under such a complete macroeconomic model, five macroeconomic sectors are brought to the model such as producers, consumers, government, banks and the central bank. The model becomes generic in the sense that diverse macroeconomic behaviors will be produced within the same model structure.

To show such a capability, some macroeconomic behaviors are discussed in this chapter. First, the existence of mostly equilibria is shown. Second, disequilibria are triggered by fixprice and business cycles caused by two different characteristics; inventory coverage period, and flexible and cost-push prices. Then, economic recession is also shown to be triggered by the credit crunch intentionally caused by the central bank.

Finally, it is demonstrated how these business cycles and economic recessions could be overcome by monetary and fiscal policies. Specifically, Keynesian monetary and fiscal policies are applied to the disequilibria caused by fixprice. In addition, accumulating government debt issue is explored.

As demonstrated by these analyses, we believe the complete macroeconomic model presented here will provide a foundation for the analysis of diverse macroeconomic behaviors.

Part III

Open Macroeconomic System

Chapter 10

Balance of Payments and Foreign Exchange Dynamics

This chapter[1] tries to model a dynamic determination of foreign exchange rate in an open macroeconomy in which goods and services are freely traded and financial capital flows efficiently for higher returns. For this purpose it becomes necessary to employ a new method contrary to a standard method of dealing with a foreign sector as adjunct to macroeconomy; that is, an introduction of another macroeconomy as a foreign sector. Within this new framework of open macroeconomy, transactions among domestic and foreign sectors are handled according to the principle of accounting system dynamics, and their balance of payments is attained. For the sake of simplicity of analyzing foreign exchange dynamics, macro variables such as GDP, its price level and interest rate are treated as outside parameters. Then, eight scenarios are produced and examined to see how exchange rate, trade balance and financial investment, etc. respond to such outside parameters. To our surprise, expectations of foreign exchange rate turn out to play a crucial role for destabilizing trade balance and financial investment. The impact of official intervention on foreign exchange and a path to default is also discussed.

10.1 Open Macroeconomy as a Mirror Image

As a natural step of the research, we are now in a position to open our macroeconomy to a foreign sector so that goods and services are freely traded and financial assets are efficiently invested for higher returns. The analytical method employed here is the same as the previous chapters; that is, the one based on the principle of accounting system dynamics.

[1]It is based on the paper: Balance of Payments and Foreign Exchange Dynamics – SD Macroeconomic Modeling (4) – in "Proceedings of the 24th International Conference of the System Dynamics Society", Boston, USA, July 29 - August 2, 2007. (ISBN 978-0-9745329-8-1)

The method requires to manipulate all transactions among macroeconomic sectors, and when applied to a foreign sector, it turns out to be necessary to introduce another macroeconomy as a reflective image of domestic macroeconomy. Contrary to a method employed in standard international economics textbooks such as [36] and [39], a foreign sector is no longer treated as an additional macroeconomic sector adjunct to a domestic macroeconomy [Companion Model: ForeignExchange.vpm].

To understand this, for instance, consider a transaction of importing goods. They add to the inventory of importers (a red disk numbered 1 in Figure 10.2 below), while the same amount is reduced from the inventory of foreign exporters (a red disk numbered 4 in Figure 10.3 below). To pay for the imported goods, importers withdraw their deposits from their bank and purchase foreign exchange, (red disks numbered 2 and 3 in Figures 10.2 and 10.5 below), which is then sent to the deposit account of foreign exporters' bank that will notify the receipts of export payments to exporters (red disks numbered 3 and 4 in Figures 10.6 and 10.3 below). In this way, a mirror image of domestic macroeconomy is needed for a foreign country as well to describe even domestic transaction processes of goods and services. Similar manipulations are also needed for the transactions of foreign direct and financial investment. Figure 10.1 expresses our image of modeling open macroeconomy by the principle of accounting system dynamics.

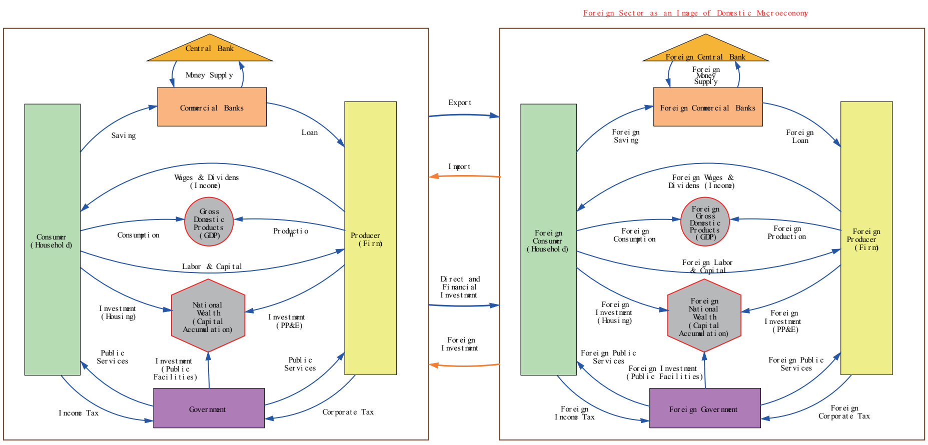

Figure 10.1: Foreign Sector as a Mirror Image of Domestic Macroeconomy

10.2 Open Macroeconomic Transactions

Modeling open macroeconomy was hitherto considered to be easily completed by merely adding a foreign sector. The introduction of a foreign country as a mirror image of domestic macroeconomy makes our analysis rather complicated. To overcome the complexity, we are forced, in this chapter, to focus only on a mechanism of the transactions of trade and foreign investment in terms of the balance of payments and dynamics of foreign exchange rate. For this purpose, transactions among five domestic sectors and their counterparts in a foreign country are simplified as follows.

Producers

Major transactions of producers are, as illustrated in Figure 10.2, summarized as follows.

- GDP (Gross Domestic Product) is assumed to be determined outside the economy, and grows at a growth rate of 2% annually.

- Produces are allowed to make direct investment abroad as well as financial investment out of their financial assets consisting of stocks, bonds and cash[2], and receive investment income from these investment abroad. Meanwhile, they are also required to pay foreign investment income (returns) to foreign investors according to their foreign financial liabilities and equity .

- Produces now add net investment income (investment income received less paid) to their GDP revenues (the added amount is called GNP (Gross National Product)), and deduct capital depreciation (the remaining amount is called NNP (Net National Product)).

- NNP thus obtained is completely paid out to consumers, consisting of workers and shareholders, as wages to workers and dividend to shareholders.

- Producers are thus constantly in a state of cash flow deficits. To make new investment, therefore, they have to borrow money from banks, but for simplicity no interest is assumed to be paid to the banks.

- Producers imports goods and services according to their economic activities, the amount of which is assumed to be 10% of GDP in this chapter.

- Similarly, their exports are determined by the economic activities of a foreign country, the amount of which is also assumed to be 10% of foreign GDP.

- Foreign producers are assumed to behave similarly as a mirror image of domestic producers as illustrated in Figure 10.3.

[2]In this chapter, financial assets are not broken down in detail and simply treated as financial assets. Hence, returns from financial investment are uniformly evaluated in terms of deposit returns.

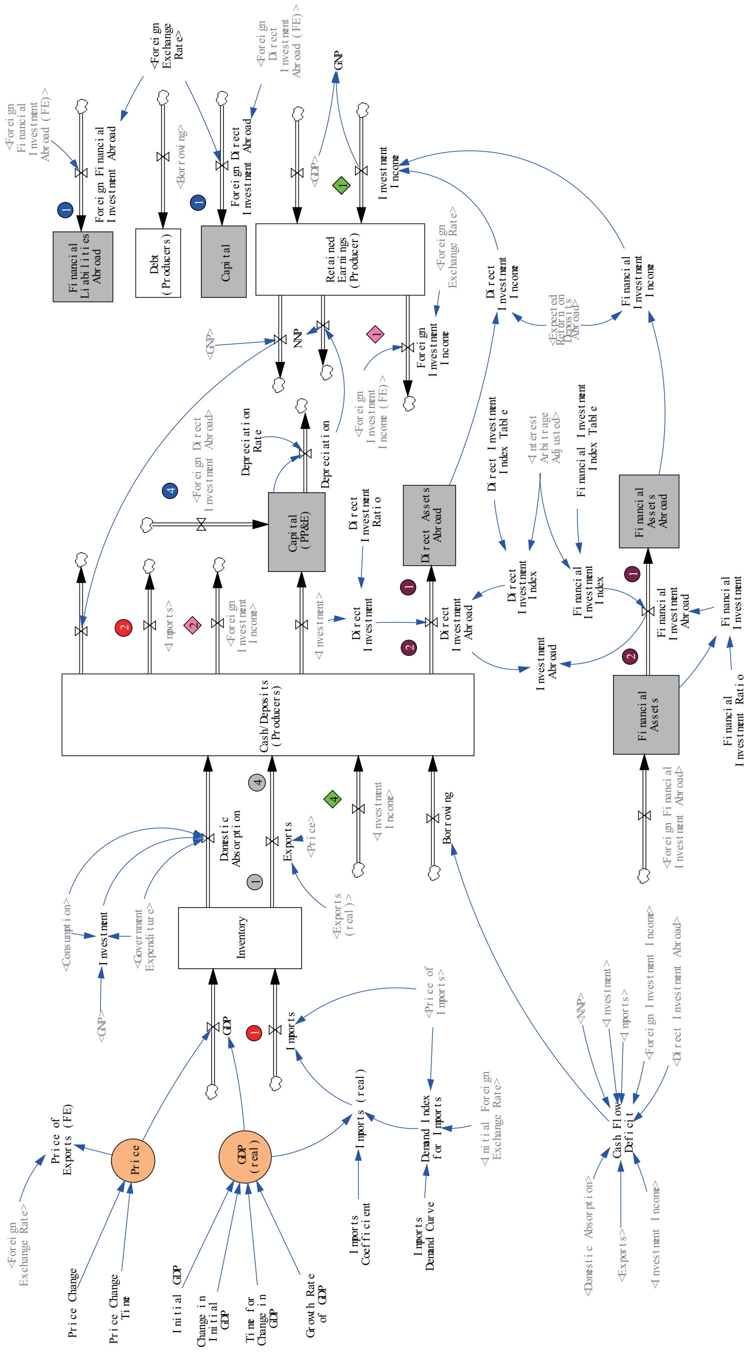

Figure 10.2: Transactions of Producers

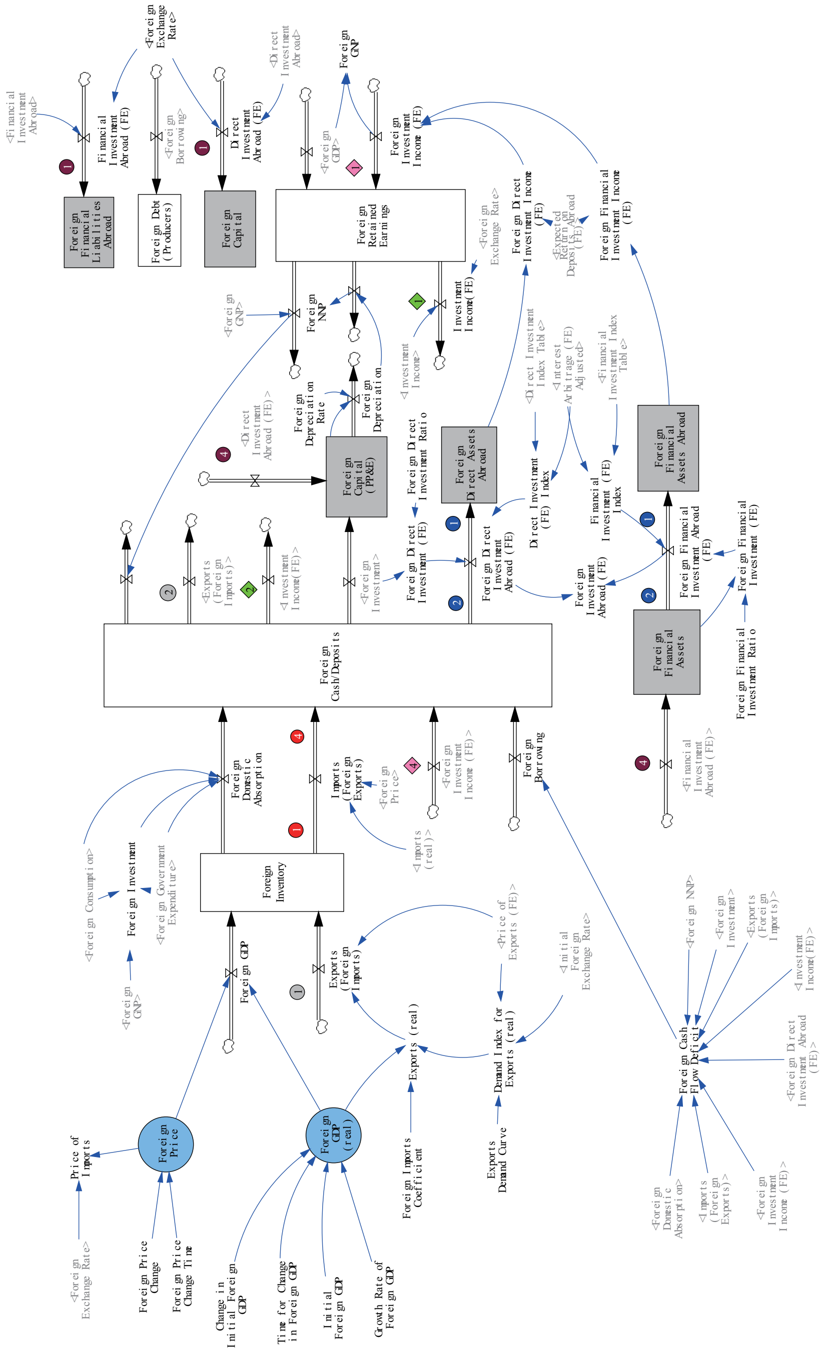

Figure 10.3: Transactions of Foreign Producers

Consumers and Government

Transactions of consumers and government are illustrated in Figure 10.4, some of which are summarized as follows.

- Consumers receive the amount of NNP as income, out of which 20% is levied by the government as income tax. The remaining amount becomes their disposable income.
- Consumers spend 60% of their disposable income and save the remaining as deposits with banks.
- Government only spends the amount it receives as income tax, and its budget is assumed to be in balance.

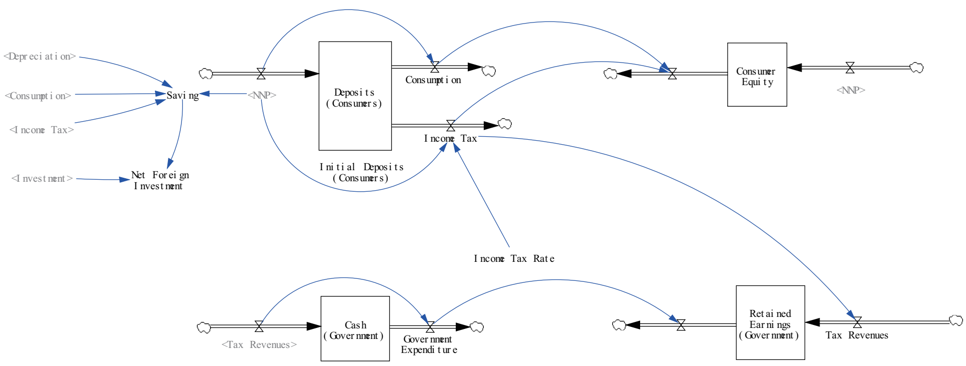

Figure 10.4: Transactions of Consumers and Government

Banks

Transactions of banks are illustrated in Figure 10.5, some of which are summarized as follows.

- Banks receive deposits from consumers and make loans to producers.
- Banks are obliged to deposit a portion of the deposits as required reserves with the central bank, but such activities are not considered in this chapter.
- Banks buy and sell foreign exchange at the request of producers and the central bank.
- Their foreign exchange are held as bank reserves and evaluated in terms of book value. In other words, foreign exchange reserves are not deposited with foreign banks. Thus net gains realized by the changes in foreign exchange rate become part of their retained earnings (or losses).

- Foreign currency is assumed to play a role of *key* currency or *vehicle* currency. Accordingly foreign banks need not set up foreign exchange account. This is a point where a mirror image of open macroeconomic symmetry breaks down, as illustrated in Figure 10.6.

Central Bank

In the integrated model of the previous chapter, the central bank played an important role of providing a means of transactions and store of value; that is, currency, and its sources of assets against which currency is issued were assumed to be gold, loan and government securities. Transactions of the central bank here are exceptionally simplified, as illustrated in Figure 10.7, so long as necessary for the analytical purpose in this chapter.

- The central bank can control the amount of money supply through monetary policies such as the manipulation of required reserve ratio and open market operations. However, such a role of money supply by the central bank is not considered here.
- The central bank is allowed to intervene foreign exchange market; that is, it can buy and sell foreign exchange to keep a foreign exchange ratio stable. These transactions are manipulated with commercial banks, which inescapably change the amount of currency outstanding and, hence, money supply. In this chapter, however, such an effect of money supply on interest rate is assumed to be out of consideration.
- Foreign exchange reserves held by the central bank is assumed to be deposited with foreign banks so that it receives interest payments.
- The central bank of foreign country is excluded simply because foreign currency is assumed to be a *vehicle* currency, and it needs not to hold foreign reserves (that is, its own currency) to stabilize its own exchange rate in this simplified open macroeconomy.

10.3 The Balance of Payments

All transactions with a foreign country such as foreign trade and foreign investment (that is, payments and receipts of foreign exchange) are booked according to a double entry bookkeeping rule, and such a bookkeeping record is called the balance of payments. According to [36] in page 295, all payments are recorded in the debit side with a minus sign, while all receipts are recorded in the credit side with plus sign. Hence, by definition, the balance of payments are kept in balance all the time. It consists of current account, capital and financial account, and net official reserve assets.

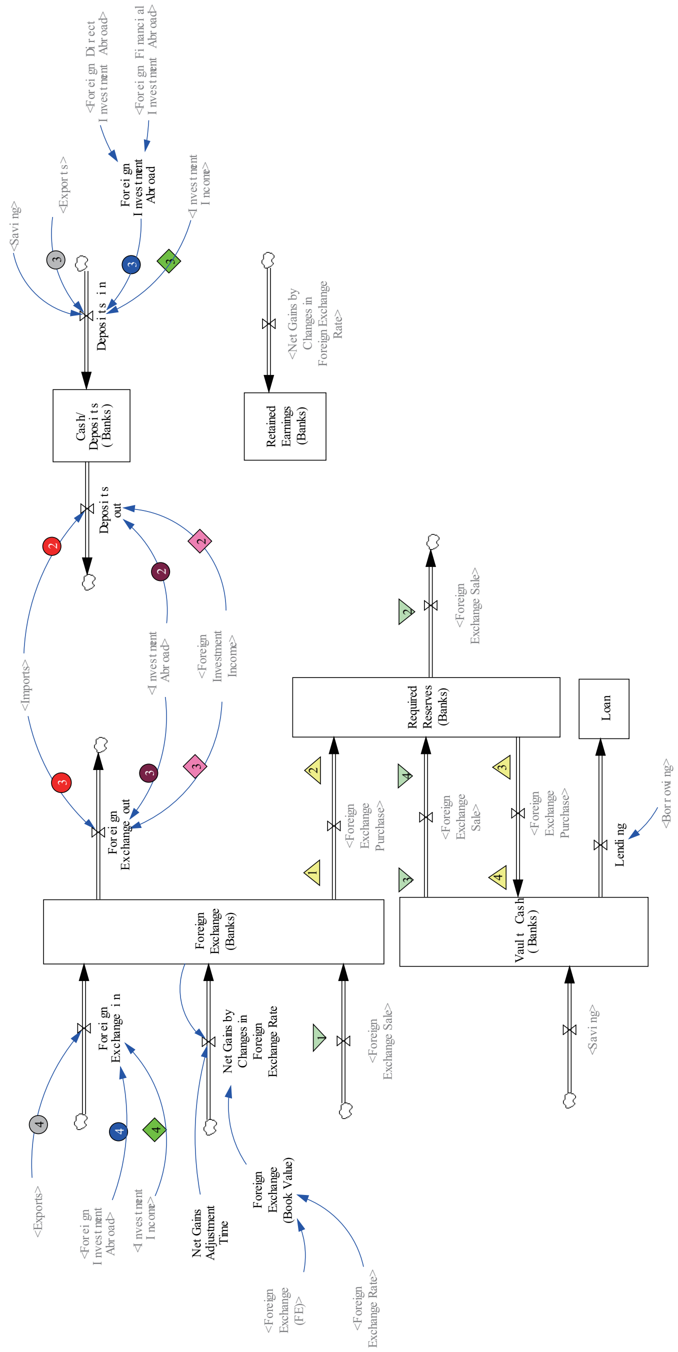

Figure 10.5: Transactions of Banks

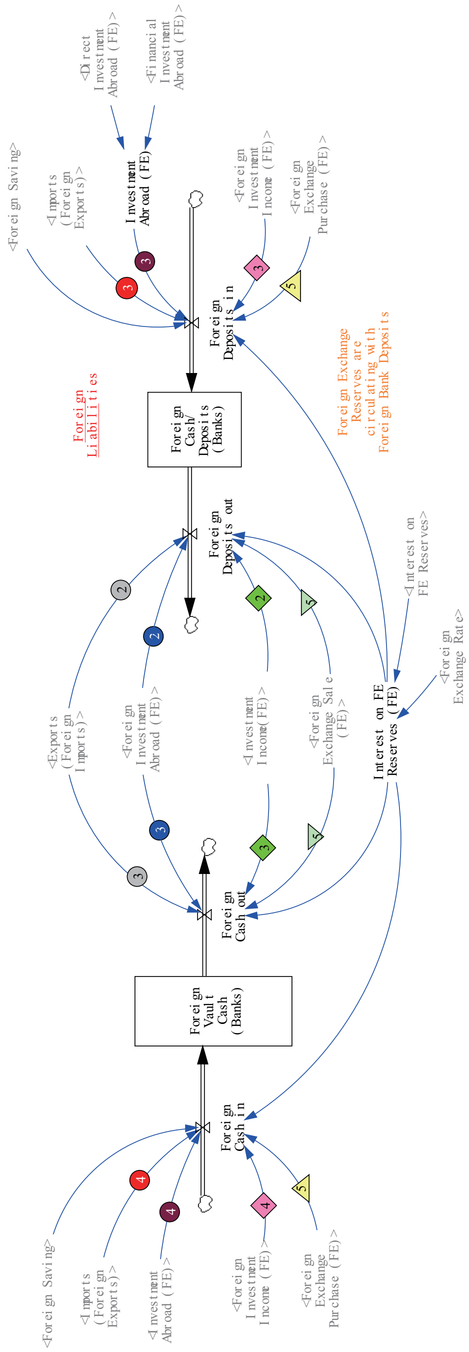

Figure 10.6: Transactions of Foreign Banks

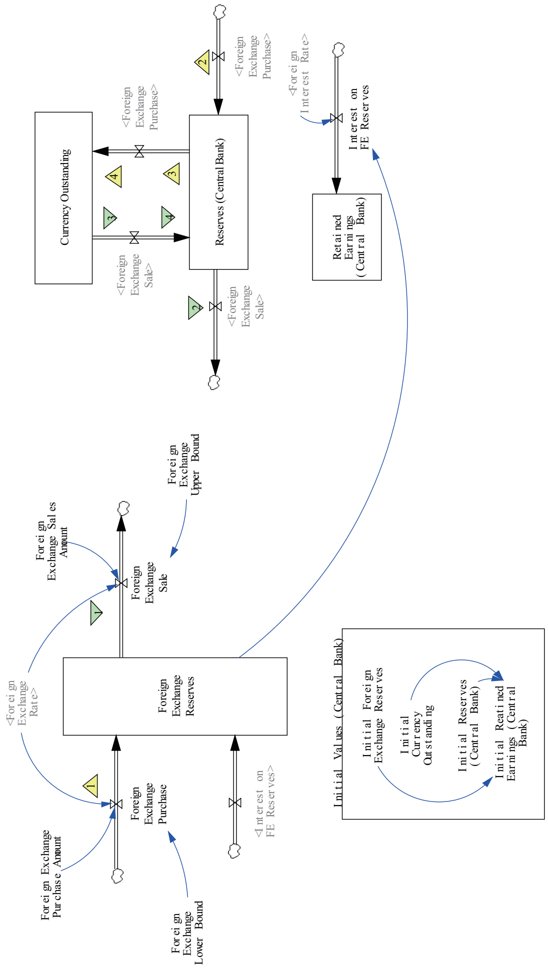

Figure 10.7: Transactions of the Central Bank

Current account consists of trade balance of goods and services and net investment income. Capital account is an one-way transfer of fund by the government that is excluded from our analysis here. Financial account consists of direct and financial foreign investment. Figure 10.8 illustrates all transactions which enter into the balance of payments account.

Figure 10.9, obtained from one of our simulation runs, displays relative positions of current account, capital and financial account, and net official reserve assets (or changes in reserve assets). A numerical value of the balance of payments is shown in the figure as being in balance all the time; that is a zero value.

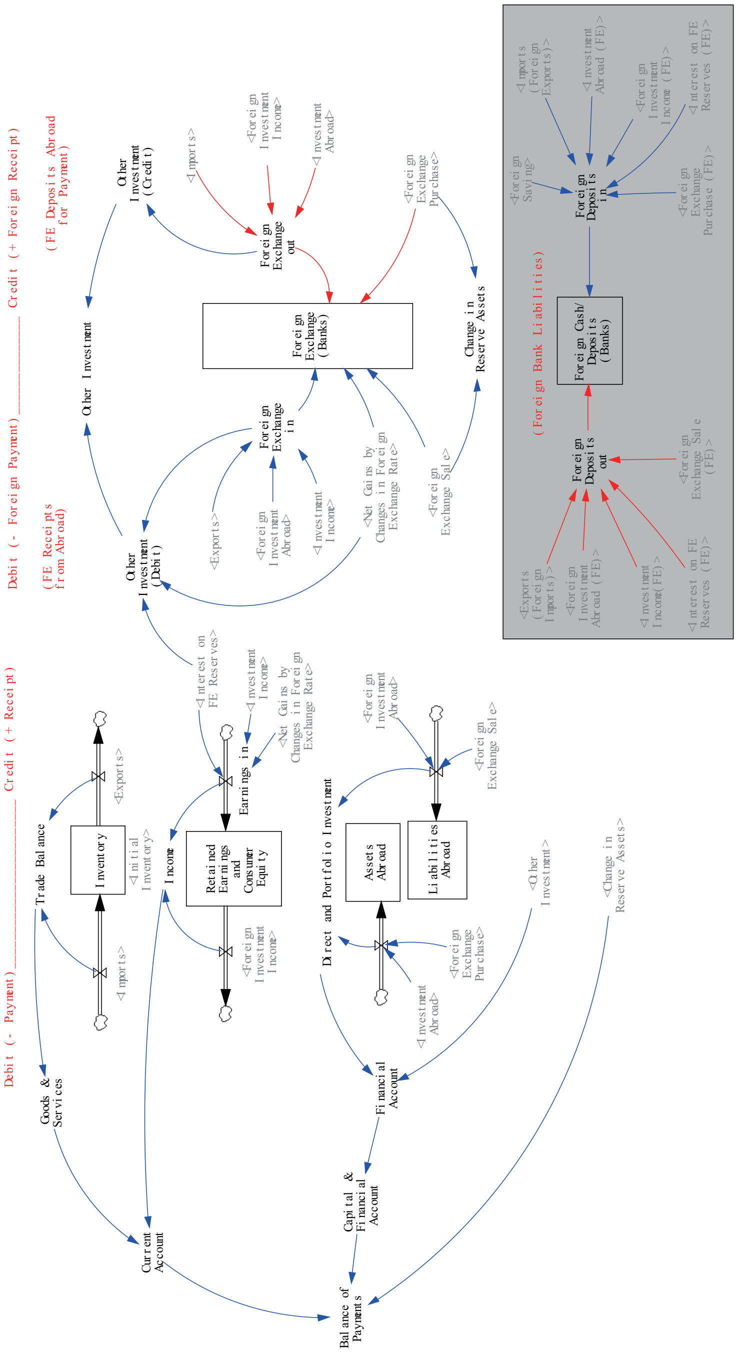

Figure 10.8: The Balance of Payments

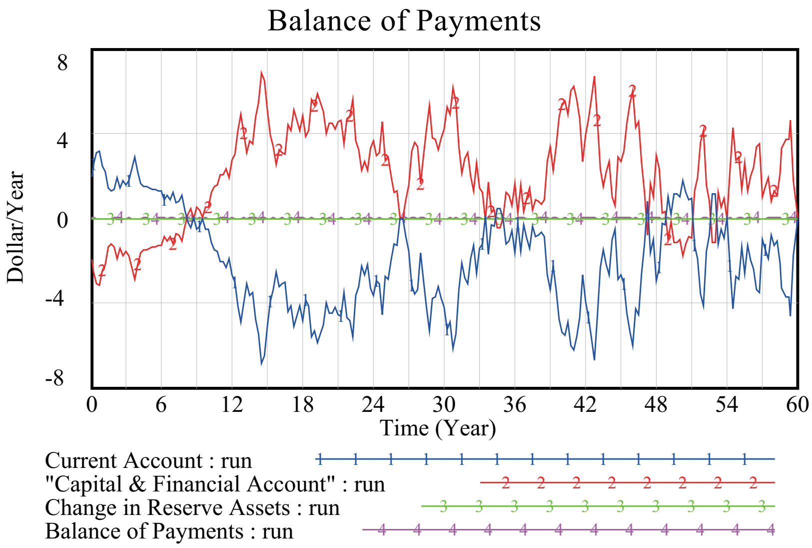

Figure 10.9: A Simulation of Balance of Payments

10.4 Determinants of Trade

Let M and X be real imports and exports, and Y and P be real GDP and its price level, respectively. Counterpart variables for a foreign country is denoted with a subscript f. A foreign exchange rate E is defined as a price of foreign currency (which has a unit of FE here) in terms of domestic dollar currency; for instance, 1.2 dollars per FE. Then, a price of imports is calculated as $P_M = P_f E$.

Imports are here simply assumed to be a function of real GDP and price of imports such that

$$M = M(Y, P_M), \text{ where } \frac{\partial M}{\partial Y} > 0 \text{ and } \frac{\partial M}{\partial P_M} < 0. \tag{10.1}$$

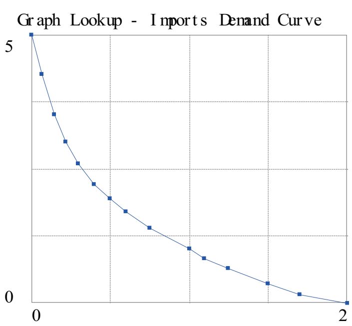

Figure 10.10: Normalized Demand Curve

This implies that imports increases as domestic economic activities, hence GDP, expand, and decreases as price of imports rises as a standard downward-sloping demand curve conjectures. Figure 10.10 illustrates one of such demand curves employed in this chapter in which demand is normalized between a scale of zero and five on a vertical axis against a price level of between zero and two on a horizontal axis.

From these simple assumptions, we can

derive the following relations:

$$M = M(Y, P_M) = M(Y, P_f E), \quad (10.2)$$

$$\frac{\partial M}{\partial P_f} = \frac{\partial M}{\partial P_M}\frac{\partial P_M}{\partial P_f} = \frac{\partial M}{\partial P_M}E < 0 \quad (10.3)$$

$$\frac{\partial M}{\partial E} = \frac{\partial M}{\partial P_M}\frac{\partial P_M}{\partial E} = \frac{\partial M}{\partial P_M}P_f < 0 \quad (10.4)$$

These relations imply that imports decrease as foreign price of imports increases and/or foreign exchange rate appreciates.

In our model, imports function is further simplified as a product of imports determined by the size of GDP and a normalized demand curve such that

$$M = M(Y, P_M) = M(Y)D(P_M) = mYD(P_f E) \quad (10.5)$$

where m is a constant coefficient of imports on GDP.

Exports are nothing but imports of a foreign country, and similarly determined as a mirror image of domestic imports function such that

$$X = X(Y_f, P_{M,f}), \text{ where } \frac{\partial X}{\partial Y_f} > 0 \text{ and } \frac{\partial X}{\partial P_{M,f}} < 0. \quad (10.6)$$

This implies that exports increase as foreign economic activities, hence foreign GDP, expand, and decreases as price of imports in a foreign country rises as a standard downward-sloping demand curve conjectures.

Price of imports in a foreign country is calculated by a domestic price and foreign exchange rate such that $P_{M,f} = P/E$. Hence, we obtain the following relations:

$$X = X(Y_f, P_{M,f}) = X(Y_f, P/E), \quad (10.7)$$

$$\frac{\partial X}{\partial P} = \frac{\partial X}{\partial P_{M,f}}\frac{\partial P_{M,f}}{\partial P} = \frac{\partial X}{\partial P_{M,f}}\frac{1}{E} < 0 \quad (10.8)$$

$$\frac{\partial X}{\partial E} = \frac{\partial X}{\partial P_{M,f}}\frac{\partial P_{M,f}}{\partial E} = \frac{\partial X}{\partial P_{M,f}}(-\frac{P}{E^2}) > 0. \quad (10.9)$$

Thus, exports decrease as a domestic price rises. Meanwhile, whenever foreign exchange appreciates, our products become cheaper in a foreign county and exports increase.

Exports are similarly broken down as a product of foreign imports and normalized demand curve of foreign country, which is assumed to be exactly the same as domestic demand curve for imports.

$$X = X(Y_f, P_{M,f}) = X(Y_f)D(P_{M,f}) = m_f Y_f D(P/E) \quad (10.10)$$

where m_f is a constant import coefficient of a foreign country.

Let us define trade balance as

$$TB(E; Y, Y_f, P, P_f) = X(E; Y_f, P) - M(E; Y, P_f) \tag{10.11}$$

Then we have

$$\frac{\partial TB}{\partial Y} = -\frac{\partial M}{\partial Y} < 0, \ \frac{\partial TB}{\partial Y_f} = \frac{\partial X}{\partial Y_f} > 0, \tag{10.12}$$

$$\frac{\partial TB}{\partial P} = \frac{\partial X}{\partial P} < 0, \ \frac{\partial TB}{\partial P_f} = -\frac{\partial M}{\partial P_f} > 0. \tag{10.13}$$

$$\frac{\partial TB}{\partial E} = \frac{\partial X}{\partial E} - \frac{\partial M}{\partial E} > 0. \tag{10.14}$$

The last relation indicates that a trade balance is an increasing function of foreign exchange rate. The relation is also confirmed in our model as illustrated in the two diagrams of Figure 10.11 in which upward-sloping blue curves are obtained from our simulation runs. As an mirror image, foreign trade balance is shown to be a decreasing function of foreign exchange rate, as indicated by downward-sloping red curves.

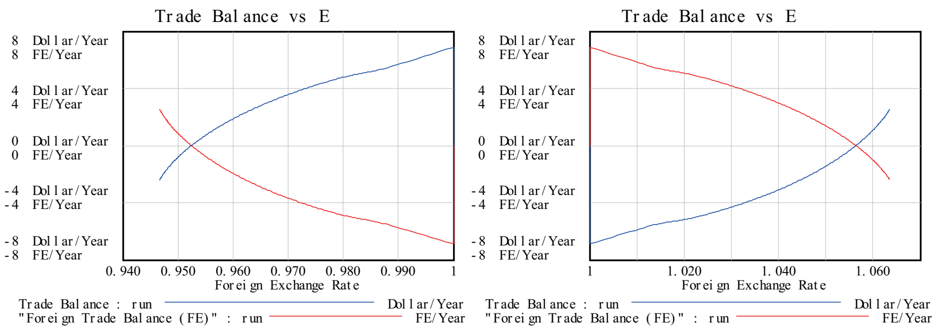

Figure 10.11: Trade Balance vs Foreign Exchange Rate

National Income Identity

Let us now briefly summarize our model in terms of national income account as follows:

$$Y = C(Y - T) + I + G + TB(E) \tag{10.15}$$

That is to say, GDP is the sum of consumption spending, investment, government expenditure and trade balance. In our model of foreign trade, investment is calculated to make this equation an identity all the time.

Private saving is defined as $S_p = Y - T - C$. Government saving is defined as $S_g = T - G$. Then national saving is obtained as a sum of these savings such that

$$S = S_p + S_g = Y - C - G, \tag{10.16}$$

which reduces to

$$S - I = TB(E). \tag{10.17}$$

Saving less investment is called net foreign investment, which is equal to trade balance. This becomes another way of describing the above national income identity in terms of net foreign investment and trade balance.

10.5 Determinants of Foreign Investment

Foreign investment consists of direct investment and financial investment such as stocks, bonds and cash, which constitute financial assets. In this chapter financial assets are not specified without losing generality as already mentioned in the footnote above. Foreign investments are here assumed to be determined on a principle of foreign exchange market efficiency under the uncovered interest rate parity (UIP) condition as explained in standard textbooks such as [36] and [44].

Let i and R be interest rate and a rate of return from financial investment, and E^e be an expected foreign exchange rate. A rate of return from a bank deposit is the same as the interest rate:

$$R = i \tag{10.18}$$

An expected return from a deposit with a foreign bank is calculated as

$$R_f = (1 + i_f)\frac{E^e}{E} - 1 \tag{10.19}$$

Thus we obtain

$$\frac{\partial R_f}{\partial E} = -\frac{(1 + i_f)E^e}{E^2} < 0 \tag{10.20}$$

$$\frac{\partial R_f}{\partial E^e} = \frac{(1 + i_f)}{E} > 0 \tag{10.21}$$

This implies that a rate of return from foreign financial investment decreases if foreign exchange rate appreciates, but it increases when foreign exchange rate is expected to appreciate.

Let us define an expected return arbitrage as

$$A(E, E^e; i, i_f) = R_f(E, E^e; i_f) - R(i) \tag{10.22}$$

and net capital flow(NCF) as

$$NCF = \text{Foreign Investment Abroad} - \text{Investment Abroad} \tag{10.23}$$

This is the amount of capital we receive from foreign country's investment less the amount we invest abroad. Under the assumption of an efficient financial market, if expected returns are greater in a foreign country and an expected return arbitrage becomes positive, then financial capital continues to outflow until the arbitrage ceases to exist. In a similar fashion, if expected returns are greater in a domestic market and an expected return arbitrage becomes negative, then financial capital continues to inflow until the arbitrage disappears. Hence, so long as a foreign exchange market is efficient, the relation between net capital flow and an expected return arbitrage become as follows:

$$\begin{cases} NCF < 0 & \text{if } A > 0 \\ NCF > 0 & \text{if } A < 0 \end{cases} \tag{10.24}$$

It is unrealistic, however, to assume an indefinite outflow of capital even if $A > 0$, or an indefinite inflow of capital even if $A < 0$. So it is assumed here that the maximum amount of direct and financial investment made available per year is a finite portion of domestic investment and financial assets. Yet, actual amount of financial investment is further assumed to be dependent on a level of an expected return arbitrage by its factor. Figure 10.12 illustrates table functions of investment levels that are assumed in our model in terms of expected return arbitrate.

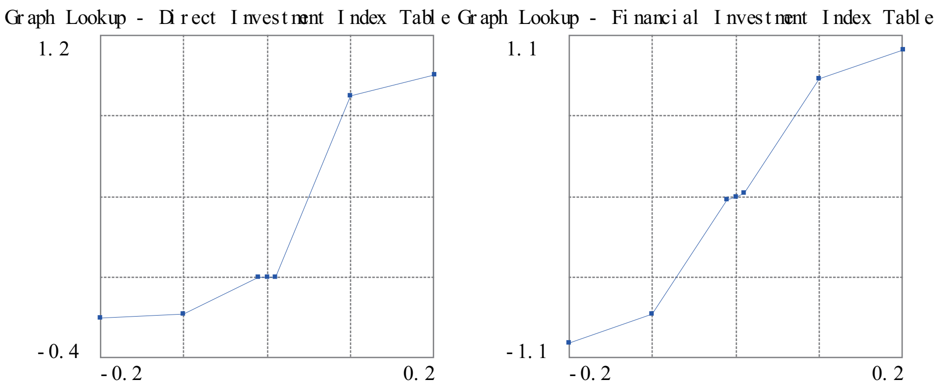

Figure 10.12: Direct and Financial Investment Indices

Specifically, left-hand diagram shows a table function of direct investment, which assumes that between the arbitrage range of -0.01 and 0.01 direct investment is not made. Right-hand diagram shows a table function of financial investment, which assumes that between the arbitrage range of -0.01 and 0.01 financial capital flows slowly between a portion of -0.02 and 0.02. These assumptions are made to reflects a realistic situation in which direct investment is not so sensitive to the arbitrage values compared with financial investment.

In this way net capital flow could be described as a function of an expected return arbitrage such that

$$NCF = NCF(A(E, E^e)), \text{ where } \frac{\partial NCF}{\partial A} < 0 \tag{10.25}$$

It is important to note, however, that this functional relation holds only in the neighborhood of equilibrium, so do the following relations as well.

$$\frac{\partial NCF}{\partial E} = \frac{\partial NCF}{\partial A}\frac{\partial A}{\partial E} = \frac{\partial NCF}{\partial A}\frac{\partial R_f}{\partial E} > 0 \tag{10.26}$$

$$\frac{\partial NCF}{\partial E^e} = \frac{\partial NCF}{\partial A}\frac{\partial A}{\partial E^e} = \frac{\partial NCF}{\partial A}\frac{\partial R_f}{\partial E^e} < 0 \tag{10.27}$$

Whenever a foreign exchange rate begins to appreciate, an expected return arbitrage declines, and capital begins to inflow, causing a positive net capital flow. When foreign exchange rate is expected to appreciate, an expected return arbitrage increases and capital begins to outflow, causing a negative net capital flow. In this way, changes in a foreign exchange rate and its expectations play a crucial role for financial investment.

It is examined in our model that these relations only hold in the neighborhood of equilibrium. In Figure 10.13, net capital flow is shown to be an

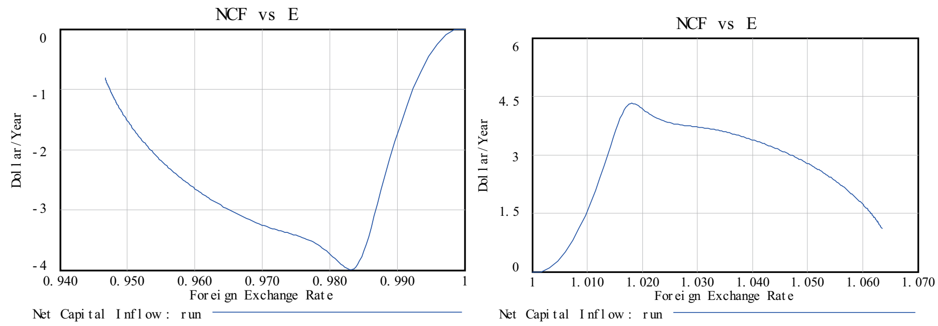

Figure 10.13: Net Capital Inflow vs Foreign Exchange Rate

increasing function only when a foreign exchange rate is around the equilibrium; that is, between 0.983 and 1.018. This may indicate a limitation of the above mathematical method of economic analysis which has been dominantly used in many textbooks. In other words, mutually interdependent economic behaviors cannot be fully captured unless they are simulated in a system dynamics model such as the one in this chapter.

10.6 Dynamics of Foreign Exchange Rates

How are the foreign exchange rate and its expectations determined, then? Foreign exchange rate is here simply assumed to be determined by the excess demand for foreign exchange; that is, a standard logic of price mechanism in economic theory. From the left-hand diagram of Figure 10.8, demand for foreign exchange is shown to stem from the need for payments due to imports, direct and financial investment abroad, and foreign investment income, as well as foreign exchange purchase by the central bank. Supply of foreign exchange results from the receipts from foreign country due to exports, foreign direct and financial investment abroad, and investment income from abroad, as well as foreign exchange sale by the central bank.

Hence, excess demand for foreign exchange is calculated as follows:

$$\begin{aligned}
& \text{Excess Demand for Foreign Exchange} \\
= \; & \text{Imports - Exports} \\
& + \text{Investment Abroad - Foreign Investment Abroad} \\
& + \text{Foreign Investment Income - Investment Income} \\
& + \text{Foreign Exchange Purchase - Foreign Exchange Sale} \\
= \; & - \text{Trade Balance } (TB) \\
& - \text{Net Capital Flow } (NCF) \\
& - \text{Net Investment Income } (NII) \\
& + \text{Net Exchange Reserves } (NER)
\end{aligned} \tag{10.28}$$

Net investment income is derived from the financial assets invested abroad and here assumed to be dependent only on domestic and foreign interest rates. Net exchange reserves depend on the official foreign exchange intervention. Therefore, NII and NER are not dependent on foreign exchange rate and its expectations.

With these relations taken into consideration, dynamics of foreign exchange rate is mathematically expressed as a function of excess demand for foreign exchange, which in turn becomes a function of E and E^e as follows:

$$\frac{dE}{dt} = \Psi(-TB(E) - NCF(E, E^e) - NII + NER) = \Psi(E, E^e) \tag{10.29}$$

On the other hand, a formation of expected foreign exchange rates is difficult to formalize. Here it is simply assumed that actual expectations of foreign exchange rate fluctuates randomly around the current exchange rate by the factor of random normal distribution of $N_{random}(m, sd)$ where (m, sd) denotes mean and standard deviation, and accordingly an expected foreign exchange rate is obtained as an adaptive expectation against the actual expectation of random normal distribution.

Mathematically, dynamics of the expected foreign exchange rate thus defined is described as

$$\frac{dE^e}{dt} = \Phi(N_{random}(m, sd)E - E^e) = \Phi(E, E^e) \tag{10.30}$$

Thus, expected foreign exchange rate can be easily adjusted to the actual trends and volatilities of various economic situations by refining values in mean and standard deviation. Figure 10.14 illustrates how foreign exchange rate and its expectation are modeled in our economy.

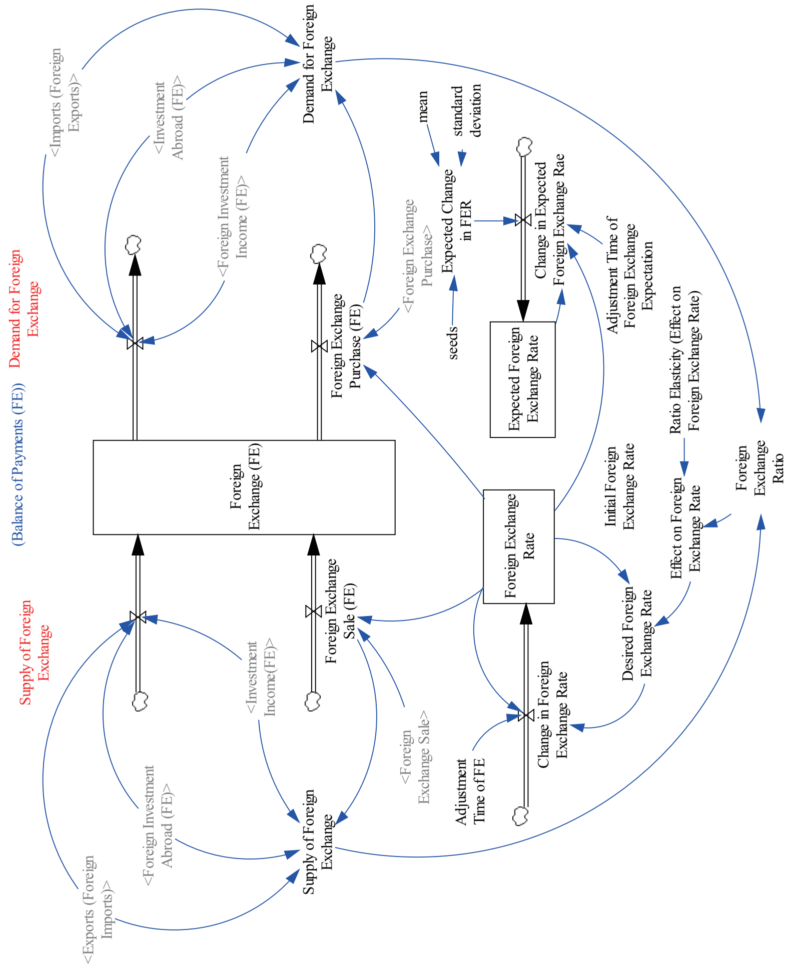

Figure 10.14: Determination of Foreign Exchange

Now dynamic modeling of foreign exchange rate in our open macroeconomy is complete. It consists of three equations: (10.15), (10.29), and (10.30), out of which three variables E, E^e and TB are determined, given parameters outside such as GDP, its price level and interest rate, as well as random normal distribution of expected foreign exchange rate. Schematically, it is written as

$$(Y, Y_f, P, P_f, i, i_f, N_{random}) \Longrightarrow (E, E^e, TB) \tag{10.31}$$

Figure 10.15 draws a theoretical gist of our open macroeconomic framework as a simplified causal loop diagram of the dynamics of foreign exchange rate in our open macroeconomy.

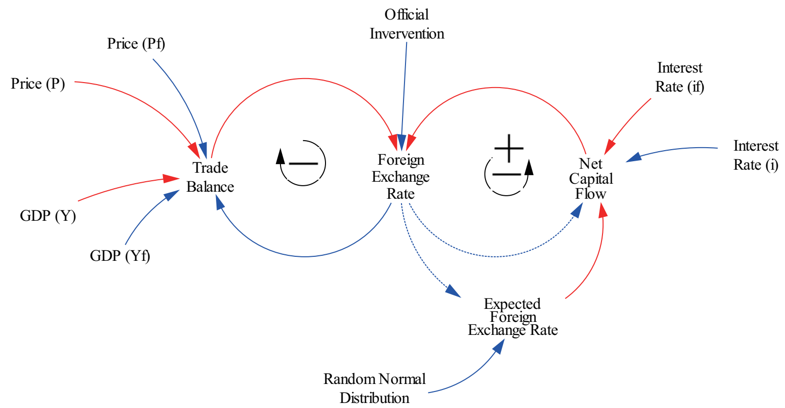

Figure 10.15: Causal Loop Diagram of the Foreign Exchange Dynamics

10.7 Behaviors of Current Account

An Equilibrium State (S)

We are now in a position to examine how our open macroeconomy behaves. Let us start with an equilibrium state of trade and foreign exchange. Domestic and foreign GDPs are assumed to grow at an annual rate of 2%. Random normal distribution for the expected foreign exchange rate is assumed to have a zero mean value and 0.1 value of standard deviation. Figure 10.16 illustrates the equilibrium state under such circumstances. Macroeconomic figures such as consumption spending, investment, government expenditures, exports and imports are shown to be growing, while trade balance is in equilibrium at a zero value in the left-hand diagram. On the other hand, a constant foreign exchange rate at one dollar per FE and its fluctuating expected rates are shown in the right-hand diagram.

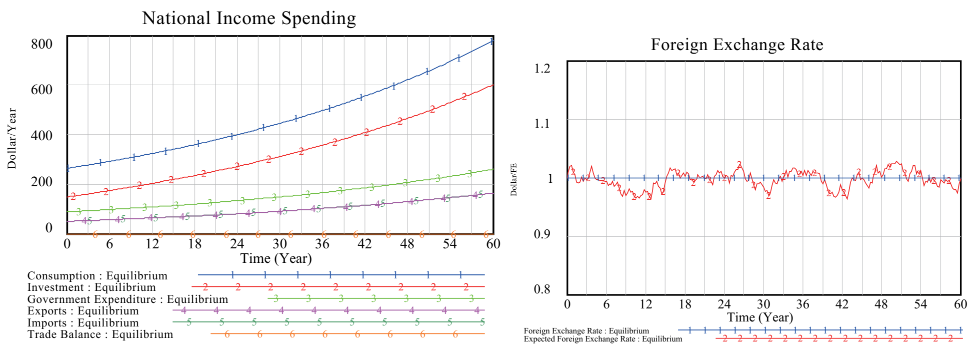

Figure 10.16: Equilibrium State of Trade and Foreign Exchange Rate (S)

In this state of equilibrium, financial investment is not yet considered. Hence, in spite of non-zero expected return arbitrate, caused by the fluctuations of estimated foreign exchange rates, capital flows are not provoked, and accordingly trade balance stays undisturbed.

Change in real GDP (S1)

Several scenarios can be considered that lead economic behaviors out of the above equilibrium state. Let us start with two simple cases in which no capital flows are allowed; that is, our dynamic system of foreign exchange rate is now simply described as

$$\frac{dE}{dt} = \Psi(-TB(E)) \tag{10.32}$$

As a first scenario, suppose a foreign real GDP decreases by 60 (billion) dollars at the year 7 due to a recession in a foreign country. The effect of this recession appears first of all as a sudden drop in our exports which are wholy dependent on foreign economic activities. This sudden plunge in exports causes a trade deficit. This will begin to increase demand for foreign exchange, because imports become relatively larger than exports, which in turn will cause foreign exchange rate to appreciate. The appreciation of foreign exchange rate makes imported goods more expensive, and eventually curbs the imports and trade balance will be gradually restored. In due course a new equilibrium state of foreign exchange rate will be attained at 1.056 dollars per FE (an appreciation rate of 5.6%)

In this way a flexible foreign exchange rate plays a decisive role of restoring trade imbalance as illustrated in Figure 10.17. Trade balance in a foreign country moves exactly into the opposite direction, so that a perfect mirror image of trade balance is created as reflected in the right-hand diagram.

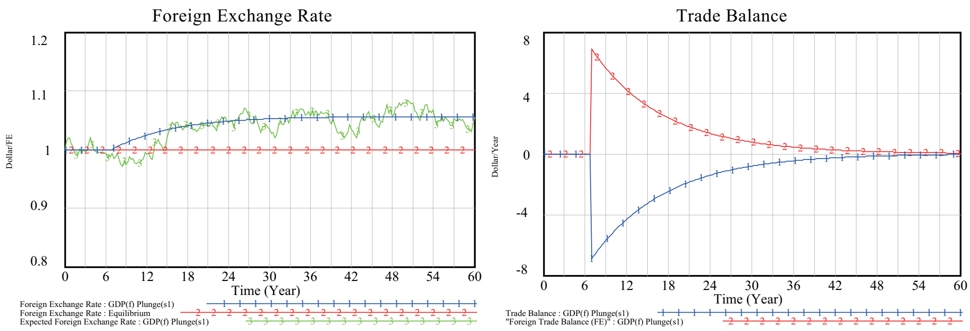

Figure 10.17: Foreign GDP Plunge and Restoring Trade Balance (S1)

Change in Price (S2)

As a second scenario, let us consider an opposite situation in which a foreign price rises by 10% due to an economic boom in a foreign country. The inflation makes imported goods more expensive and imports are suddenly suppressed, causing a surplus trade balance. Trade surplus will bring in more foreign exchange, causing a foreign exchange rate to depreciate. The depreciated foreign exchange rate now makes imported goods relatively cheaper and stimulates imports again. In this way trade balance will be restored and a new level of exchange rate is attained in due course at 0.97 dollars per FE (a depreciation rate of 3 %) as illustrated in Figure 10.18.

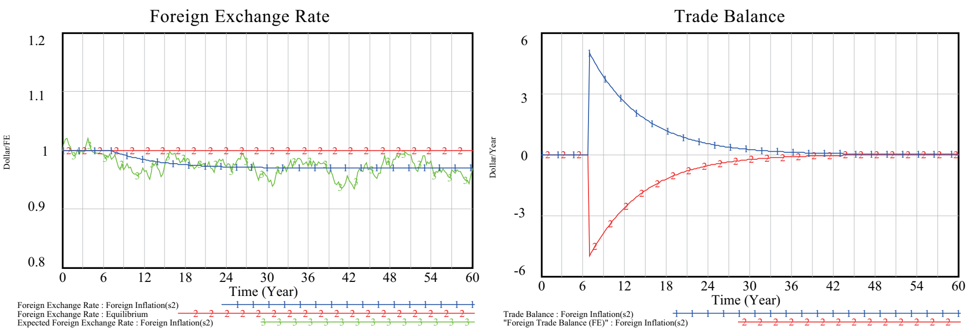

Figure 10.18: Foreign Inflation and Restoring Trade Balance (S2)

10.8 Behaviors of Financial Account

Expectations and Foreign Investment (S3)

In the above equilibrium state, standard deviation of random normal distribution is assumed to be 0.1, and expected foreign exchange rates are allowed to move randomly. Accordingly, non-zero return arbitrage caused by such fluctuations of foreign exchange rate could have triggered capital inflows and outflows under the assumption of efficient financial market. Yet, in order to see the effect of economic activities and price levels on trade balance and exchange rate, financial investment is excluded from the analysis. In this sense, the equilibrium state discussed above is not a real equilibrium state under free capital flows.

From now on let us consider three cases in which free capital flows are allowed for higher returns. In other words, behaviors of three variables E, E^e and TB are fully analyzed under the three equations: (10.15), (10.29), and (10.30).

As a scenario 3, let us consider the original equilibrium state again and see what will happen if free capital flows are additionally allowed for higher returns. As a source of financial investment, 20 % of domestic investment is assigned to direct investment abroad, and 30 % of financial assets are allowed for financial investment for both economies. The actual financial investment, however, depends on the scale of investment indices illustrated in Figure 10.12 above.

Figure 10.19 illustrates a revised equilibrium state under free flows of capital. Top-right figure shows the existence of the expected return arbitrage under the fluctuations of expected foreign exchange rates. The emergence of the arbitrage undoubtedly trigger capital flows of financial investment for higher returns, breaking down the original equilibrium state of trade balance, as shown in the bottom two diagrams. In this way, the original equilibrium state of trade is easily thrown out of balance by merely introducing random expectations of foreign exchange rate under an efficient capital market. In other words, random expectations among financial investors are shown to be a cause of trade turbulence, and hence economic fluctuations of boom and bust in international trade. A flexible foreign exchange rate can no longer restore a trade balance. This is an unexpected and surprising simulation result in this chapter.

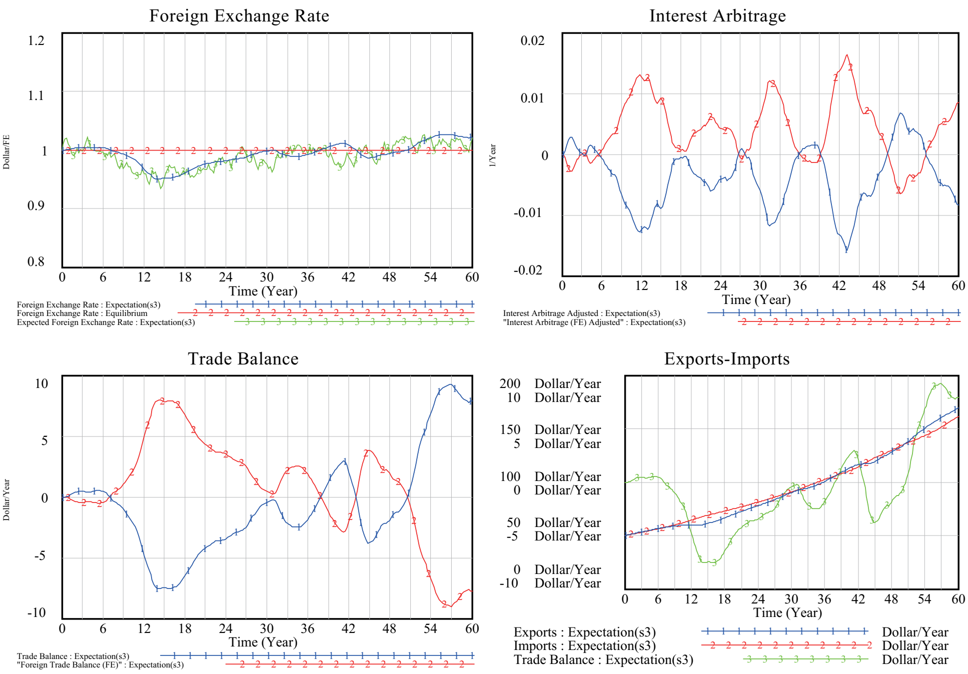

Figure 10.19: Random Expectations and Foreign Investment (S3)

Change in Interest Rate (S4)

Under the situation of the above scenario 3, let us additionally suppose, as scenario 4, that a domestic interest rate suddenly plummets by 2% and becomes 1%

from the original 3% at the year 7. This drop may be caused by an increase in money supply. The lowered interest rate surely drives capital outflows abroad. This in turn will increase the demand for foreign exchange, and a foreign exchange rate will begin to appreciate. The appreciation of foreign exchange rate makes exports price relatively cheaper, and trade balance turns out to become surplus. Figure 10.20 illustrates how a plummet of interest rate appreciates foreign exchange rate and improve a trade balance.

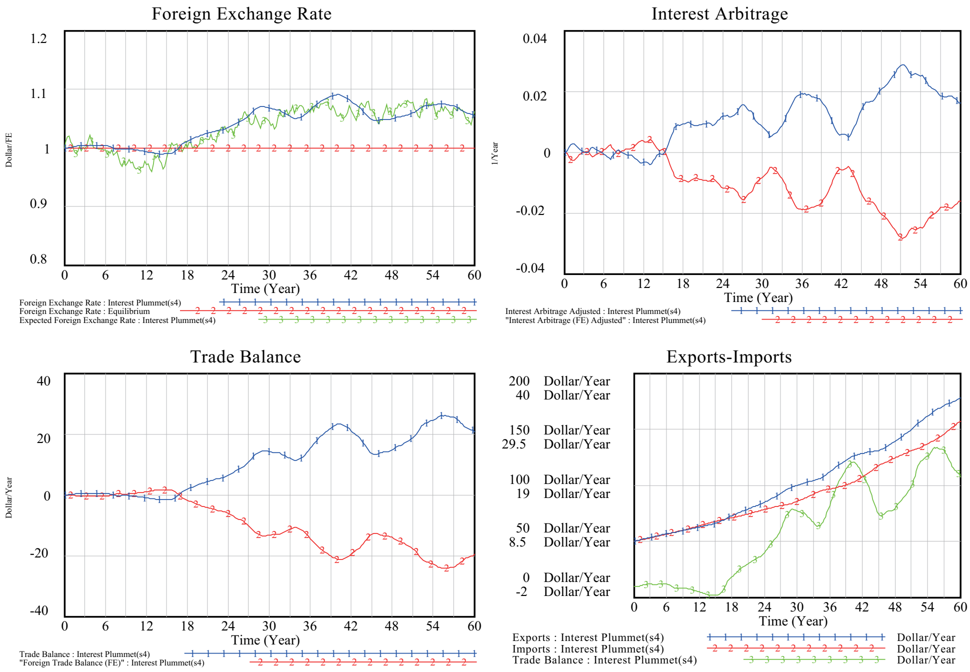

Figure 10.20: Interest Plummet under Random Expectations (S4)

Left-hand diagram of Figure 10.21 illustrates the balance of payments under the original equilibrium state (scenario 3). Current account is shown to be in deficit all the time, and in order to finance it financial account has to be in surplus. Under the same situation, a domestic interest rate is additionally lowered (scenario 4). Right-hand diagram indicates how lowered interest rate stimulates the economy and improves a deficit state of the balance of payments.

Change in GDP and Free Capital Flow (S5)

Let us revisit the scenario 3. Then as a scenario 5, let us additionally assume a decrease in foreign GDP by 60 (billion) dollars at the year 7 due to a recession in a foreign country as in the scenario 1. Furthermore, the central bank is now assumed to hold foreign exchange reserves of 100 (billion) dollars that are deposited with foreign banks.

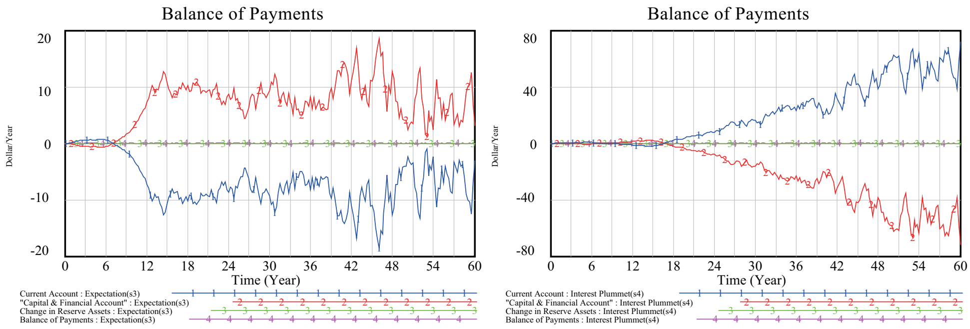

Figure 10.21: Comparison of the Balance of Payments between S3 and S4

As already discussions in the scenario 1, foreign exchange rate continues to appreciate, yet trade balance is no longer attained and trade deficits continues for a foreseeable future due to the disturbance caused by free capital flows as explored in the scenario 3. Top diagrams of Figure 10.22 illustrate these situations. Bottom-left diagram indicates current account deficits in the balance of payments, which has to be offset by the net inflow of capital.

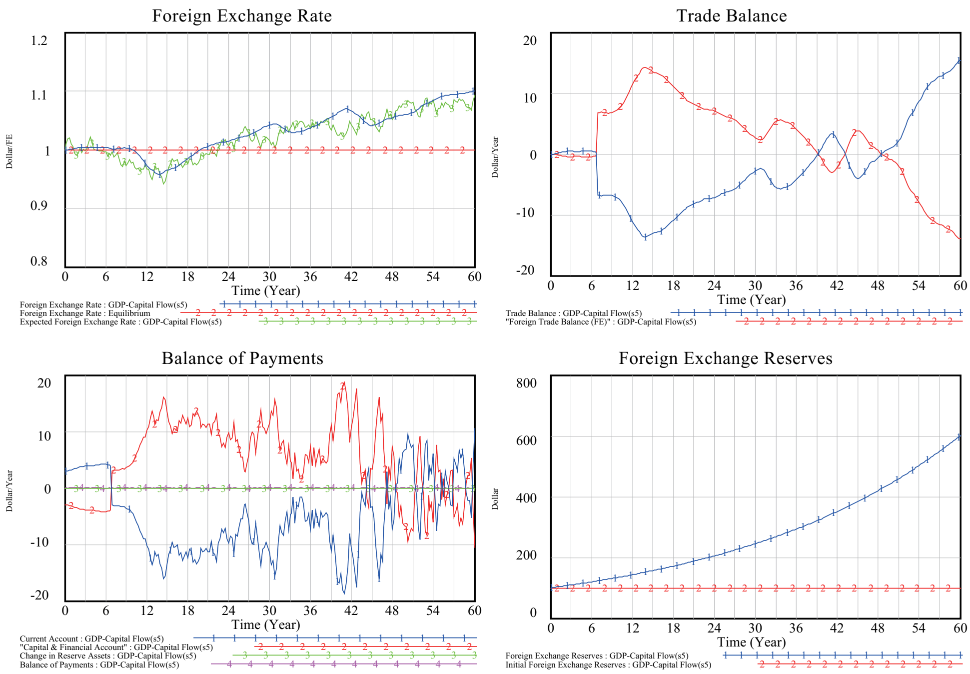

Figure 10.22: Foreign GDP Plunge and Foreign Investment (S5)

Bottom-right diagram shows that foreign exchange reserves by the central bank continues to grow at a rate of the foreign interest rate of 3 %. From a well-known principle of a doubling time of exponential growth, the reserves keep doubling approximately every 23 years.

10.9 Foreign Exchange Intervention

Official Intervention and Default (S6)

In the scenario 5 above, our macroeconomy continues to suffer from a continual depreciation of domestic currency (or an appreciation of foreign exchange rate), and deficits in trade and accordingly in current account. Surely, such a critical macroeconomic situation in a competitive international economic environment cannot be left uncontrolled. To prevent such an economic crisis let us introduce, as scenario 6, an official intervention to the foreign exchange market; specifically, the central bank (and government) begins to sell foreign exchange in order to reduce foreign exchange rate, say, to 1.02 dollars per FE; that is, by 2 % of the original equilibrium exchange rate.

As Figure 10.23 illustrates, even under such circumstances trade and current account deficits continue to persist. Gradually, the foreign exchange reserves begins to decline due to the official intervention, and becomes lower than the original reserve level of 100 (billion) dollars around the year 40 and completely gets depleted around the year 50, as indicated in the bottom right-hand diagram. This implies the government is forced to declare financial *default*, that is, an economic destruction, unless successfully eliciting an emergent loan from the international institutions such as the IMF.

Zero Interest Rate and Default (S7)

To avoid such financial default, now suppose, as scenario 7, money supply is increased to stimulate the economy and a domestic interest rate is lowered by 3%; that is, a zero interest rate is introduced from the original 3% at the year 3. This policy of zero interest rate surely improves trade balance and the balance of payments as Figure 10.24 indicates. Yet, under the official intervention of keeping a foreign exchange rate below 1.02 dollars per FE, the central bank (and the government) is forced to keep selling foreign exchange reserves[3]. The original 100 (billion) dollars of foreign exchange reserves will be completely depleted around the year 11 as the bottom right-hand diagram indicates. Therefore, this zero interest policy does not work unless the government can successfully borrow foreign exchange from the international institutions such as the IMF.

[3]To be precise, for maintaining the rate below this level, the central bank (and the government) has to keep selling 60 (billion) dollars of foreign exchange annually instead of 20 (billion) dollars in the previous scenario

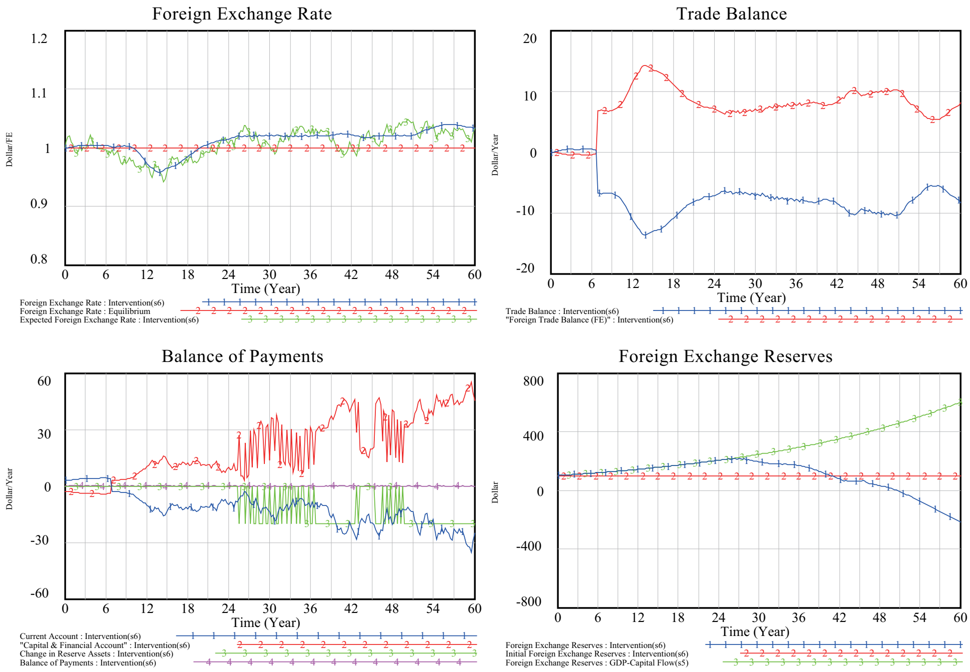

Figure 10.23: Official Intervention and Default (S6)

No Official Intervention (S8)

Let us further suppose that the central bank (and the government) gives up official intervention and stops selling foreign exchange to avoid a depletion of its foreign reserves. This scenario 8 surely brings about a further appreciation of foreign exchange rate. But to our surprise, after attaining a highest value of 1.225 dollars at the year 41, it begins to depreciate as the top left-hand diagram of Figure 10.25 illustrates. Moreover, trade balance and the balance of payments are getting improved, and foreign exchange reserves keeps growing according to the same figure. This is another counter-intuitive result in a sense that official intervention to foreign exchange market won't work to save the economic crisis.

In this way, so long as the working of our domestic macroeconomy is concerned, combined policies of zero interest rate and no official intervention seem to work. Yet, from a foreign country's point of view, the same policies worsen its economy as a mirror image of our economy. Hence, a so-called trade war becomes unavoidable in the international macroeconomic framework. Our simple open macroeconomic model has successfully exposed one of the fundamental causes of economic conflicts among nations.

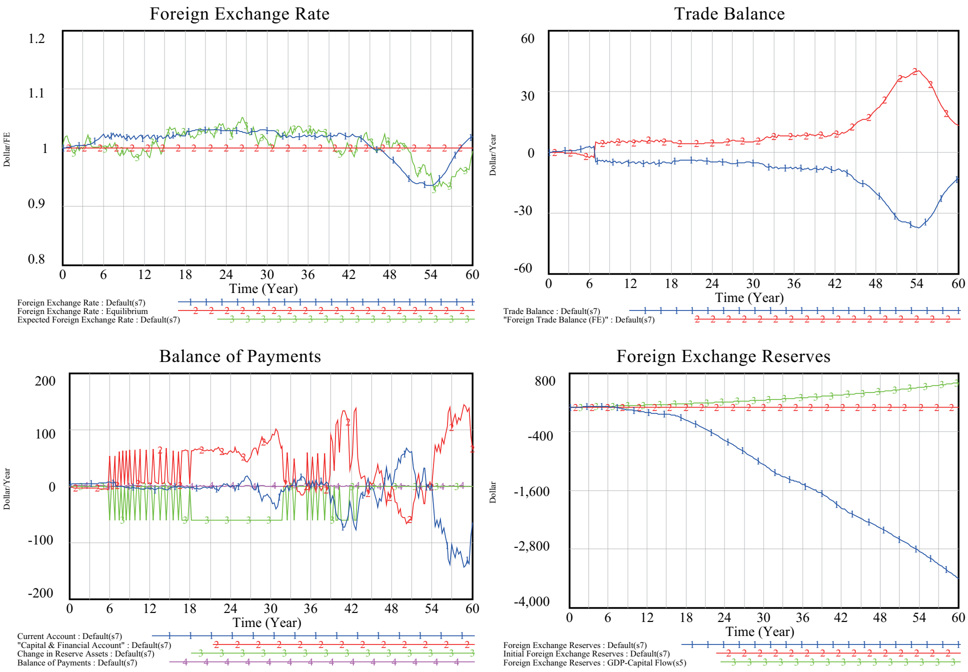

Figure 10.24: Zero Interest Rate and Default (S7)

10.10 Missing Feedback Loops

We have now presented eight different scenarios of international trade and financial investment, which indicates capability of our open macroeconomic modeling. Yet, our generic model is far from a complete open macroeconomy, because significant economic variables such as GDP, its price level and interest rate are treated as outside parameters, and no feedback loops exist in the sense that they are affected by the endogenous variables such as a foreign exchange rate and its expectations. Schematically, one-way direction of decision-making in the equation (10.31) has to be made two-way such that

$$(Y, Y_f, P, P_f, i, i_f, N_{random}) \Longleftrightarrow (E, E^e, TB) \tag{10.33}$$

Mundell-Fleming Model

Compared with our model, one of the repeatedly used open macroeconomic model in standard international economics textbooks is the Mundell-Fleming

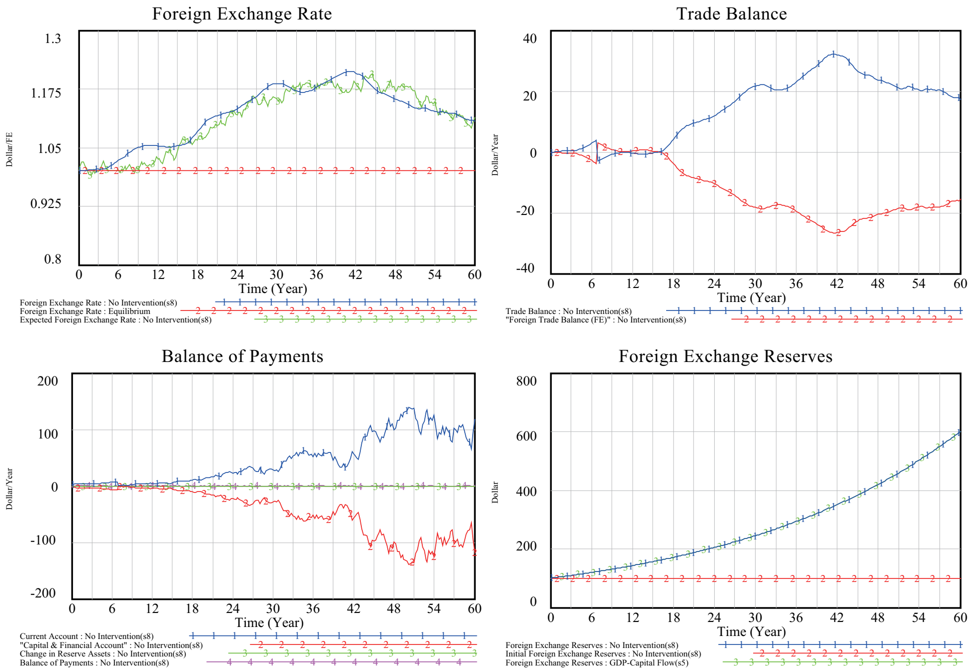

Figure 10.25: No Official Intervention (S8)

model that is described, according to [39], as

$$Y = C(Y-T) + I(i)) + G + TB(E) \tag{10.34}$$

$$\frac{M^s}{P} = L(i, Y) \tag{10.35}$$

$$i = i_f \tag{10.36}$$

This macroeconomic model indeed determines Y, E and i. In other words, significant economic variables such as GDP and interest rate are simultaneously determined in the model, though interest rate is restricted by a competitive world interest rate. In comparison, our model consisting of the three equations: (10.15), (10.29), and (10.30), determines only three variables E, E^e and TB, and fails to determine Y and i.

Hence, Mundel-Fleming model could be said to be a better presentation of open macroeconomy. Yet, it lacks a mechanism of determining money supply M^s and a price level P. In this sense, it is still far from a complete open macroeconomic model.

Missing Loops

It is now clear from the above arguments that for a complete open macroeconomic model some missing feedback loops have to be supplemented. They could constitute the following in our model:

- Imports and exports are assumed to be determined by the economic activities of GDPs, which are in turn affected by the size of trade balance. Yet, they are missing.
- Foreign exchange intervention by the central bank (and the government) such as the purchase or sale of foreign exchange surely changes the amount of currency outstanding and money supply, which in turn must affect an interest rate and a price level. Yet, they are being fixed.
- A change in interest rates affects investment, which in turn determines the level of GDP. Yet, investment is not playing such a role.
- A change in price level must also affect consumption spending and hence real GDP. Yet, these loops are missing.
- Official intervention must influence speculations and estimations on foreign exchange and investment returns among international financial investors. Yet, these fluctuations are only given by outside random normal distribution.

If we could add these missing feedback loops to the causal loop diagram of the foreign exchange dynamics in Figure 10.15, then we can obtain a complete feedback loop diagram as illustrated in Figure 10.26.

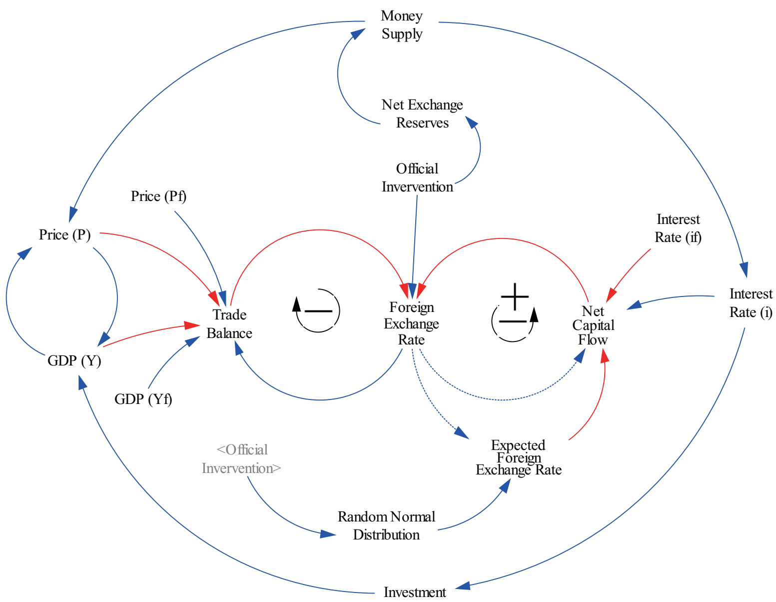

Figure 10.26: Missing Feedback Loops Added to the Foreign Exchange Dynamics

Obviously, our open macroeconomic model is not complete until these missing loops are incorporated in the model. Specifically, the previous chapter has presented a model of macroeconomic system which determines GDP, money supply, a price level, investment and interest rate, to name but a few. Therefore, our next challenge is to integrate the model with our present foreign exchange model by crating a whole image of domestic macroeconomy as its foreign sector macroeconomy.

10.11 Conclusion

Our open macroeconomic modeling turned out to need another model of the balance of payments and dynamics of foreign exchange rate. Consequently, the approach in this chapter led by the logic of accounting system dynamics became an entirely new one in the field of international economics.

Under the framework, a double-booking accounting of the balance of payments is modeled. Then determinants of trade and foreign direct and financial investment are analytically examined together with an introduction of differential equations of foreign exchange rate and its expected rate.

Upon a completion of the model, eight scenarios are are produced and examined by running various simulations to obtain some behaviors observed in actual international trade and financial investment. It is a surprise to see how an equilibrium state of trade balance is easily disturbed by merely introducing random expectations among financial investors under the assumption of efficient financial market. To indicate the capability of our model furthermore, the impact of official intervention on foreign exchange and a path to default is discussed.

Finally, several missing feedback loops in our model are pointed out for making it a complete open macroeconomic model. This task of completion will inevitably lead to our next research in this system dynamics macroeconomic modeling series in the next chapter.

Chapter 11

Open Macroeconomies as A Closed Economic System

This chapter[1] tries to expand the integrated model to the open macroeconomies according to the framework developed in the previous chapter. It provides a complete generic model of open macroeconomies as a closed system, consisting of two economies, a foreign economy as an image of domestic economy. As a demonstration of its analytical capability, a case of credit crunch is examined to show how domestic macroeconomic behaviors influence foreign macroeconomy through trade and financial capital flows.

11.1 Open Macroeconomic System Overview

This chapter finalizes our series of macroeconomic modeling on the basis of the principle of accounting system dynamics. Chapters 5 and 6 constructed a model of money supply and its creation process, followed by the introduction of interest rate to the model. Chapter 7 modeled dynamic determination processes of GDP, interest rate and price level. For its analysis four sectors of macroeconomy were introduced such as producers, consumers, banks and government. Chapter 8 and 9 integrated real and monetary sectors that had been analyzed separately in chapters 5, 6 and 7, by adding the central bank, then labor market. Chapter 10 built a model of a dynamic determination of foreign exchange rate in open macroeconomies in which goods and services are freely traded and financial capital flows efficiently for higher returns. For this purpose a new method was needed contrary to the standard method of dealing with a foreign sector as adjunct to macroeconomy; that is, an introduction of another macroeconomy as a foreign sector.

[1] It is based on the paper:Open Macroeconomies as A Closed Economic System – SD Macroeconomic Modeling Completed – that is presented at the 26th International Conference of the System Dynamics Society, Athens, Greece, July 23-27, 2008.

In this chapter, the integrated macroeconomy in chapter 9 is opened to foreign economy through trade and financial capital flows according to the framework developed in the previous chapter. In other words, a complete mirror economy is created as a foreign economy as illustrated in Figure 11.1.

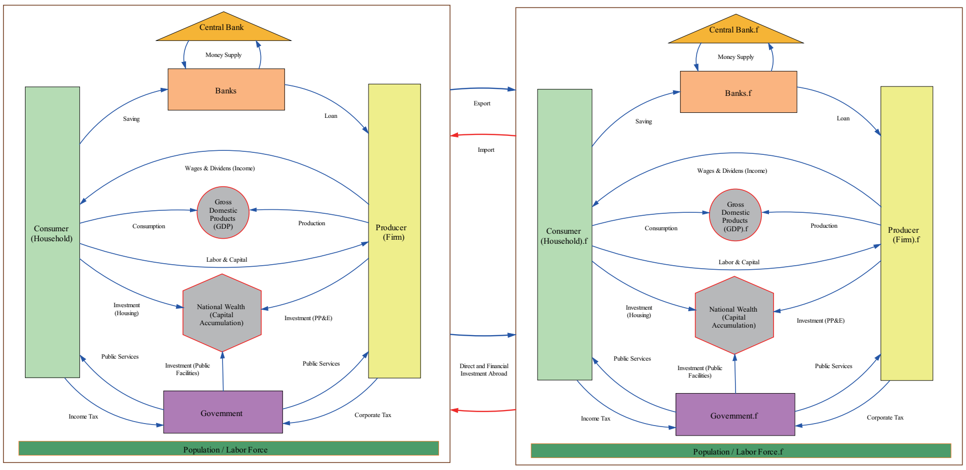

Figure 11.1: Open Macroeconomic System Overview

The only exception is the banking sectors such as commercial and central banks. Specifically, it is assumed that all foreign exchange transactions are done through domestic banks to meet the demand for foreign exchange services by consumers and producers.

11.2 Transactions in Open Macroeconomies

We are now in a position to open our integrated macroeconomy to foreign trade and direct and financial investment abroad [Companion Model: MacroDynamics2-3.vpm]. According to our method in the previous chapter, this is nothing but a process of creating another macroeconomy as an image economy of domestic macroeconomy. All variables of the foreign economy, then, are renamed with a suffix of .f; for instance, GDP.f, consumption.f, etc.

To avoid analytical complication, we have picked up the existing currency units of yen and dollar, among which dollar is assumed to plays a role of key currency. We have further assumed that a domestic economy has yen currency, and a foreign economy has dollar currency.

Nominal foreign exchange rate FE (merely called foreign exchange rate here) is now the amount of yen in exchange for one unit of foreign currency; that is, dollar as assumed above, and has a unit of *Yen/Dollar*. At this stage of building a generic open macroeconomies, the initial foreign exchange rate is assumed to be one; that is, one yen is exchanged for one dollar. Foreign exchange rate thus defined does not by all means reflect the ongoing current exchange rate in the real world economy.

Real foreign exchange rate (RFE) is the amount of real goods worth per unit of the equivalent foreign real goods such that

$$RFE = \frac{FE * P_f}{P} \tag{11.1}$$

which has a unit of *YenReal/DollarReal*.

Let us now describe main transactions of the open macroeconomies by producers, consumers, government, banks and the central bank.

Producers

Main transactions of producers are summarized as follows. They are also illustrated in Figure 11.15 in which stocks of gray color are newly added for open economies.

- Out of the GDP revenues producers pay excise tax, deduct the amount of depreciation, and pay wages to workers (consumers) and interests to the banks. The remaining revenues become profits before tax.
- They pay corporate tax to the government out of the profits before tax.
- The remaining profits after tax are paid to the owners (that is, consumers) as dividends, including dividends abroad. However, a small portion of profits is allowed to be held as retained earnings.
- Producers are thus constantly in a state of cash flow deficits. To make new investment, therefore, they have to borrow money from banks and pay interest to the banks.
- Producers imports goods and services according to their economic activities, the amount of which is assumed to be a portion of GDP in our model, though actual imports are also assumed to be affected by their demand curves.
- Similarly, their exports are determined by the economic activities of a foreign economy, the amount of which is also assumed to be a portion of foreign GDP.
- Produces are also allowed to make direct investment abroad as a portion of their investment. Investment income from these investment abroad are paid by foreign producers as dividends directly to consumers as owners of assets abroad. Meanwhile, producers are required to pay foreign investment income (returns) as dividends to foreign investors (consumers) according to their foreign financial liabilities.
- Foreign producers are assumed to behave in a similar fashion as a mirror image of domestic producers

Consumers

Main transactions of consumers are summarized as follows. They are also illustrated in Figure 11.16 in which stocks of gray color are newly added for open economies.

- Sources of consumers' income are their labor supply, financial assets they hold such as bank deposits, shares (including direct assets abroad), and deposits abroad. Hence, consumers receive wages and dividends from producers, interest from banks and government, and direct and financial investment income from abroad.

- Financial assets of consumers consist of bank deposits and government securities, against which they receive financial income of interests from banks and government.

- In addition to the income such as wages, interests, and dividends, consumers receive cash whenever previous securities are partly redeemed annually by the government.

- Out of these cash income as a whole, consumers pay income taxes, and the remaining income becomes their disposal income.

- Out of their disposable income, they spend on consumption. The remaining amount is either spent to purchase government securities or saved.

- Consumers are now allowed to make financial investment abroad out of their financial assets consisting of stocks, bonds and cash. For simplicity, however, their financial investment are assumed to be a portion out of their deposits . Hence, returns from financial investment are uniformly evaluated in terms of deposit returns.

- Consumers now receive direct and financial investment income. Similar investment income are paid to foreign investors by producers and banks. The difference between receipt and payment of those investment income is called income balance. When this amount is added to the GDP revenues, GNP (Gross National Product) is calculated. If capital depreciation is further deducted, the remaining amount is called NNP (Net National Product).

- NNP thus obtained is completely paid out to consumers, consisting of workers and shareholders, as wages to workers and dividends to shareholders, including foreign shareholders.

- Foreign consumers are assumed to behave in a similar fashion as a mirror image of domestic consumers.

Government

Transactions of the government are illustrated in Figure 11.17, some of which are summarized as follows.

- Government receives, as tax revenues, income taxes from consumers and corporate taxes from producers.
- Government spending consists of government expenditures and payments to the consumers for its partial debt redemption and interests against its securities.
- Government expenditures are assumed to be endogenously determined by either the growth-dependent expenditures or tax revenue-dependent expenditures.
- If spending exceeds tax revenues, government has to borrow cash from consumers and banks by newly issuing government securities.
- Foreign government is assumed to behave in a similar fashion as a mirror image of domestic government.

Banks

Main transactions of banks are summarized as follows. They are also illustrated in Figure 11.18 in which stocks of gray color are newly added for open economies.

- Banks receive deposits from consumers and consumers abroad as foreign investors, against which they pay interests.
- They are obliged to deposit a portion of the deposits as the required reserves with the central bank.
- Out of the remaining deposits, loans are made to producers and banks receive interests to which a prime rate is applied.
- If loanable fund is not enough, banks can borrow from the central bank to which discount rate is applied.
- Their retained earnings thus become interest receipts from producers less interest payment to consumers and to the central bank. Positive earnings will be distributed among bank workers as consumers.
- Banks buy and sell foreign exchange at the request of producers, consumers and the central bank.
- Their foreign exchange are held as bank reserves and evaluated in terms of book value. In other words, foreign exchange reserves are not deposited with foreign banks. Thus net gains realized by the changes in foreign exchange rate become part of their retained earnings (or losses).

- Foreign currency (dollars in our model) is assumed to play a role of *key* currency or *vehicle* currency. Accordingly foreign banks need not set up foreign exchange account. This is a point where a mirror image of open macroeconomic symmetry breaks down.

Central Bank

Main transactions of the central bank are summarized as follows. They are also illustrated in Figure 11.19 in which stocks of gray color are newly added for open economies.

- The central bank issues currencies against the gold deposited by the public.
- It can also issue currency by accepting government securities through open market operation, specifically by purchasing government securities from the public (consumers) and banks. Moreover, it can issue currency by making credit loans to commercial banks. (These activities are sometimes called *money out of nothing*.)
- It can similarly withdraw currencies by selling government securities to the public (consumers) and banks, and through debt redemption by banks.
- Banks are required by law to reserve a certain amount of deposits with the central bank. By controlling this required reserve ratio, the central bank can control the monetary base directly.
- The central bank can additionally control the amount of money supply through monetary policies such as open market operations and discount rate.
- Another powerful but hidden control method is through its direct influence over the amount of credit loans to banks (known as *window guidance* in Japan.)
- The central bank is allowed to intervene foreign exchange market; that is, it can buy and sell foreign exchange to keep a foreign exchange ratio stable (though this intervention is actually exerted by the Ministry of Finance in Japan, it is regarded as a part of policy by the central bank in our model).
- Foreign exchange reserves held by the central bank is usually reinvested with foreign deposits and foreign government securities, which are, however, not assumed here as inessential.

Missing Loops Fixed

In the previous chapter five loops below are pointed out as missing. To repeat,

- Imports and exports are assumed to be determined by the economic activities of GDPs, which are in turn affected by the size of trade balance. Yet, they are missing.

- Foreign exchange intervention by the central bank (and the government) such as the purchase or sale of foreign exchange surely changes the amount of currency outstanding and money supply, which in turn must affect an interest rate and a price level. Yet, they are being fixed.
- A change in interest rates affects investment, which in turn determines the level of GDP. Yet, investment is not playing such a role.
- A change in price level must also affect consumption spending and hence real GDP. Yet, these loops are missing.
- Official intervention must influence speculations and estimations on foreign exchange and investment returns among international financial investors. Yet, these fluctuations are only given by outside random normal distribution.

Our open macroeconomies have now successfully augmented these missing feedback loops except the last loop of speculation. Figure 11.2 illustrates newly fixed feedback loops.

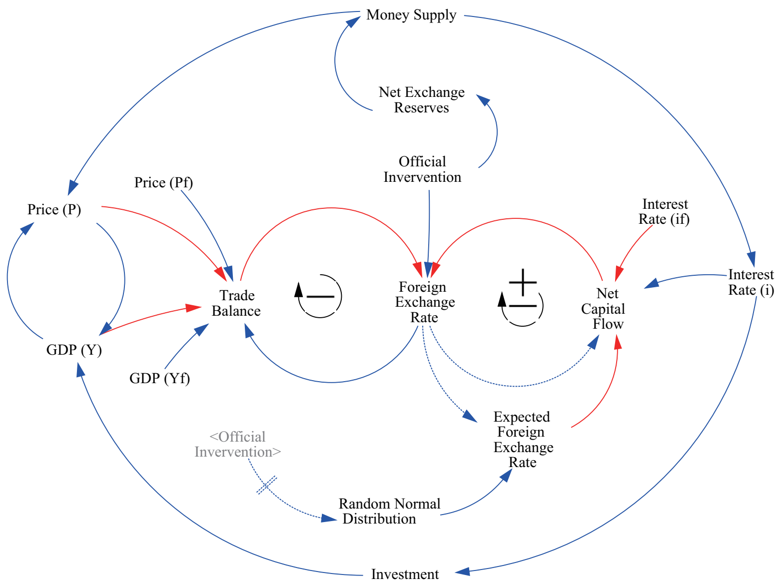

Figure 11.2: Fixed Missing Loops in the Foreign Exchange Dynamics Model

11.3 Behaviors of Open Macroeconomies

Mostly Equilibria under Trade and Capital Flows

The construction of open macroeconomies is now completed. There are three channels to open a domestic economy to a foreign economy. Trade channel is opened by allowing producers to import a portion of its GDP for domestic production and distribution. Capital flows have two channels. First, producers are allowed to make direct investment abroad as a portion of their domestic investment. Secondly, consumers are allowed to make deposits abroad out of their domestic deposits as a financial portfolio investment. (For simplicity, portfolios among deposits, shares and securities are not considered here.) These capital flows by direct and financial investment are determined by the interest arbitrage as analyzed in the previous chapter.

Let us now open all three channels by setting the values of import coefficient, direct investment ratio, and financial investment ratio to be the same 10%, respectively. Under the international activities of such trade and capital flows, Figure 11.3 demonstrates that our open macroeconomies can attain mostly equilibrium states.

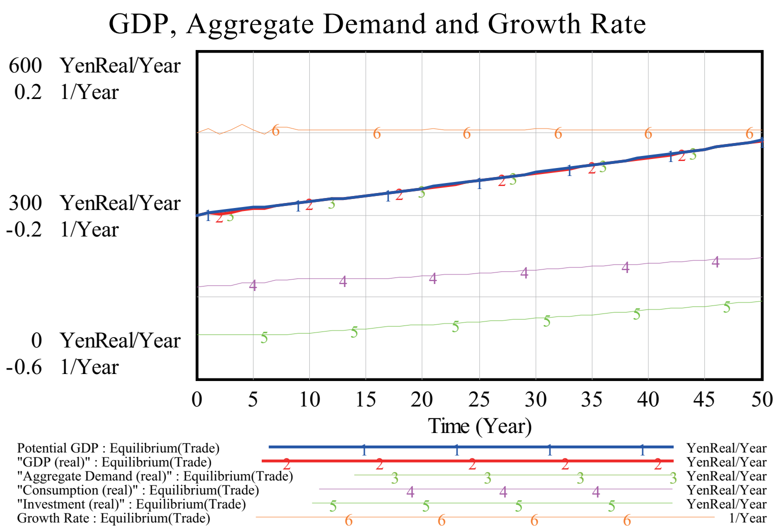

Figure 11.3: Mostly Equilibria under Trade and Capital Flows

Mostly equilibria thus obtained, however, do not imply balances of trade and capital flows. In fact, very small amount of trade imbalance is still observed as illustrated in Figure 11.4. Moreover, alternating interest arbitrages generate very small amounts of capital inflows and outflows as illustrated in Figure 11.5 due to the different interest rates prevailing over two economies, and random normal distribution that is exerted on the expected foreign exchange rate. Compared with the size of GDP, however, these variances trade and capital flows are within the range of less than 0.5% of GDP.

Inventory Business Cycles under Trade and Capital Flows

Our generic model of open macroeconomies could be applied in many different ways to the economic analyses of specific issues. In chapter 9, two types of

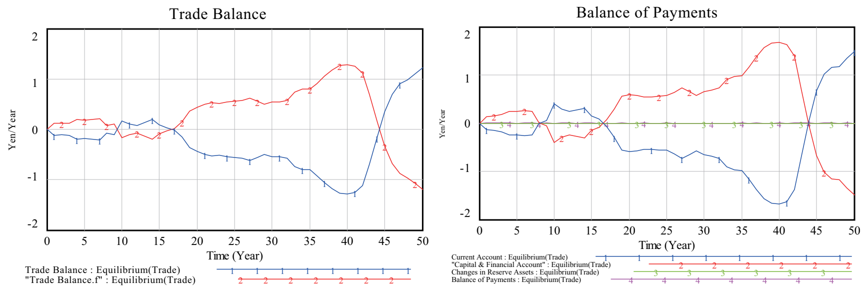

Figure 11.4: Trade Balance and Balance of Payments

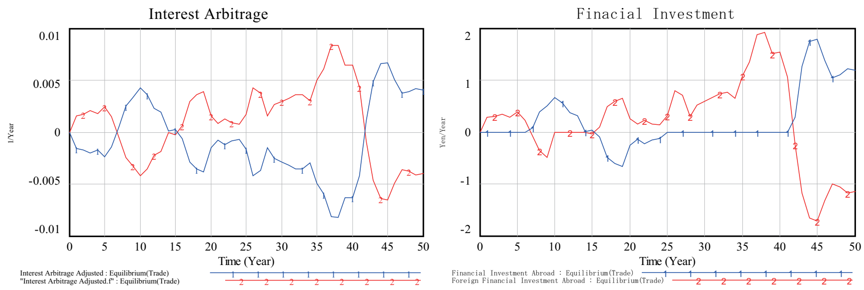

Figure 11.5: Interest Arbitrage and Financial Investment

business cycles are triggered out of the mostly equilibrium states such as the ones by inventory coverage, price fluctuation and cost-push wages, as well as an economic recession by credit crunch. It would be interesting, as a continuation of our discussions, to examine how these domestic business cycles and recession affect foreign macroeconomies through trade and capital flows.

Let us first consider a business cycle caused by inventory coverage. Suppose a normal inventory coverage is set to be 0.7 or 8.4 months instead of the initial value of 0.1 or 1.2 month as done in chapter 9. As expected again a similar business cycle is being generated in the domestic economy as illustrated in Figure 11.6.

Does this business cycle affect a foreign economy? Figure 11.7 illustrates the foreign country's GDP and its growth rate. It clearly displays that business cycles are being exported to the foreign economy through trade and capital flows. This means vice versa that our domestic economy cannot be also free from the influence of foreign economic behaviors. In this sense, open macroeconomies can be said to be mutually interdependent and constitute indeed a closed economic system as a whole.

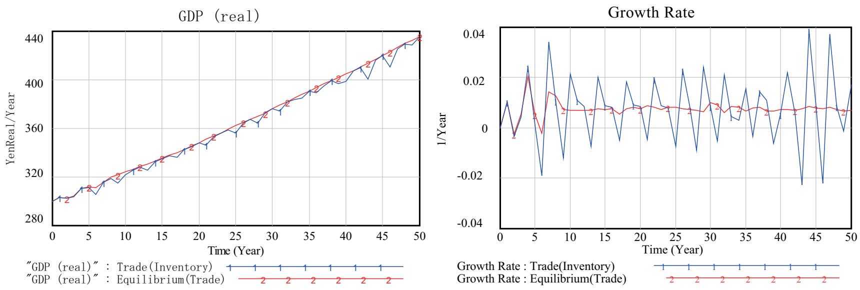

Figure 11.6: Inventory Business Cycles under Trade and Capital Flow

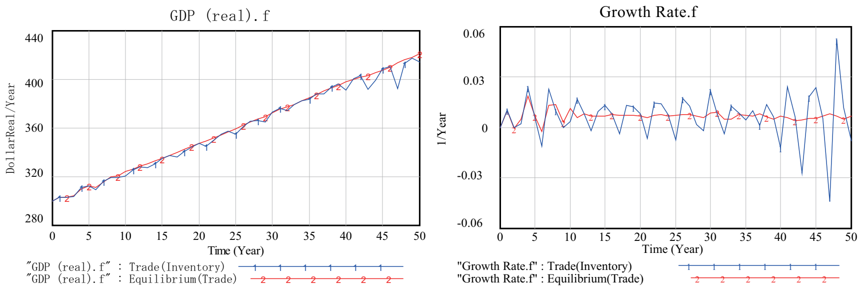

Figure 11.7: Inventory Business Cycles Affecting Foreign Macroeconomy

Credit Crunch under Trade and Capital Flows

Now let us examine an economic recession triggered by the credit crunch. For this purpose, let us now assume that the central bank reduces the amount of credit loans by 40%. An economic recession is similarly generated again in the domestic economy as illustrated in Figure 11.8.

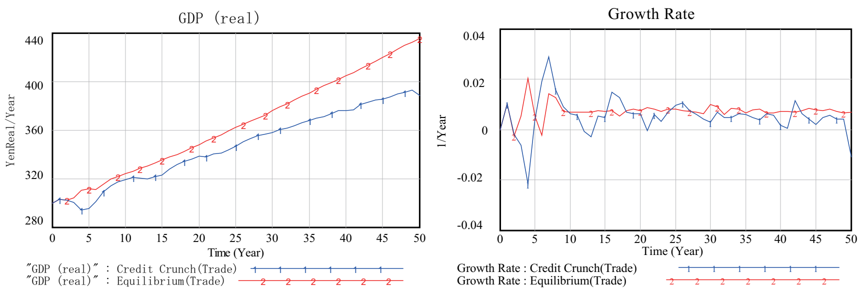

Figure 11.8: Credit Crunch under Trade and Capital Flow

Does this domestic recession affect the foreign economy? Figure 11.9 illustrates the foreign country's GDP and its growth rate. It clearly displays that economic recession is being exported to the foreign economy through trade and capital flows. This means vice versa that our domestic economy cannot be also free from the influence of foreign economic behaviors. In this sense, open macroeconomies can be said to be mutually interdependent and constitute indeed a closed economic system as a whole.

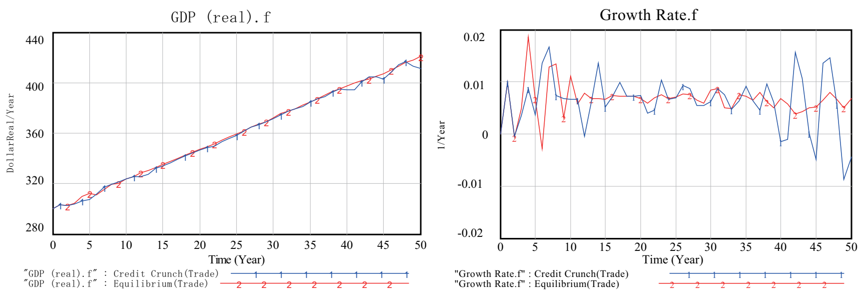

Figure 11.9: Credit Crunch Affecting Foreign Macroeconomy

11.4 Where to Go from Here?

Robust Foundation of the Model

Our macroeconomic model building is based on the following two well-established scientific methods;

- Double-entry accounting system: a foundation of social science
- Theory of differential equations: a foundation of natural science

Accounting system has been said to be the most rigorous methodology in social science, and widely used since ancient times to keep orderly records of chaotic market transactions. Differential equations have been, since Newton, widely applied to describe dynamic movements in natural science as the most fundamental tool for dynamical analysis. System dynamics is in a sense a computer-based tool for the numerical computation of differential equations.

These two well-established scientific methods are consolidated as the method of accounting system dynamics [63], and have been applied in our model building of macroeconomic system. Hence, our model could be said to have been built on the robust foundation, and in this sense may provide a generic framework for further analysis in macroeconomic theory. Where should we go from here then? At least four roads seem to lie ahead of us.

Road 1: Unified Macroeconomic Systems View

By its nature as a generic model, our model could be refined to clarify the fundamental causes of disputes among different schools of economic thoughts; for instance, in the line of unification among Neoclassical, Keynesian and Marxian schools in [58]. It is our belief that their differences are those of the assumptions made in the model, not the framework of the model itself. If this is right, the model could provide a common framework for further theoretical discussions among economists. Accordingly, depending on the economic issues for clarification the model could be fine-tuned for sharing various economic views.

Following are some of these fine-tuning directions for further analysis of the economic issues if they are the focus of macroeconomic controversies.

1. Portfolio decisions for financial assets and wealth among cash, shares and securities are not yet incorporated.

2. Housing investment and real estate transactions by consumers are not treated. This could be an interesting extension to analyze the subprime housing investment bubbles, followed by the housing crisis in 2007, and financial crisis in 2008.

3. Consumption is a function of basic consumption C_0, income Y and Price P, but interest i and wealth effect W_e are still not considered such that

$$C = C(C_0, Y, P, i, W_e) \tag{11.2}$$

4. An interdependent relation between money supply and inflation isXS weak, and only the following causal route is covered;
   Money(↑) ⟶ Investment(↑) ⟶ Desired Output(↑) ⟶ Price(↑).
   Moreover, inflation in financial assets and real estate is not treated.

5. Proportionate movement of price and wages is weak.

6. Comparative advantage theory of international trade is not handled.

Road 2: Japanese and US Macroeconomic Modeling

The series of macroeconomic modeling was originally intended to construct a Japanese macroeconomic model for strategic applications among business executives and policy makers. And we still believe it's the road we should take as a next step. Specifically, we'd like to analyze the world largest and second largest[2] economies in terms of GDP simultaneously; that is, Japanese macroeconomy as a domestic economy and US macroeconomy as a foreign economy along the framework of our open macroeconomies.

For this purpose, actual macroeconomic data have to be incorporated into the model. It would be very interesting to see, out of many possible behaviors the model can produce like chaos out of a simple deterministic equation, which possibility is to be chosen historically by the real economy.

[2]Chinese GDP surpassed Japanese GDP in 2010, and Japan is now the third largest economy.

Road 3: Systems of National Account

Our modeling method turns out to be along the United Nations *System of National Accounts 1993*, known as 'the SNA93', though in a more wholistic way. Accordingly, it could be extended closer to the complete SNA93 in a systemic way.

Road 4: Green Macroeconomies

Macroeconomic activities cannot sustain themselves, and have to be supported by environment. In this sense, it is important to consider a long-term sustainability of macroeconomic activities. I have contributed a sustainability modeling in chapter 3 of "Handbook of Sustainable Development Planning" [42], which is in turn based on my step-by-step definition of sustainability in terms of physical, social and ecological reproducibilities [61]. According to this line of argument, it is crucial to explore some features and frameworks that have to be established for attaining green macroeconomies.

Considering the research time left for me, I have decided to follow the road 4, in part IV, in search for designing a new macroeconomic system for financial stability and sustainability.

11.5 Conclusion

This is the final chapter that completes our series of building macroeconomic system. The integrated model in chapter 9 is extended to the open macroeconomies on the basis of the balance of payment in the previous chapter. Its main feature is that two similar macroeconomies are needed to analyze international trade and capital flows through direct and financial investment.

With a completion of building the open macroeconomies this way, many possibilities are made available for the analysis of economic issues. Our analyses are confined to the issues of inventory business cycles and credit crunch. Then, it is shown that a business cycle and an economic recession triggered in the domestic economy causes similar recessions in a foreign economy through the transactions of trade and capital flows. In this sense, open macroeconomies are indeed demonstrated to be a closed system in which economic behaviors are reciprocally interrelated.

Though the model is still far from being complete and genetic by its nature, it's time, I believe, to make it open to the public for further fine-tunings and revisions by the peers and macroeconomists as well as those who are interested in our approach of macroeconomic system. The following appendix presents our complete macroeconomic dynamic model.

Appendix: MacroDynamics 2.2 Illustrated[3]

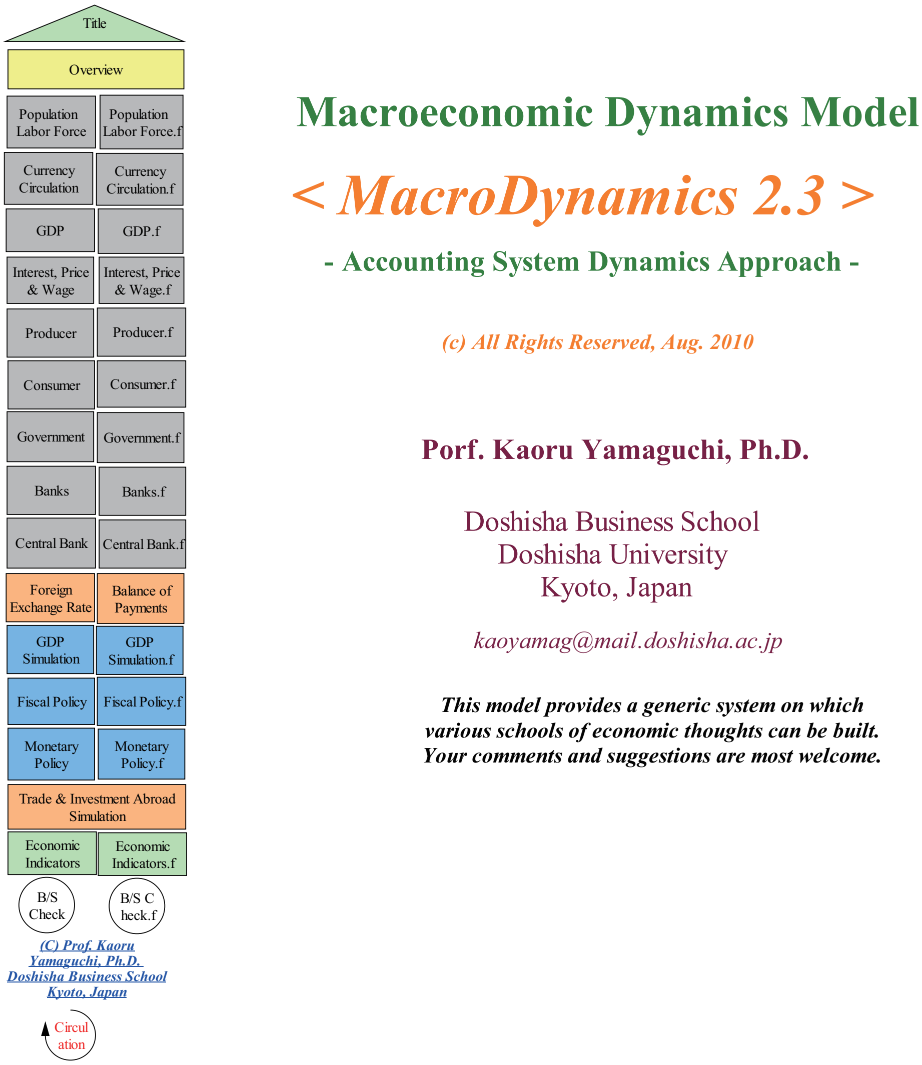

Figure 11.10: Title Page of the MacroDynamics Model

[3]In this illustrated section of the model, only domestic macroeconomy is presented. The model called MacroDynamics version 2.3 is available in the attached CD in this book.

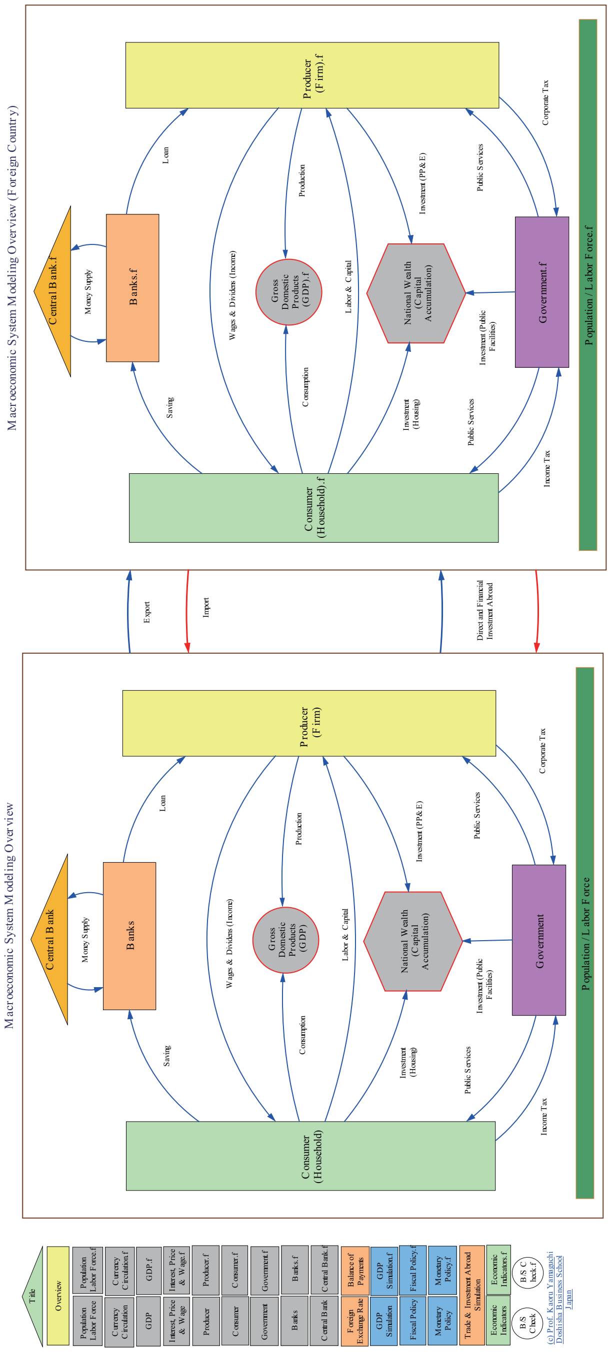

Figure 11.11: Model Overview

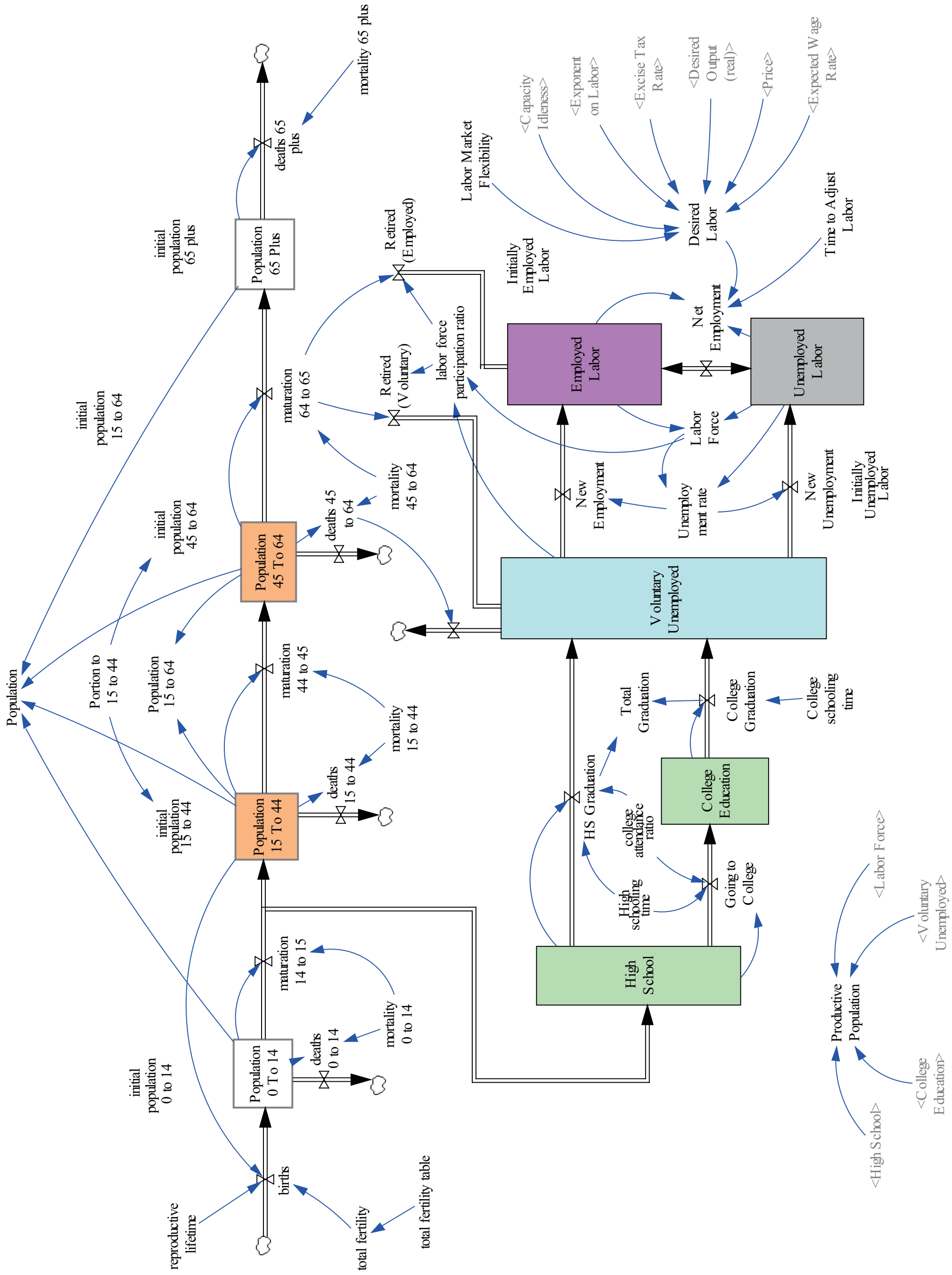

Figure 11.12: Population and Labor Force

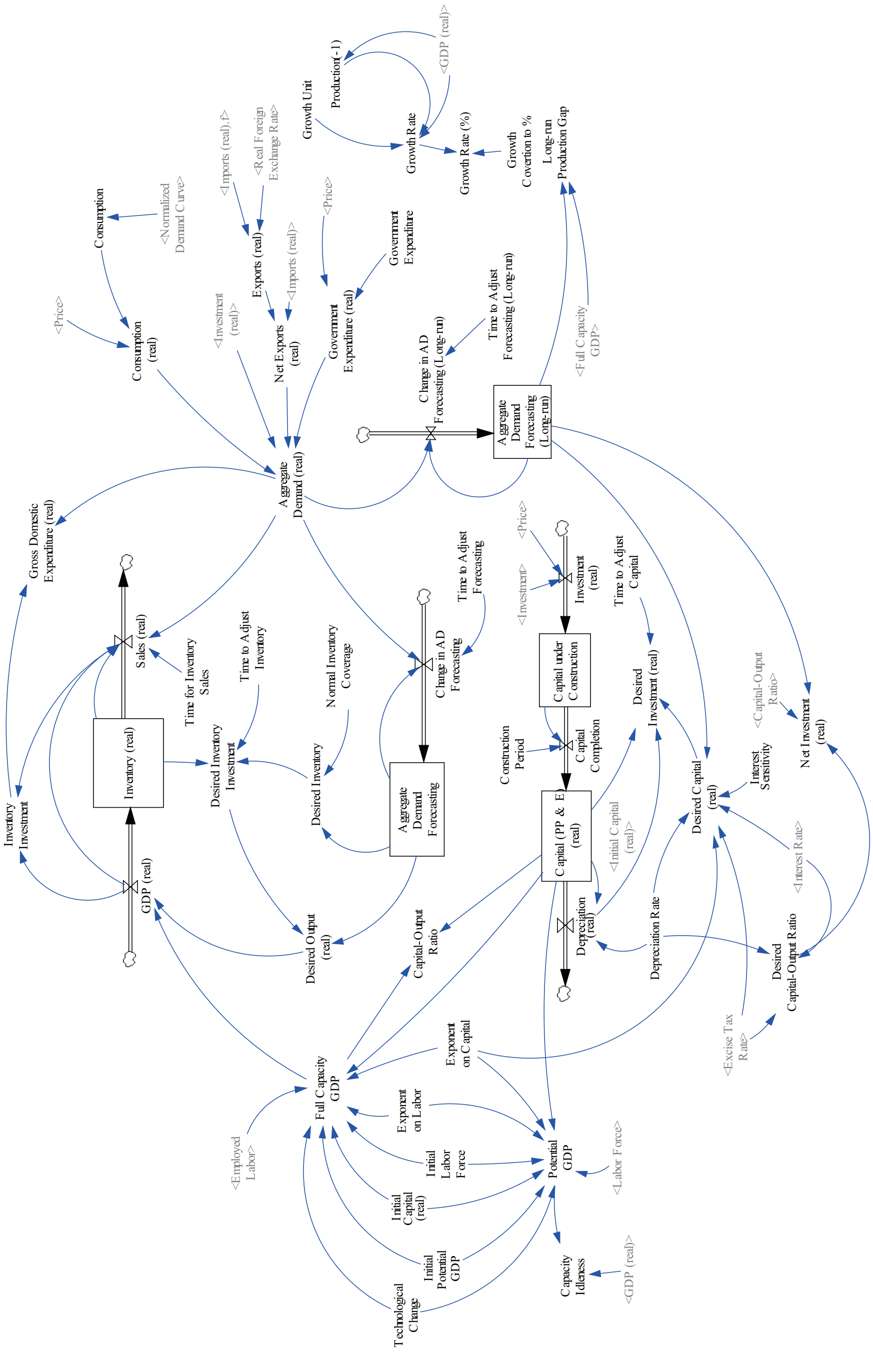

Figure 11.13: GDP Determination

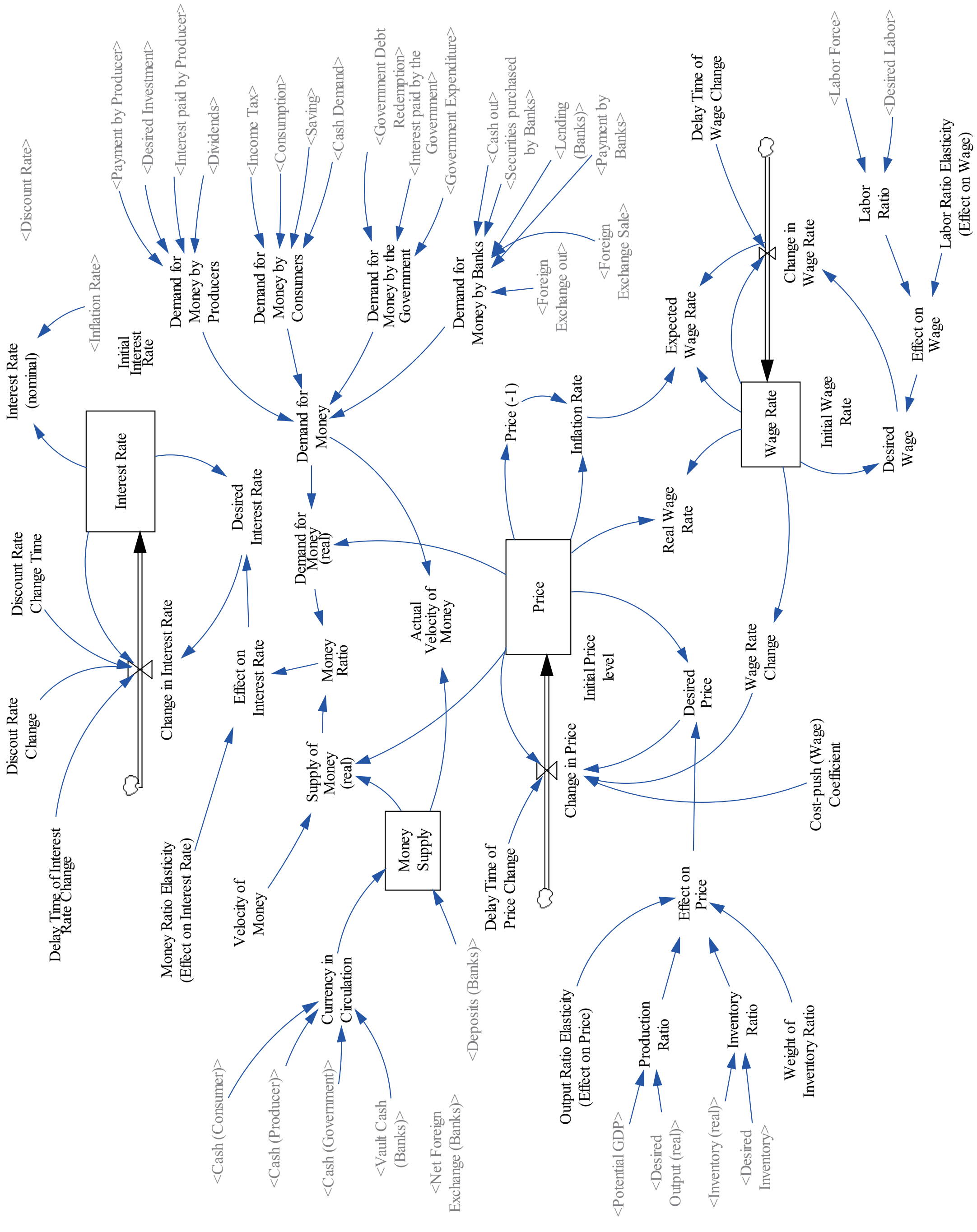

Figure 11.14: Interest Rate, Price and Wage Rate

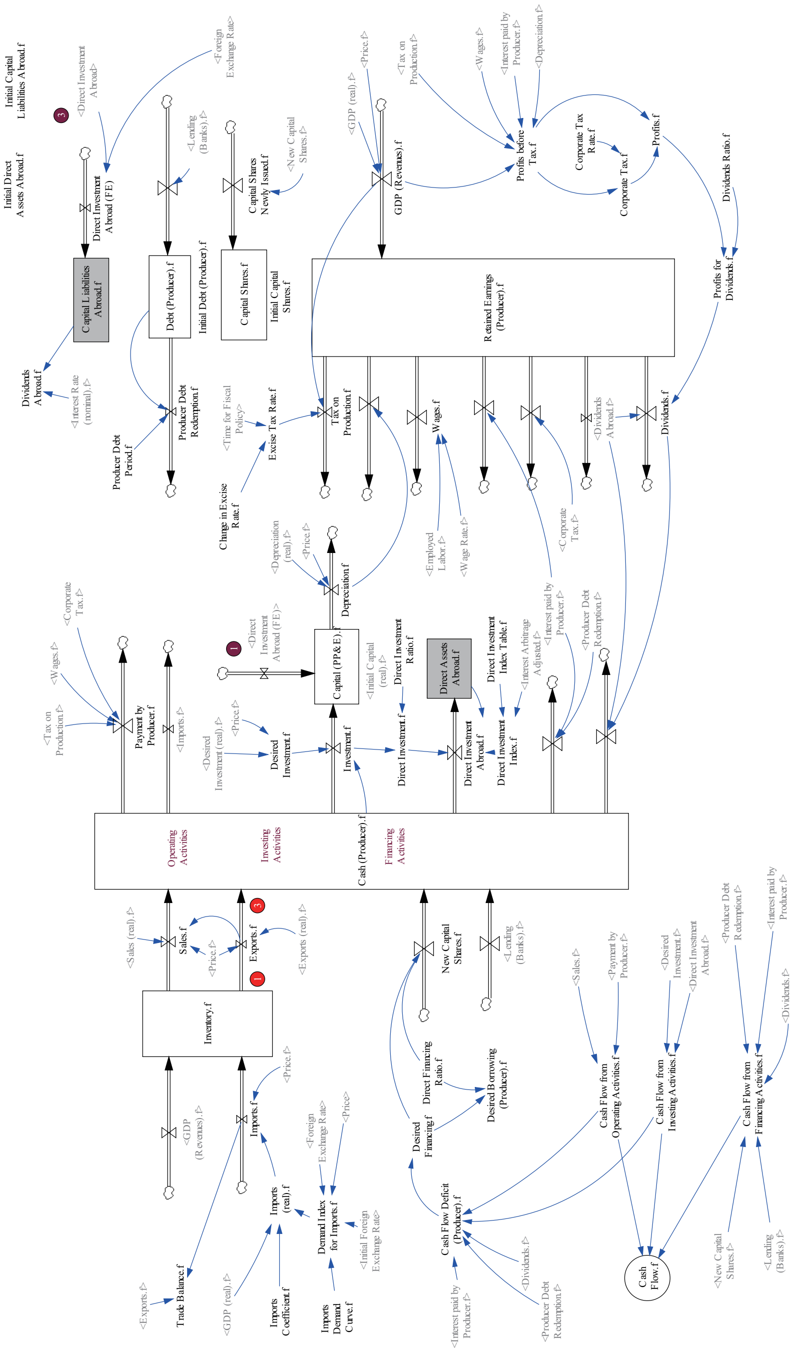

Figure 11.15: Transactions of Producers

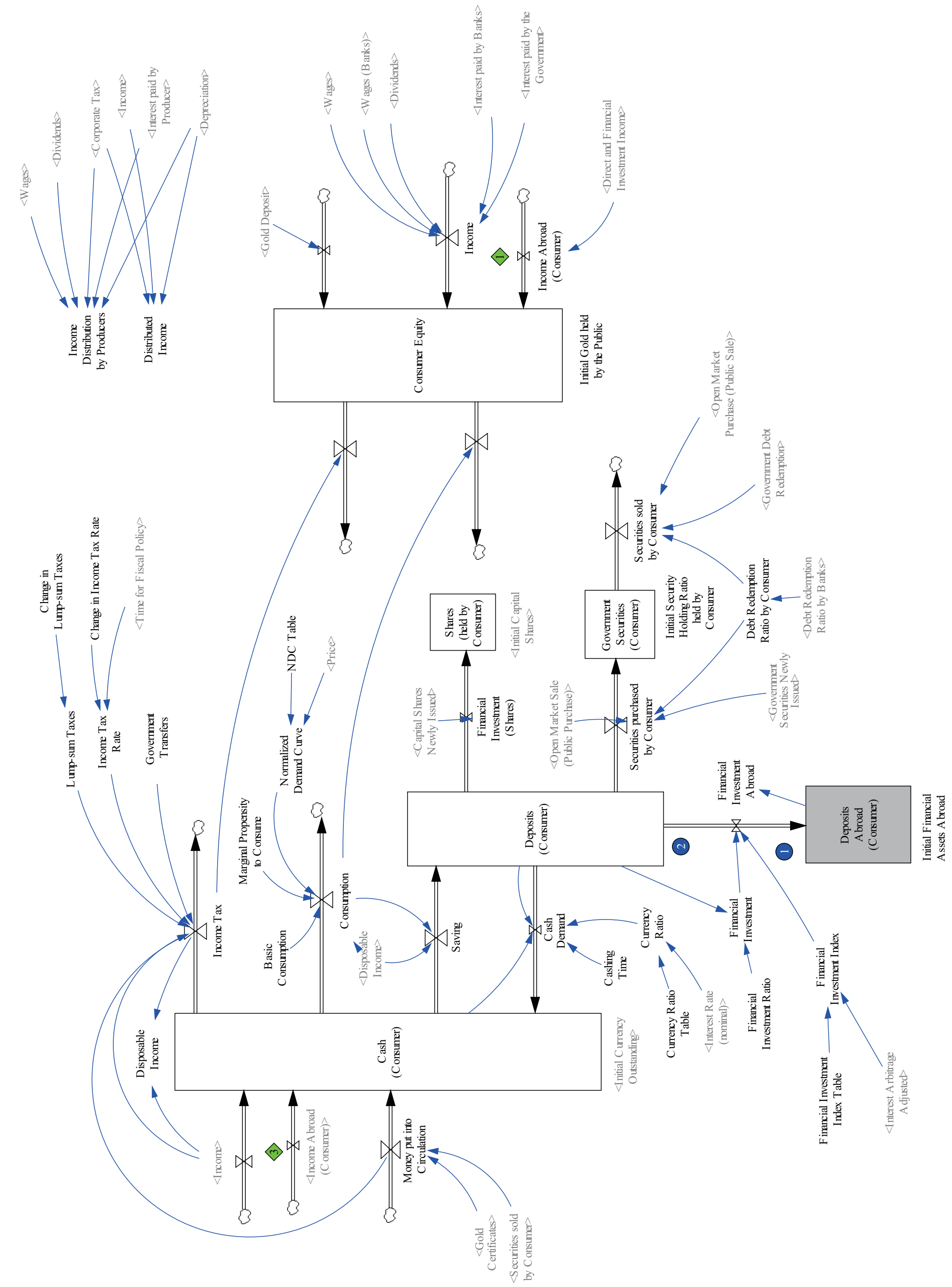

Figure 11.16: Transactions of Consumers

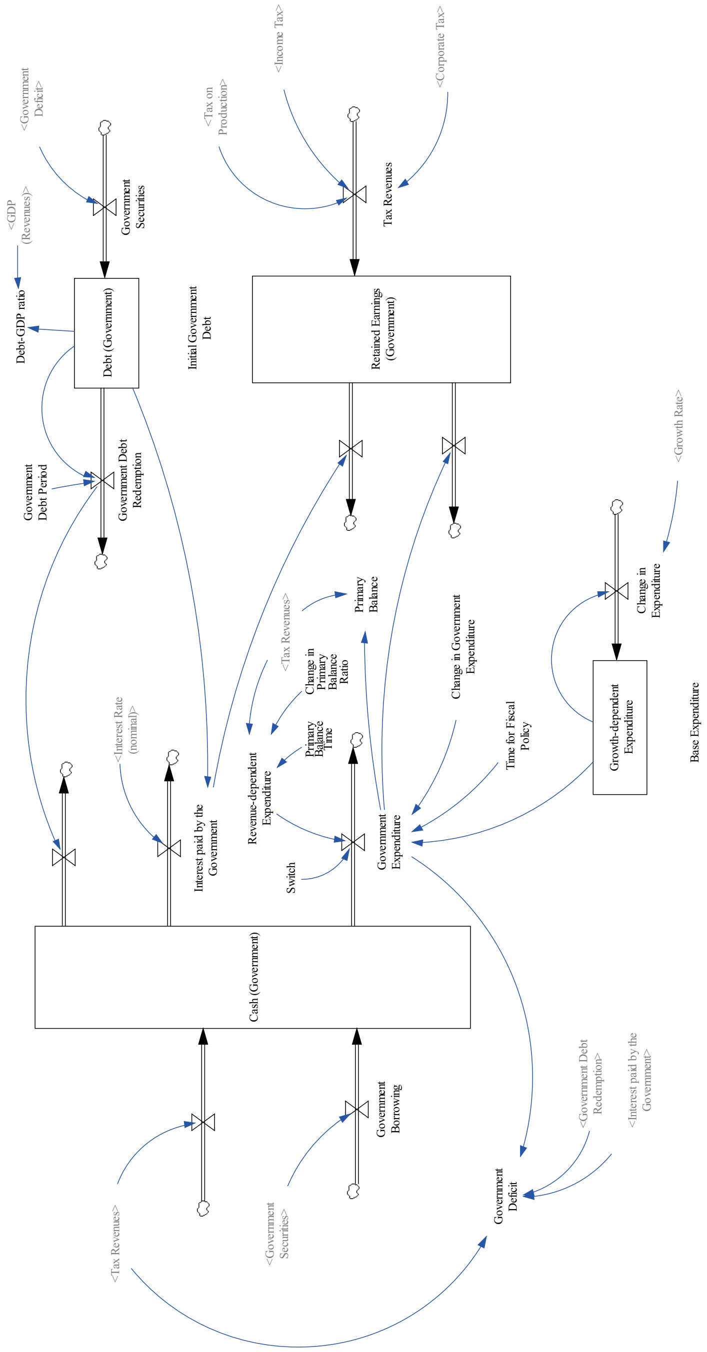

Figure 11.17: Transactions of Government

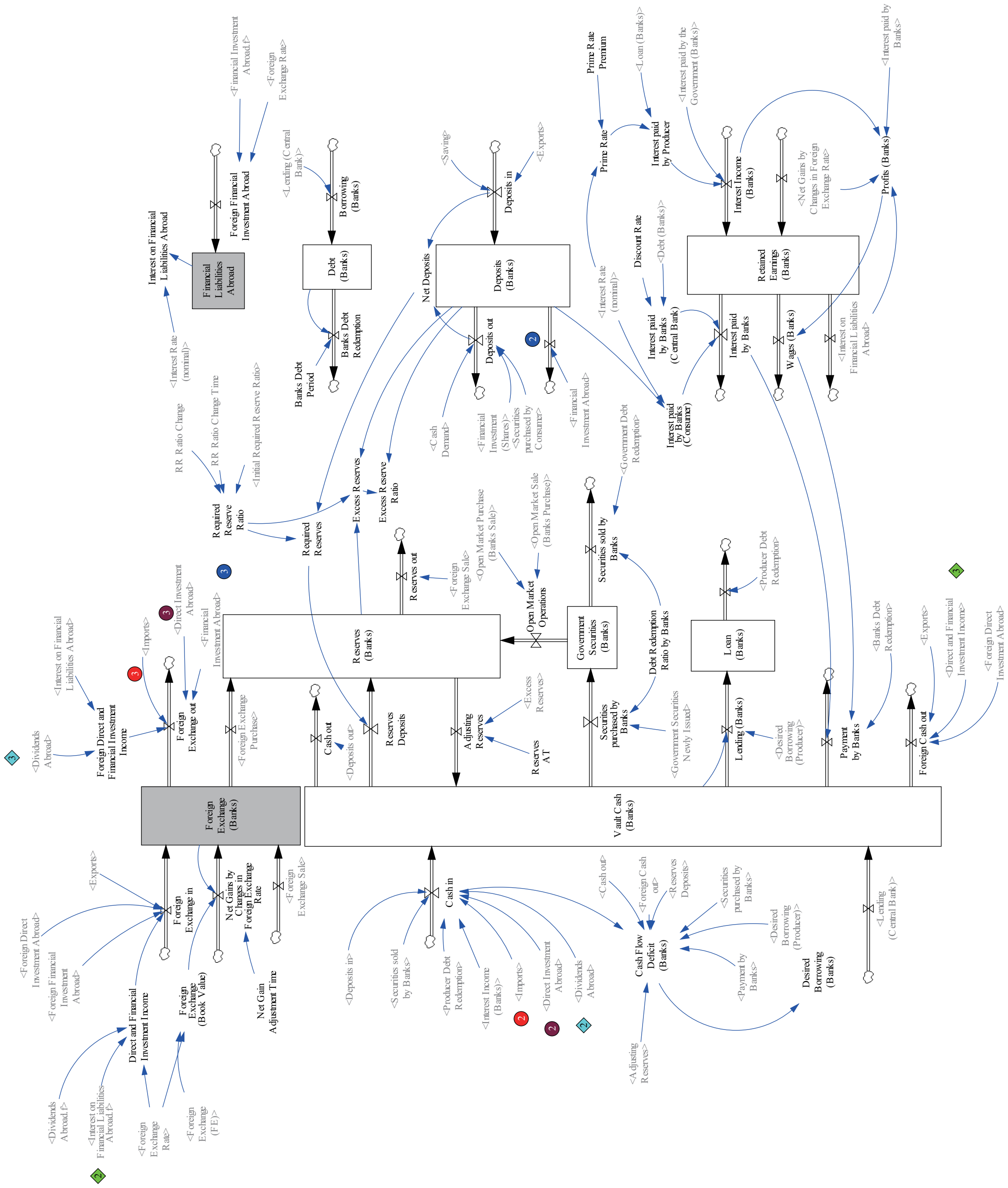

Figure 11.18: Transactions of Banks

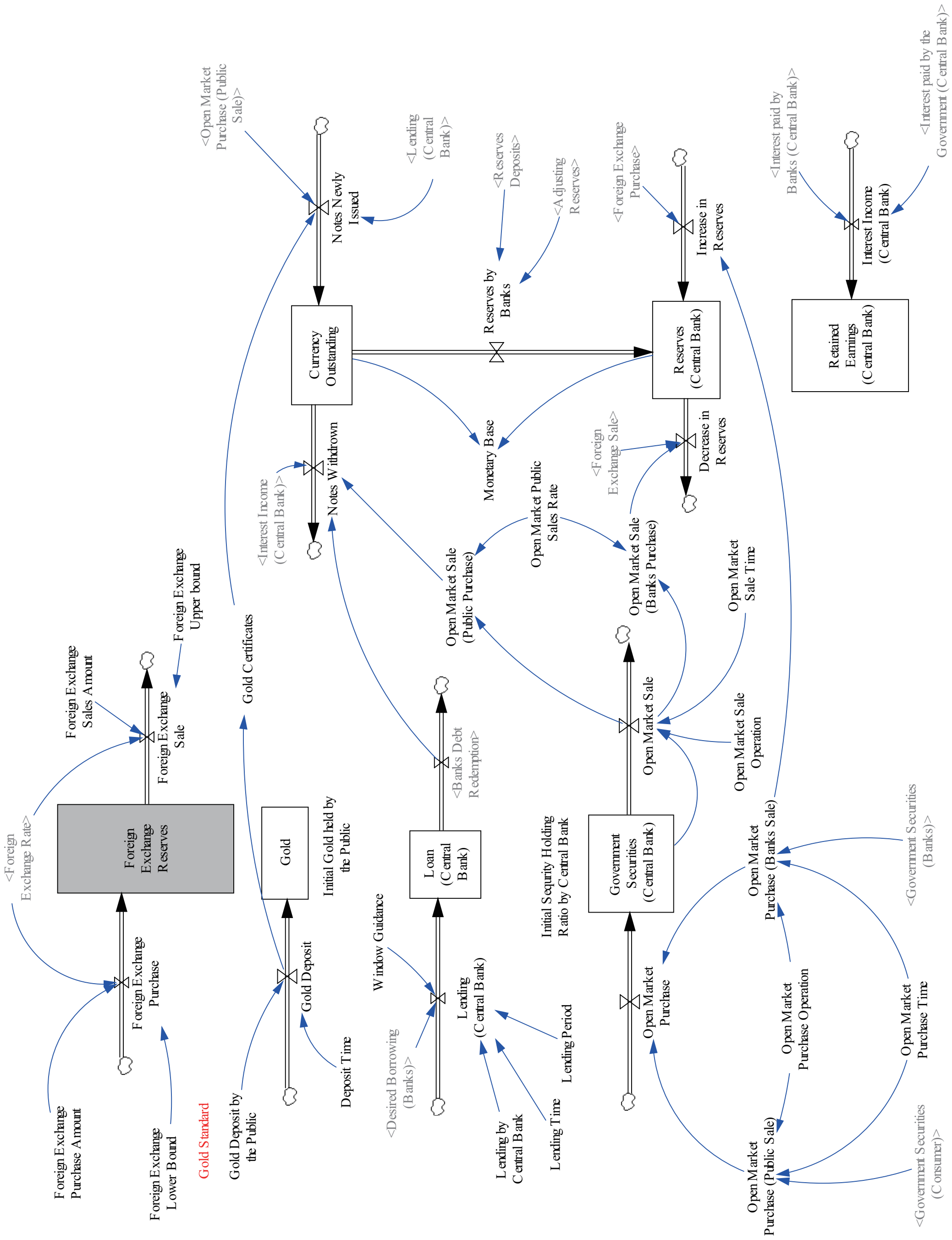

Figure 11.19: Transactions of Central Bank

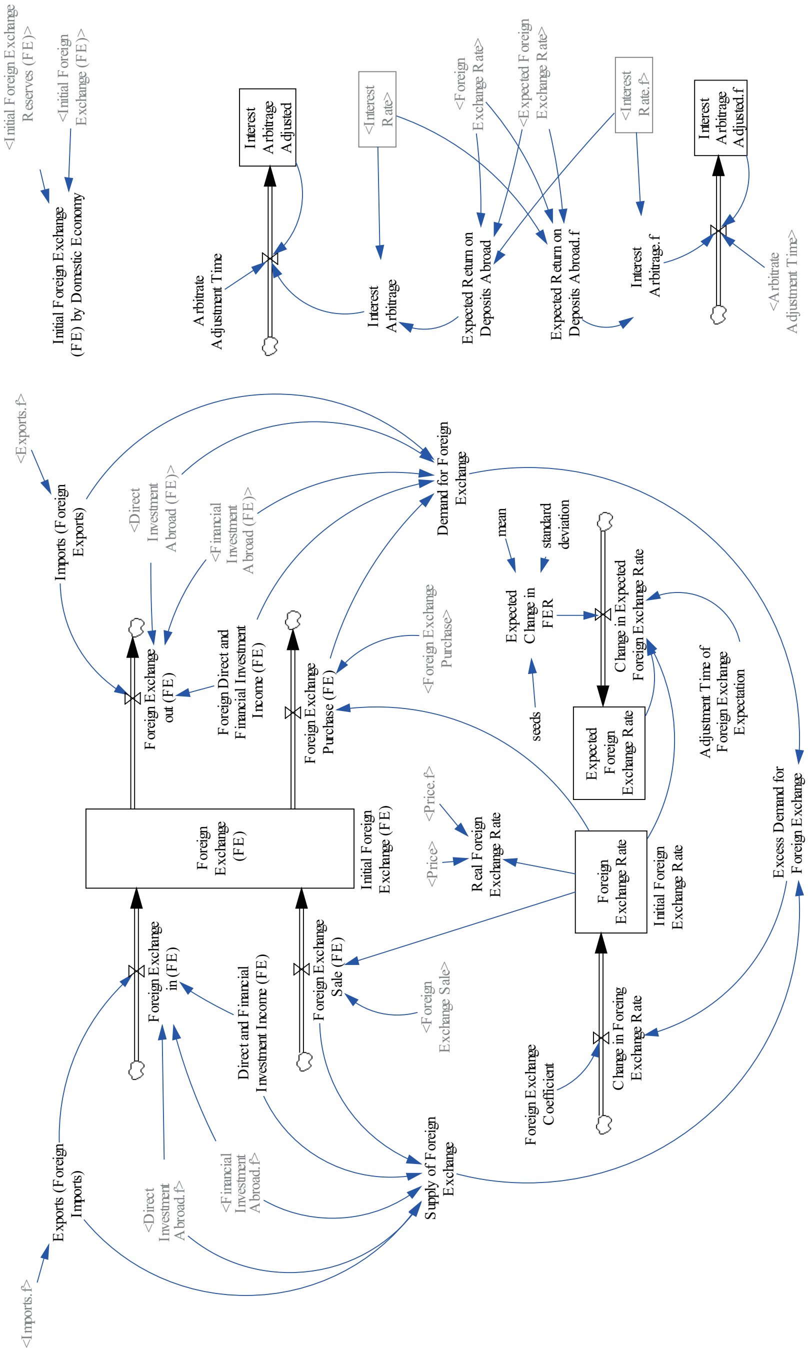

Figure 11.20: Foreign Exchange Market

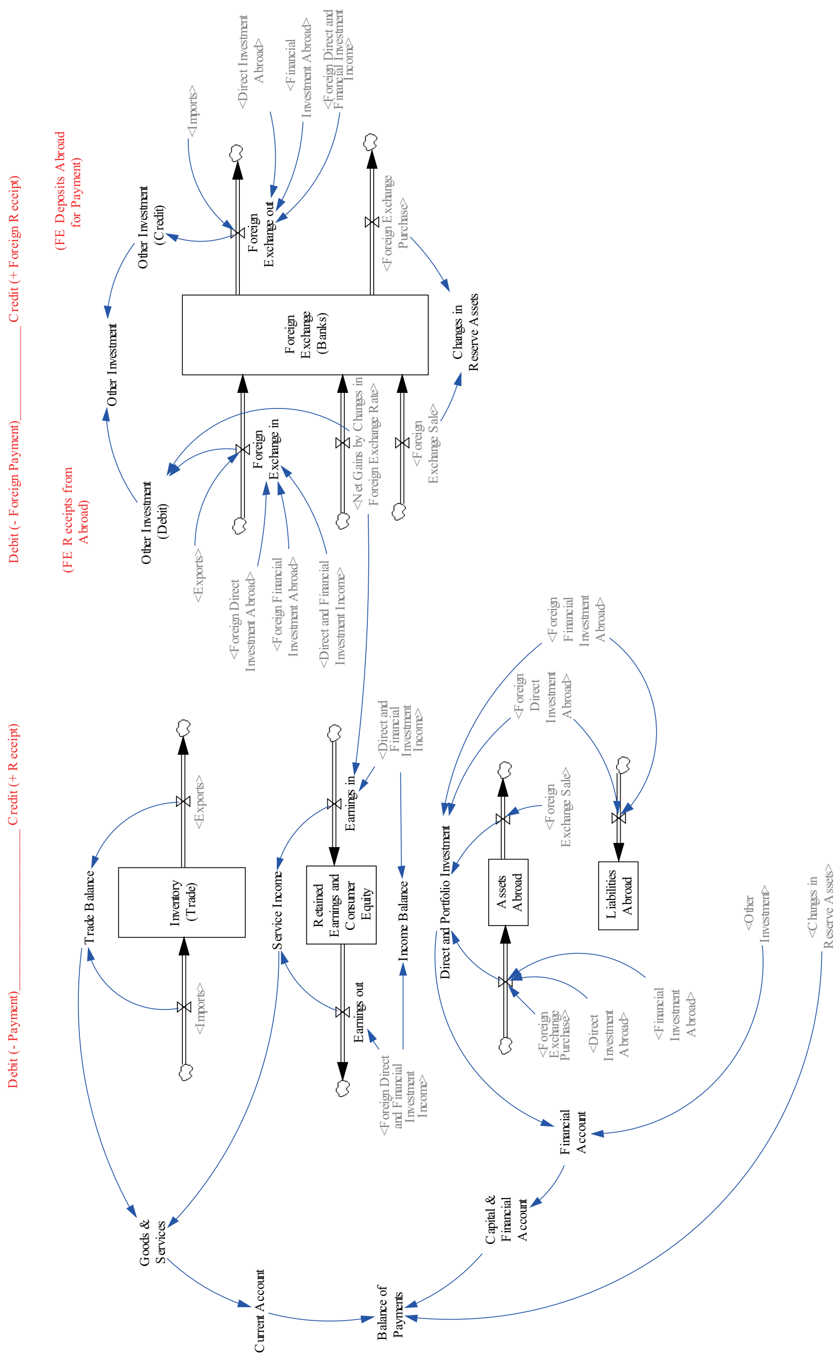

Figure 11.21: Balance of Payment

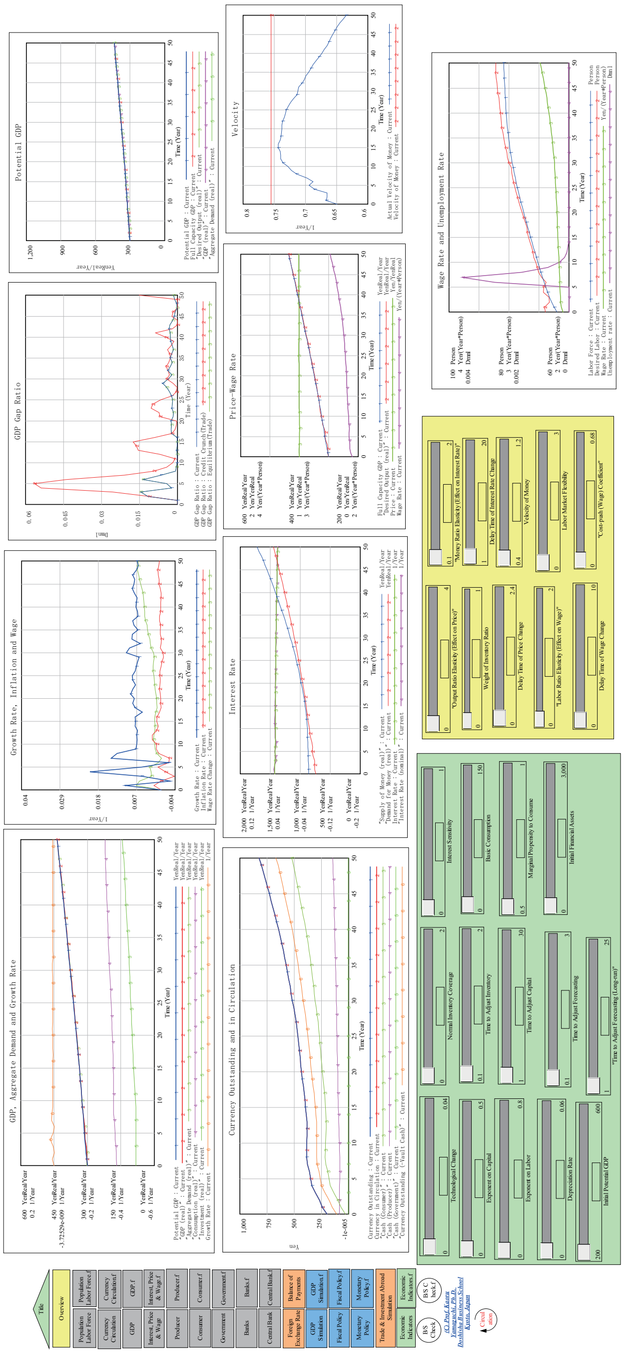

Figure 11.22: Simulation Panel of GDP

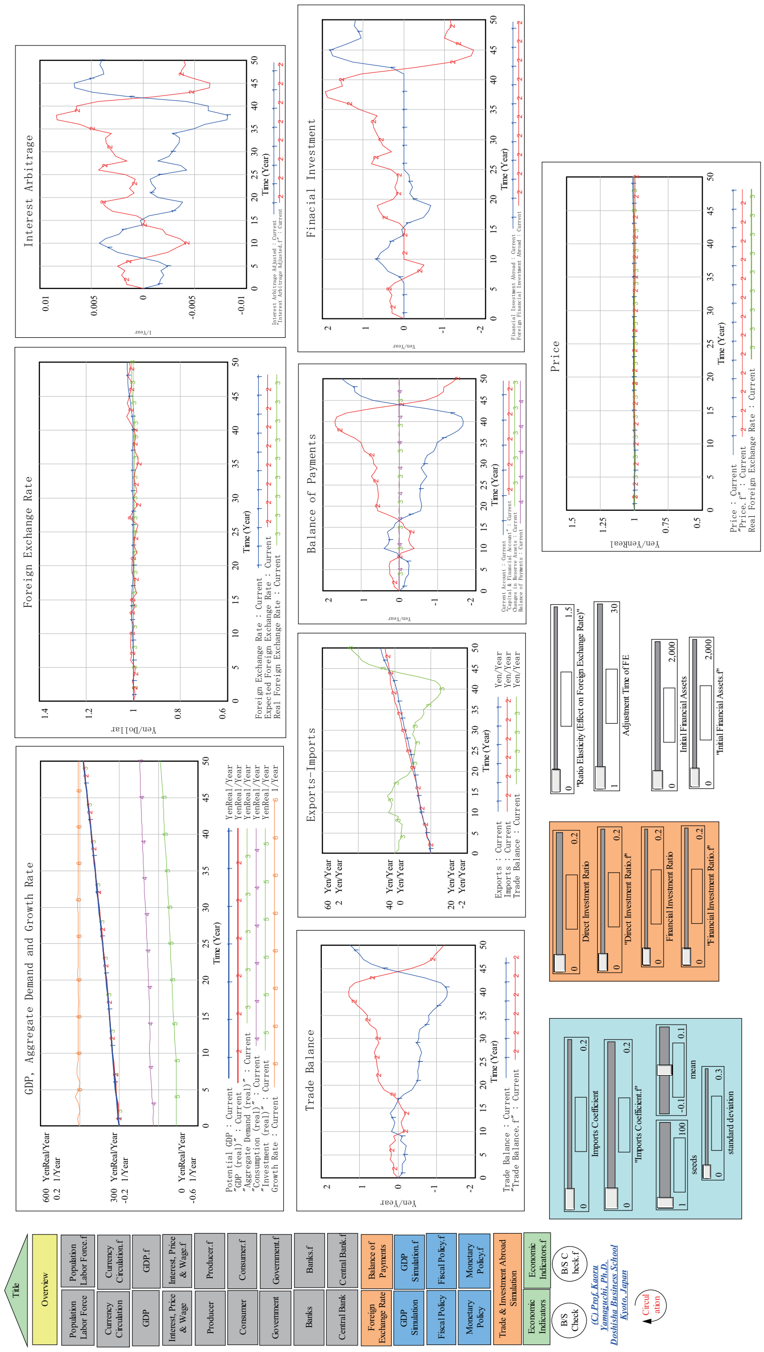

Figure 11.23: Simulation Panel of Trade and Investment Abroad

Part IV

New Macroeconomic System

Chapter 12

Designing A New Macroeconomic System

This chapter [1] first examines that the government debt is structurally built in the current macroeconomic *system of money as debt* which is founded on the Keynesian macroeconomic framework, and it becomes very costly to reduce it, within the scheme, by raising tax or cutting expenditure. Then, it demonstrates how the government debt could be liquidated without cost under an alternative macroeconomic *system of debt-free money* that is proposed by the American Monetary Act. Finally, it is posed that debt-free macroeconomic system is far superior to the debt-burden current macroeconomic system in a sense that it can not only liquidate government debt but also attain higher economic growth.

12.1 Introduction

While my macroeconomic modeling was advancing, the current world-wide financial crises were triggered by the bankruptcy of Lehman Brothers in September, 2008, and the US government has been forced to bail out the troubled financial institutions with $800 billion out of taxpayers' pockets, which in turn caused furious angers among American people.

These financial turmoils gave me, as a system dynamics researcher, a chance to re-think about the effectiveness of current macroeconomic system as a system design, since system dynamics is a methodology to help design a better system as Jay Forrester, founder of system dynamics, emphasized in 1961:

> Labor turmoil, bankruptcy, inflation, economic collapse, political unrest, revolution, and war testify that we are not yet expert enough

[1] This chapter is based on the paper: On the Liquidation of Government Debt under A Debt-Free Money System – Modeling the American Monetary Act – in "Proceedings of the 28th International Conference of the System Dynamics Society", Seoul, Korea, July 25 - 29, 2010. ISBN .

in the design and management of social systems [11, p.1].

System of Money as Debt

Books such as [29] and [70] enlightened me as being an *expert enough* to understand that the currently dominant macroeconomic system has been founded on the basis of money as debt. In the United States, this monetary framework was instituted by the Federal Reserve Act in 1913, which allows the 100% privately-owned Federal Reserve Board to play a role of the central bank and issue money. Without exception almost all of macroeconomic textbooks such as [39], [40], [41], [30], which have been refereed to my modeling works, justify the current macroeconomic system without mentioning an alternative system, if any.

Specifically, the current macroeconomic system can be defined as a system of money as a debt in which government has no control over the issue of money, except a small amount of metal coinage. Under the system only the central bank has a control to issue money against debt borrowed by the government or commercial banks. See chapter 5 for a detailed process of creating money. Growing economies constantly need to put newly issued money into circulation, yet under the current system it can only be fulfilled by the deficit of government with interest payment out of taxes. In other words, the government is systemically obliged to accumulate debt, as analyzed in chapters 8 and 9, to sustain economic growth.

Japanese government owns 55% of the shares of the central bank, that is, the Bank of Japan, yet it has no control over the issue of money. Accordingly, the government has to keep borrowing from the central bank with interest payment, though indirectly through the open market operations. In this sense, the Japanese macroeconomic system can also be said to be a system of money as debt.

What's wrong with this system? Nothing may be wrong if the system provides economic stability, full employment, fair income distribution and environmental sustainability. On the contrary, the system behaves oppositely, as theoretically analyzed in the model in chapter 11, and historically evidenced by the Great Depression in 1929 and the recent financial crises, to pick up some major ones. Accordingly, seven reasons to abolish the system were articulated in [29], which seem convincing and hard to refute by rational economists. Due to the limitation of space in this chapter they are not quoted here, but the reader is encouraged to investigate them in detail. Moreover, voices seem to be surfacing increasingly in the Internet against the current monetary system as a fundamental cause of the recent financial crises, specifically against the privately-owned and controlled central bank, that is, the Fed in the United States[2].

The current macroeconomic system has been structurally fabricated by the Keynesian economics, in which it is proposed that the additional government expenditure can rescue the troubled economy from recession. Yet, it fails to

[2]For instance, http://www.themoneymasters.com and http://www.monetary.org.

analyze why this fiscal policy is destined to accumulate government debt as mentioned above. In fact, even though GDP gap is very huge in Japan, yet due to the fear of runaway accumulation of debt, the government is very reluctant to stimulate the economy and, in this sense, it seems to have totally lost the discretion of public policies for the welfare of people even though production capacities and workers have been sitting idle and ready to be called in service. In other words, Keynesian fiscal policy cannot be applied to the troubled Keynesian macroeconomy. Isn't this an irony of the Keynesian theory? Macroeconomic system of money as a debt seems to have fallen into the dead-end trap.

The first objective of this chapter is accordingly to reconfirm the fact that the accumulation of the government debt is built in the Keynesian macroeconomic system of money as debt.

System of Debt-Free Money

As a system dynamics modeler and designer, I have now to pose a question if there exits an alternative macroeconomic system that helps escape from the above dead-end trap of the current system? One of the answers seem to be presented in [70, Chap. 24] and [71] as a macroeconomic system of debt-free money. This alternative system can be compactly defined as having the following three features:

- Governmental control over the issue of money
- Abolishment of credit creation
- Constant inflow of money to sustain economic growth and welfare

Accordingly, the second objective of this chapter is to construct a macroeconomic model which incorporates the above three features and examine if this alternative system could help liquidate government debt or not. Before moving on, let us take a closer look at these features in detail.

Governmental control over the issue of money

In macroeconomics, the amount of money to be issued by the central bank is called *monetary base* or *high-powered money.* In order for the government to control the issue of money and monetary base, the American Monetary Act suggests as follows:

> First, the Federal Reserve system becomes incorporated into the U.S. Treasury. This nationalizes the money system, not the banking system. Banking is *not a proper function of government*, but control and oversight of the money system *must be done by government* [71, p.12].

In Japan, the government owns 55% of the shares of the Bank of Japan. Accordingly, its incorporation to the government could be rather smoothly done, though the government, its major shareholder, is currently prohibited from the bank's decision-making process by law. In Europe, two incorporation processes could be possible. First, EU member countries are politically integrated into, say, the United States of Europe, which in turn establishes its own federal European government and incorporate the current European Central Bank into its branch. Or the ECB is once again disintegrated and incorporated into the governments of member countries, respectively.

Abolishment of credit creation

Credit can be created by the lending of commercial banks, and becomes a part of money supply, because it plays a role of means of exchange. The credit creation has been called *money out of nothing* or *money out of thin air* by Keynes. It is made possible because banks are required to hold only a fraction of deposits (with the central bank) and can lend the remaining larger portion. This system is called *a fractional reserve banking system.* Heavily-criticized practice of leveraged investment that led to the recent financial crises is made possible by the credit creation. Under a debt-free money system, this fractional reserve banking system is abolished; that is, a fractional reserve ratio has to be 100%. The American Monetary Act suggests as follows:

> Second, the accounting privilege banks now have of creating money through fractional reserve lending of their credit is stopped entirely, once and for all. Banks remain private companies and are encouraged to act as intermediaries between their clients who want a return on their savings and those clients willing to pay for borrowing those savings, but they may no longer create any part of the nation's money supply [71, p.12].

It will be worthwhile to clarify a position of money in an economy. Suppose there exists N commodities in an economy. Under gold standard or commodity money standard, one of the commodities becomes money against which the remaining N-1 commodities are exchanged. Hence, quantity of money is limited by the production of gold or commodity money. Under a fractional reserve banking system, credit is created and used as *low-powered* money in addition to monetary base or high-powered money. In other words, 2 types of money are being used for the exchange of N commodities; that is, currency in circulation and deposits. Finally, under a system of debt-free money, only the government-issued fiat money is used to exchange for N commodities. Schematically, positions of money under different monetary system is summarized as follows:

Gold or Commodity Money Standard
(N - 1) + Gold

Fiat Money as Debt
N + Currency in Circulation + Credits(Deposits)

Debt-free Fiat Money
N + Money

Constant inflow of money

Growing economy demands for a growing amount of money as a means of exchange if monetary value is to be sustained. This can be easily verified from the following quantity theory of money:

$$MV = PT = kPY \tag{12.1}$$

where M is money supply, V is a velocity of money, P is a price level, T is the amount of transaction, Y is real GDP, and k is a so-called Marshall's k.

Assuming that V and k are constant, we have

$$\frac{\dot{M}}{M} = \frac{\dot{P}}{P} + \frac{\dot{Y}}{Y} \tag{12.2}$$

Thus, to sustain a monetary value by avoiding inflation or deflation, we have to attain $\dot{P}/P = 0$. This implies that $\dot{M}/M = \dot{Y}/Y$; that is, money has to be issued and put into circulation in accordance with the economic growth.

Under a system of money as debt, the injection of new money into circulation has only been carried out by the privately-controlled central bank at its discretion and for its interest. Under a debt-free money system, two channels for money injection becomes available. First, the government can directly distribute newly issued money into circulation as an additional expenditure according to its public policies supported by voters in the field of infrastructures, education, medical care, green technologies and environment. Second, the central bank, now as a part of government, can make loans to commercial banks, free of interest, according to a guideline set by governmental growth strategies for the interest and welfare of people.

As to the new issue of money, American Monetary Act suggest as follows:

> Third, new money is introduced into circulation by government spending it into circulation starting with the $2.2 trillion the engineers tell us is needed for infrastructure repair and renewal. In addition, health care and education are included as human infrastructure. Everyone supports the infrastructure, but they worry how to pay for it. That becomes possible with the passage of the American Monetary Act [71, p.12].

Battles to Control the Money System

Human history could be, in a sense, said to be a history of battles to control the issue of money; that is, the battles between a system of money as debt and a system of debt-free money in our terminology here, or between interest-bearing money and interest-free money.

Science of money, according to [70], was founded by Aristotle (384-322 BC). He viewed that "Money exists not by nature but by law(nomos)" . His view has been supported through the church's condemnation of usury up to the work by the philosopher George Berkeley[3] in his 1735 book of questions "Querest". Neglecting his work, Adam Smith, father of economics, ended the battle by supporting the Bank of England, founded in 1694, as a system of money as debt in his book *The Wealth of Nations* in 1776.

> This battle is summarized as Aristotle's science of money vs. Adam Smith's metallic view of money. ⋯ Whether money should be tangible wealth and thereby be privately controlled to benefit the wealthy (Smith), or be an abstract legal fiat power publicly controlled to promote the general welfare (Aristotle) [71, p.6].

In the United States, this battle was finalized when the Federal Reserve Act was approved and system of money as debt has been introduced [29]. Since then science of money has been lost and economists showed no doubt on the role of money as debt, including Keynes. The lost science has been reflected in many macroeconomics textbooks, including the ones as briefly mentioned above.

The dominance of current macroeconomic system is being challenged again with the introduction of the American Monetary Act under the recent financial crises. As a system dynamics researcher and a professionally trained economist, I'm of the opinion that it is a reclaiming process of *the lost science of money* to construct macroeconomic models which enable us to compare these two systems and evaluate them impartially in terms of economics as a science.

12.2 Macroeconomic System of Money as Debt

For the comparative analysis of the two macroeconomic systems, the integrated macroeconomic model developed in chapter 8 is revisited in this chapter[4]. According to the discussions above, the model is classified as a macroeconomic system of money as debt, in which five macroeconomic sectors are assumed to play interdependent activities simultaneously; that is, producers, consumers, banks, government and the central bank. Foreign sector is excluded from the model.

Under the current macroeconomic system of money as debt, transactions by producers, consumers, government, banks and the central bank remain the same as already explained in Chapter 8. Yet, transactions of government, banks and central bank are repeated here as a comparative reference to the revised transactions under a macroeconomic system of debt-free money to be presented below.

[3] University of California at Berkeley, where I studied mathematical economics in late 70's and early 80's, was named after the philosopher Berkeley.

[4] This choice of the model is in accordance with my viewpoint that labor market should be abolished from a market economy as a better economic system, which is proposed as the MuRatopian economy in [58]. The MuRatopian economy is presented as an alternative system, beyond capitalist market economy and planned socialist economy, suitable for the information age of the 21st century. However, money system is missing in the economy, I admit. While writing this chapter, I become convinced that a debt-free money system should be indeed a monetary system of the MuRaopian economy.

Government

Transactions of the government are illustrated in Figure 12.1, some of which are summarized as follows.

- Government receives, as tax revenues, income taxes from consumers and corporate taxes from producers as well as excise tax on production.
- Government spending consists of government expenditures and payments to the consumers for its partial debt redemption and interests against its securities.
- Government expenditures are assumed to be endogenously determined by either the growth-dependent expenditures or tax revenue-dependent expenditures.
- If spending exceeds tax revenues, government has to borrow cash from banks and consumers by newly issuing government securities.

Banks

Transactions of banks are illustrated in Figure 12.2, some of which are summarized as follows.

- Banks receive deposits from consumers, against which they pay interests.
- They are obliged to deposit a fraction of the deposits as the required reserves with the central bank (which is called a fractional reserve banking system).
- Out of the remaining deposits loans are made to producers and banks receive interests for which a prime rate is applied.
- Their retained earnings thus become interest receipts from producers less interest payment to consumers. Positive earning will be distributed among bank workers as consumers.

Central Bank

The central bank plays an important role of issuing money or currency. Sources of its assets against which money is issued are simply confined to gold, discount loans and government securities. The central bank can control the amount of money supply through the amount of monetary base consisting of currency outstanding and bank reserves. This monetary control can be carried out through monetary policies such as a manipulation of required reserve ratio and open market operations as well as direct control of lending to the banks. Transactions of the central bank are illustrated in Figure 12.3, some of which are summarized as follows.

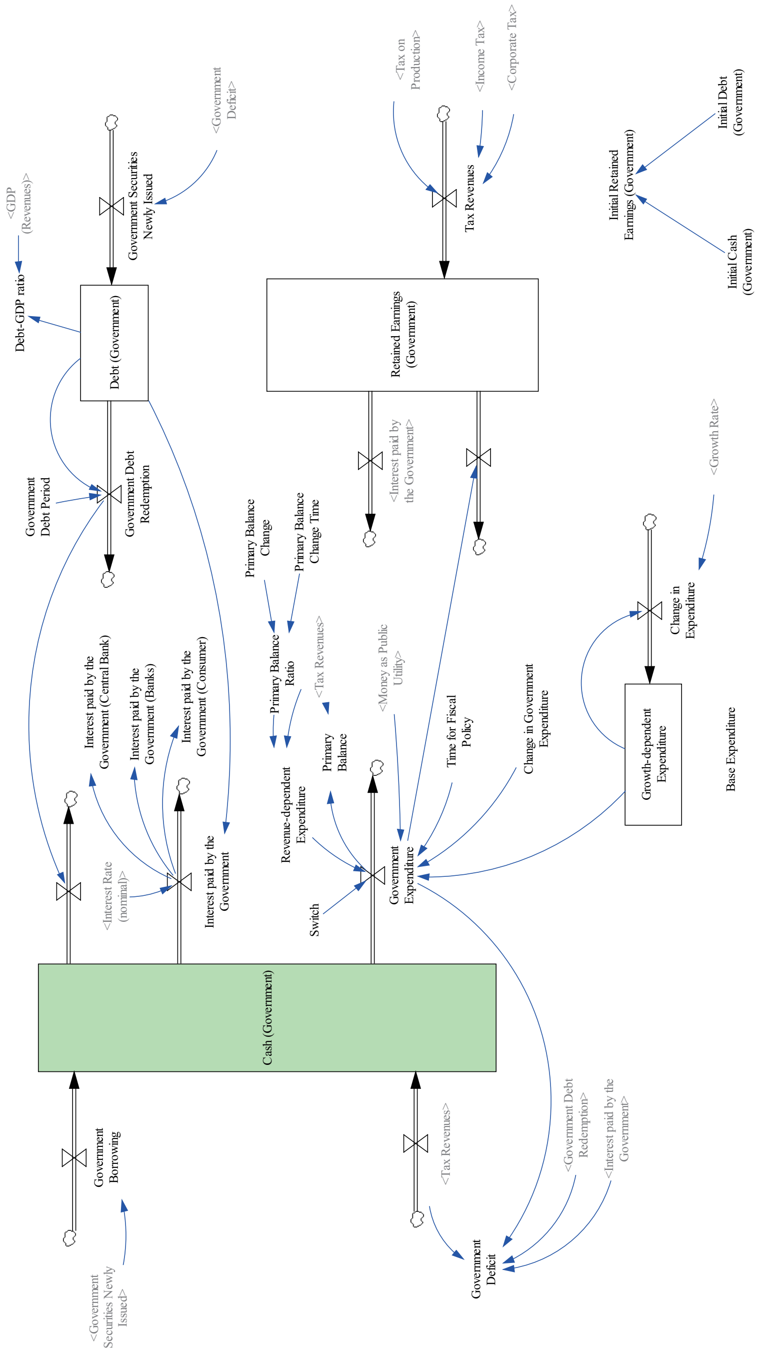

Figure 12.1: Transactions of Government

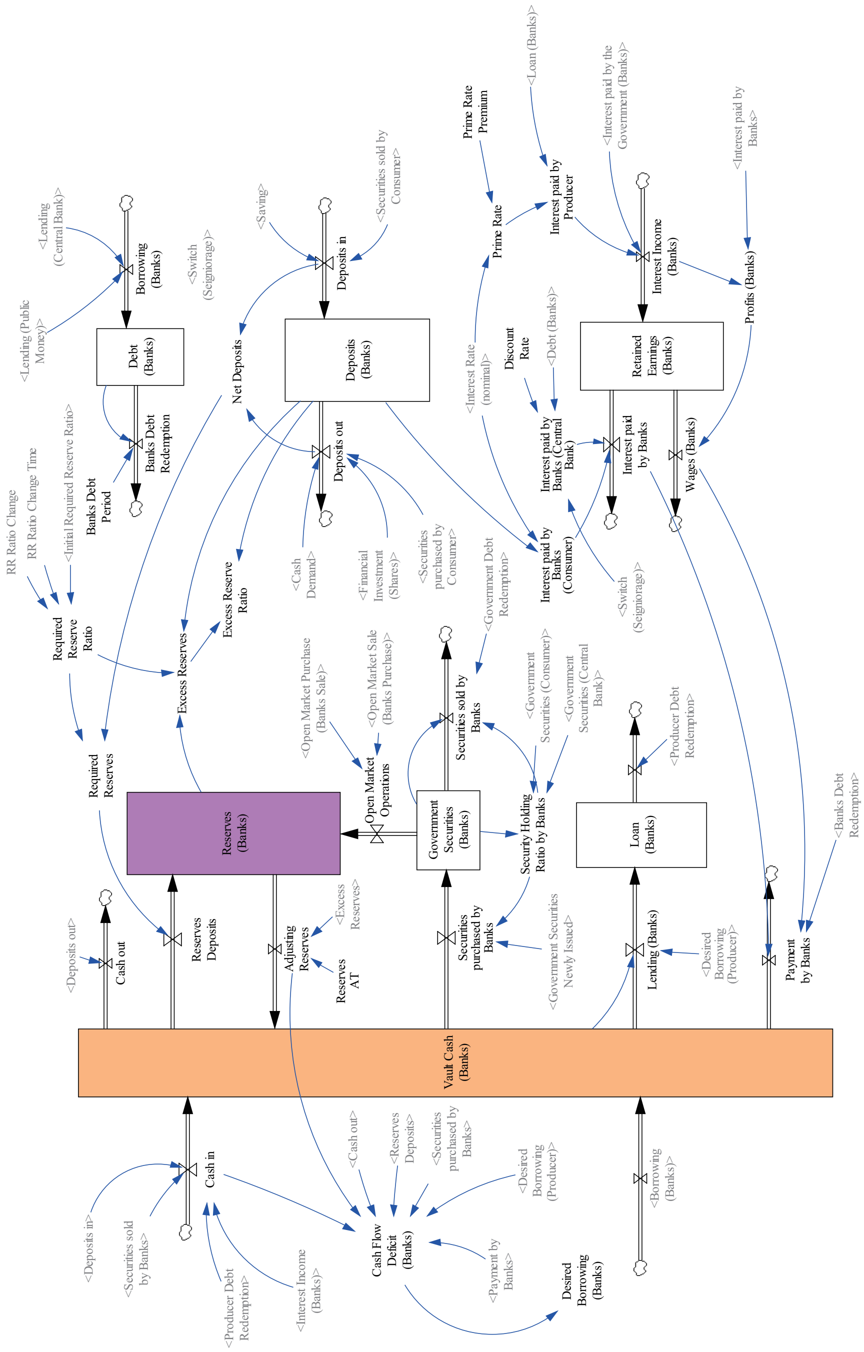

Figure 12.2: Transactions of Banks

- The central bank issues money (historically gold certificates) against the gold deposited by the public.
- It can also issue money by accepting government securities through open market operation, specifically by purchasing government securities from the public (consumers) and banks. Moreover, it can issue money by making discount loans to commercial banks. (These activities are sometimes called creation of *money out of nothing*.)
- It can similarly withdraw money by selling government securities to the public (consumers) and banks, and through debt redemption by banks.
- Banks are required by law to reserve a certain fraction of deposits with the central bank. By controlling this required reserve ratio, the central bank can control the monetary base directly.
- The central bank can, thus, control the amount of money supply through monetary policies such as open market operations, reserve ratio and discount rate.
- Another powerful but hidden control method is through its direct influence over the amount of discount loans to banks (known as *window guidance* in Japan.)

12.3 System Behaviors of Money as Debt

Mostly Equilibria in the Real Sector

The macroeconomic model of money as debt is now complete. It is a generic model, out of which diverse macroeconomic behaviors will be generated. Let us only focus on an equilibrium growth path of the macroeconomy. As already discussed in chapter 8, an equilibrium state is called *a full capacity aggregate demand equilibrium* if the following three output and demand levels are met:

$$\text{Full Capacity GDP} = \text{Desired Output} = \text{Aggregate Demand} \tag{12.3}$$

By trial and error, mostly equilibrium states are acquired, as in chapter 8, when a ratio elasticity of the effect on price e is 1, and a weight of inventory ratio ω is 0.1, as illustrated in Figure 12.4.

These equilibrium states are used in chapter 8 as a benchmarking state for comparisons with various disequilibrium cases such as fix-price disequilibria, business cycles caused by inventory coverage and elastic price fluctuation, and economic recession caused by credit crunch. Moreover, these disequilibria are shown to be fixed toward equilibria through monetary and fiscal policies. In this chapter, our analysis is confined only to the case of liquidation of government debt.

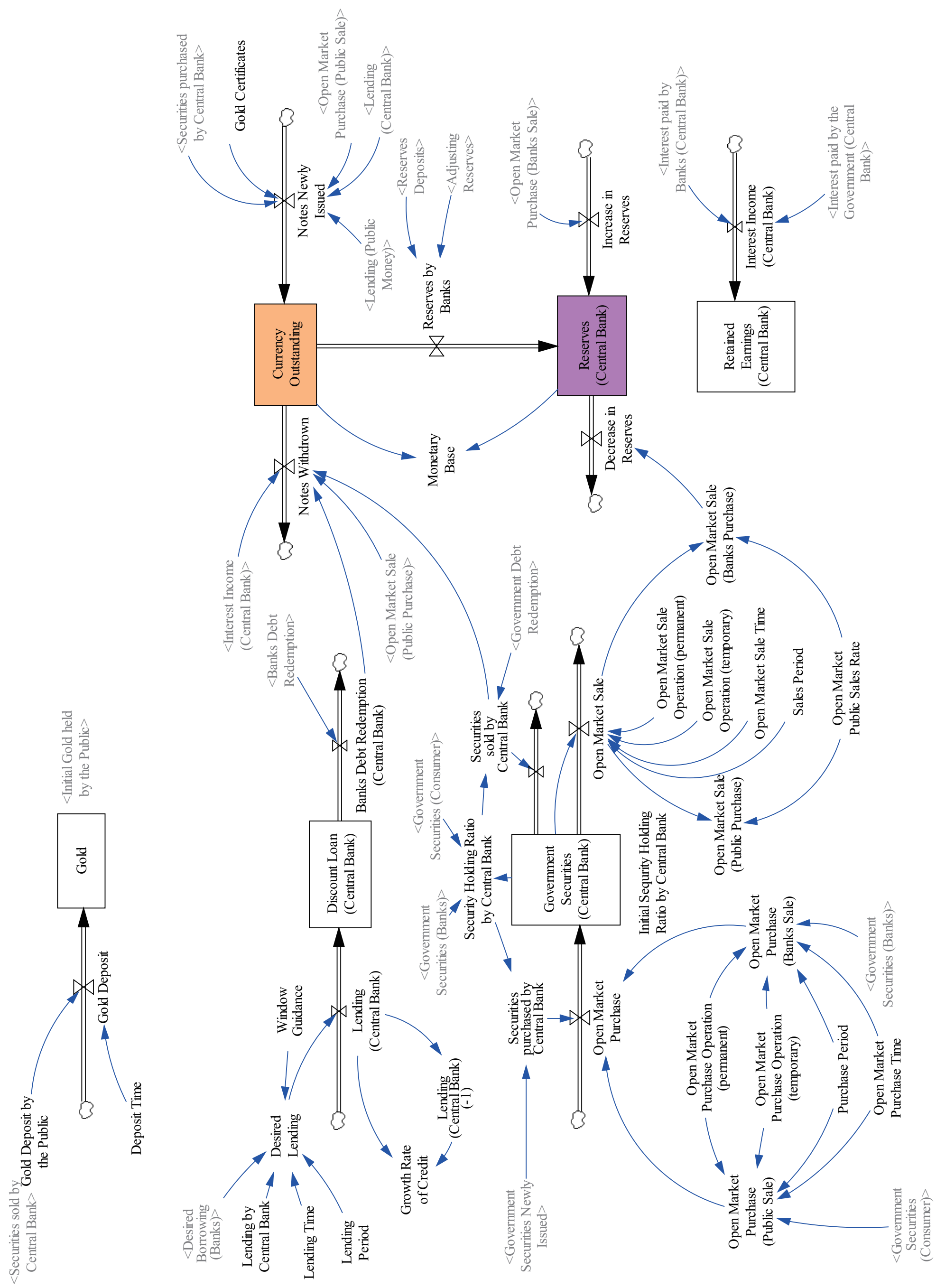

Figure 12.3: Transactions of Central Bank

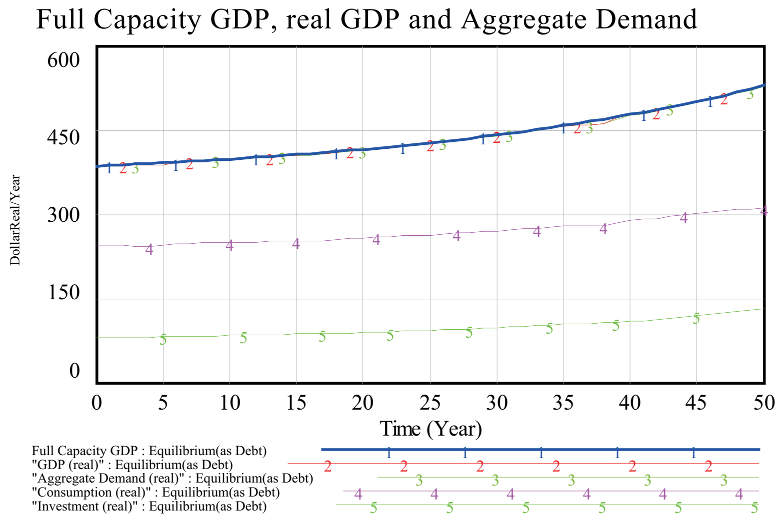

Figure 12.4: Mostly Equilibrium States

Money as Debt

For the attainment of mostly equilibria, enough amount of money has to be put into circulation to avoid recessions caused by credit crunch as analyzed in chapter 8. Demand for money mainly comes from banks and producers. Banks are assumed to make loans to producers as much as desired so long as their vault cash is available. Thus, they are persistently in a state of shortage of cash as well as producers. In the case of producers, they could borrow enough fund from banks. From whom, then, should the banks borrow in case of cash shortage?

In a closed economic system, money has to be issued or created within the system. Under the current financial system of money as debt, only the central bank is endowed with a power to issue money within the system, and make loans to the commercial banks directly and to the government indirectly through the open market operations. Commercial banks then create credits under a fractional reserve banking system by making loans to producers and consumers. These credits constitute a major portion of money supply. In this way, money and credits are only crated when commercial banks and the government as well as producers and consumers come to borrow at interest. If all debts are repaid, money ceases to exit. This is an essence of a system of money as debt. This process of creating money is known as *money out of nothing*.

Figure 12.5 indicates unconditional amount of annual discount loans and its growth rate by the central bank at the request of desired borrowing by banks. In other words, money has to be incessantly created and put into circulation in order to sustain an economic growth under mostly equilibrium states. Roughly speaking, a growth rate of credit creation by the central bank has to be in average equal to or slightly greater than the economic growth rate as suggested by the right hand diagram of Figure 12.5.

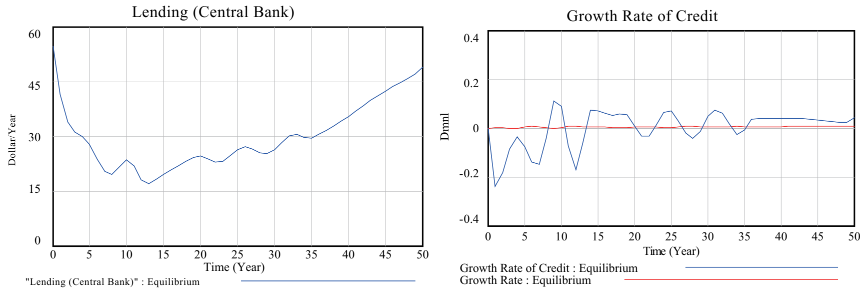

Figure 12.5: Lending by the Central Bank and its Growth Rate

In this way, the central bank begins to exert an enormous power over the economy through its credit control. What happens if the central bank fails to supply enough currency intentionally or unintentionally? An economic recession by credit crunch as analyzed in chapter 8. An influential role of the central bank which caused economic bubbles and the following burst in Japan during 1990's is completely analyzed by Warner in [54] and [55].

Government Debt

So long as the mostly equilibria are realized in the economy, through monetary and fiscal policies in the days of recession, no macroeconomic problem seems to exist. This is a positive side of Keynesian macroeconomic theory. Yet behind the full capacity aggregate demand growth path in Figure 12.4 government debt continues to accumulate as the line 1 in the left diagram of Figure 12.6 illustrates. This is a negative side of the Keynesian theory. Yet most macroeconomic textbooks neglect or less emphasize this negative side, partly because their macroeconomic frameworks cannot handle this negative side of the system of money as debt.

Primary balance ratio is initially set to be one and balanced budget is assumed here; that is, government expenditure is set to be equal to tax revenues, and no deficit seems to arises. Why, then, does the government continue to accumulate debt? Government deficit is, as discussed in chapter 8, precisely defined as

$$\text{Deficit} = \text{Tax Revenues - Expenditure - Debt Redemption - Interest} \quad (12.4)$$

Therefore, even if balanced budget is maintained, the government still has to keep paying its debt redemption and interest. This is why it has to keep borrowing and accumulating its debt. Initial GDP in the model is obtained to be 386, while government debt is initially set to be 200. Hence, the initial debt-GDP ratio is around 0.52 year (similar to the current ratios among EU member countries). Yet, the ratio continues to increase to 1.45 year at the year 50 in

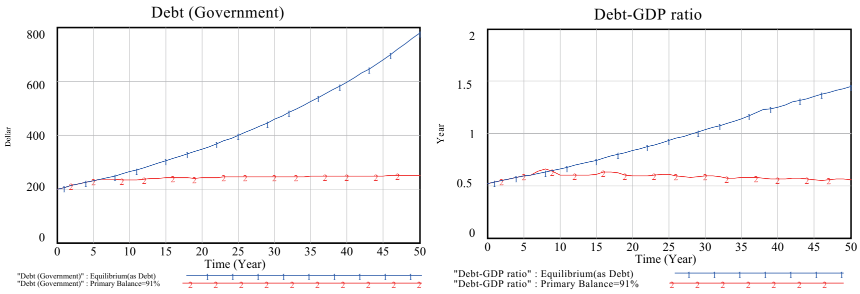

Figure 12.6: Accumulation of Government Debt and Debt-GDP Ratio

the model as illustrated by the line 1 in the right diagram of Figure 12.6. This implies the government debt becomes 1.45 years as high as the annual level of GDP.

Can such a high debt be sustained? Absolutely no. Eventually this runaway accumulation of government debt may cause nominal interest rate to increase, because the government may be forced to keep borrowing by paying higher interests, which may eventually cause hyper-inflation[5].

Higher interest rates may in turn trigger a sudden drop of government security price, deteriorating values of financial assets owned by banks, producers and consumers. The devaluation of financial assets may force some banks and producers to go bankrupt eventually. In this way, another financial crisis becomes inevitable and government is eventually destined to collapse as well. This is one of the hotly debated scenarios about the consequences of the rapidly accumulating debt in Japan, whose debt-GDP ratio in 2009 was 1.893 years; the highest among OECD countries! Compared with this, debt-GDP ratios in the model seem to be still modest.

Remarks: if this scenario of financial breakdown due to the runaway accumulation of debt fails to be observed in the near future, still there exit some legitimate reasons to stop the accumulating debt. First, it continues to create unfair income distribution in favor of bankers and financial elite, causing inefficient allocation of resources and economic performances, and eventually social turmoils by the poor. Second, forced payment of interest forces the indebted producers to continue incessant economic growth to the limit of environmental carrying capacity, which eventually leads to the collapse of environment. In short, system of money as debt is unsustainable as an economic system.

Liquidation of Government Debt

Let us now consider how to avoid such a financial crisis and collapse. At the face of the financial crisis as discussed above, suppose that the government is forced to reduce its debt-GDP ratio to less than 0.6 by the year 50, as required to all

[5]This feedback loop from the accumulating debt to the higher interest rate is not yet fully incorporated in the model.

EU members by the Maastricht treaty. To attain this goal, a primary balance ratio has to be reduced to 0.91 in our economy. In other words, the government has to make a strong commitment to repay its debt annually by the amount of 9 % of its tax revenues. Let us assume that this reduction is put into action at the year 6. Under such a radical financial reform, as illustrated by the line 2 in the right diagram of Figure 12.6, debt-GDP ratio will be reduced to around 0.56, and the accumulation of debt will be eventually curved (line 2 in the left diagram).

Even so, this radical financial reform becomes very costly to the government and its people as well. At the next year of the implementation of 9 % reduction of a primary balance ratio, growth rate is forced to drop to minus 5 %, and the economy fails to sustain a full capacity aggregate demand equilibrium of line 1 as illustrated by the line 2 in Figure 12.7. Compared with the mostly equilibrium path of line 1, debt-reducing path of line 2 causes economic recession, followed by business cycles.

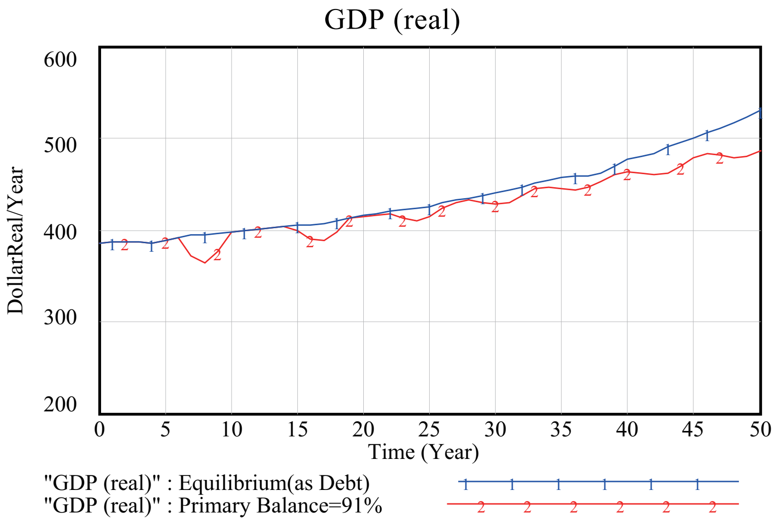

Figure 12.7: Comparison of GDP paths

12.4 Macroeconomic System of Debt-Free Money

We are now in a position to implement the alternative macroeconomic system discussed in the introduction, as proposed by the American Monetary Act, in which the central bank is incorporated into the government and a fractional reserve banking system is abolished. Let us call this new system a macroeconomic system of debt-free money. Money issued in this new system plays a role of public utility of medium of exchange. Hence the newly incorporated institution may be appropriately called *the Public Money Administration*.

Under this incorporation, transactions of the government, commercial banks and the public money administration (formally the central bank) need to be revised slightly. Let us start with the description of the revised transactions of the government.

Government

- Balanced budget is assumed to be maintained; that is, a primary balance ratio is unitary. Yet the government may still incur deficit due to the debt redemption and interest payment.
- Government now has the right to newly issue money whenever its deficit needs to be funded. The newly issued money becomes seigniorage inflow of the government into its equity or retained earnings account.
- The newly issued money is simultaneously deposited with the reserve account of the Public Money Administration. It is also booked to its deposits account of the government assets.
- Government could further issue money to fill in GDP gap.

Revised transaction of the government is illustrated in Figure 12.8. Green stock box of deposits is newly added to the assets.

Banks

Revised transactions of commercial banks are summarized as follows.

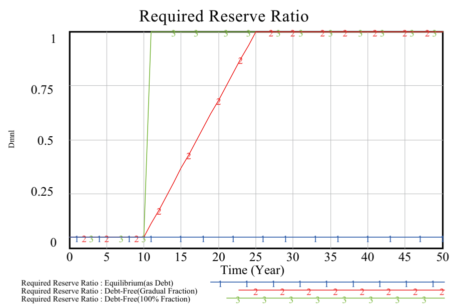

Figure 12.9: Required Reserve Ratios

- Banks are now obliged to fully deposit the amount of deposits they owe as the required reserves with the public money administration. Time deposits are excluded from this obligation.
- When the amount of time deposits is not enough to meet the demand for loans from producers, banks are allowed to borrow from the public money administration free of interest; that is, a former discount rate is now zero. Allocation of loans to the banks will be prioritized according to the public policies of the government. (This constitutes a market-oriented issue of new money. Alternatively, the government can issue new money directly through its public policies to fill in a GDP gap, if any, as already discussed above.)

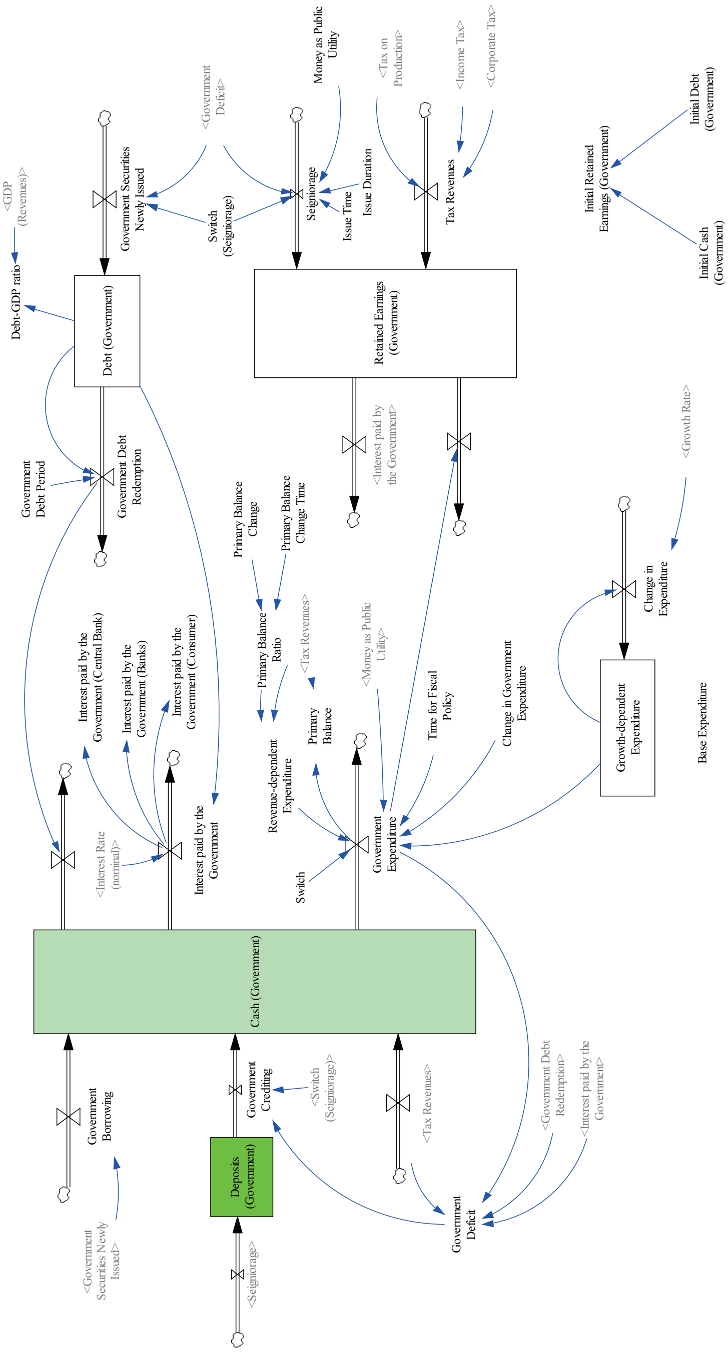

Figure 12.8: Transactions of Government

Line 1 in Figure 12.9 illustrates the originally required reserve ratio of 5%. In addition, two different ways of abolishing a fractional reserve banking system are assumed. Line 2 shows that the 100% fraction (full reserves) is gradually attained in 10 years at the year 25, starting from the year 10, meanwhile line 3 indicates that the 100% fraction is attained immediately at the year 10.

Public Money Administration (Formerly Central Bank)

The central bank now gets incorporated as one of the governmental organizations which is here called the Public Money Administration (PMA). Its revised transactions now become as follows.

- The PMA accepts newly issued money of the government as seigniorage assets and enter the same amount into the government reserve account. Under this transaction, the government needs not print hard currency, instead it only sends digital figures of the new money to the PMA.
- When the government wants to withdraw money from their reserve accounts at the PMA, the PMA could issue new money according to the requested amount. In this way, for a time being, former central bank note and government money coexist in the market.
- With the new issue of money the PMA meets the demand for money by commercial banks, free of interest, according to the guideline set by the government public policies.

Under the revised transactions, open market operations of sales and purchases of government securities become ineffective, simply because government debt gradually diminishes to zero. Furthermore, discount loan is replaced with interest-free loan. This lending becomes a sort of open and public *window guidance*, which once led to the rapid economic growth after World War II in Japan [54]. Accordingly, interest incomes from discount loans and government securities are reduced to be zero eventually. Transactions of the public money administration are illustrated in Figure 12.10. Green stock boxes of seigniorage assets and government reserves are newly added.

12.5 System Behaviors of Debt-Free Money

Liquidation of Government Debt

Under this alternative macroeconomic system of debt-free money, the accumulated debt of the government gets gradually liquidated as demonstrated in Figure 12.11, which is the same as Figure 12.6 except that lines 3 and 4 are added here. Recollect that line 1 was a benchmark debt of the mostly equilibria under the system of money as debt, while line 2 was the decreased debt when debt-ratio is reduced under the same system. Now newly added line 3 indicates that the government debt continues to decline when a gradual fraction ratio

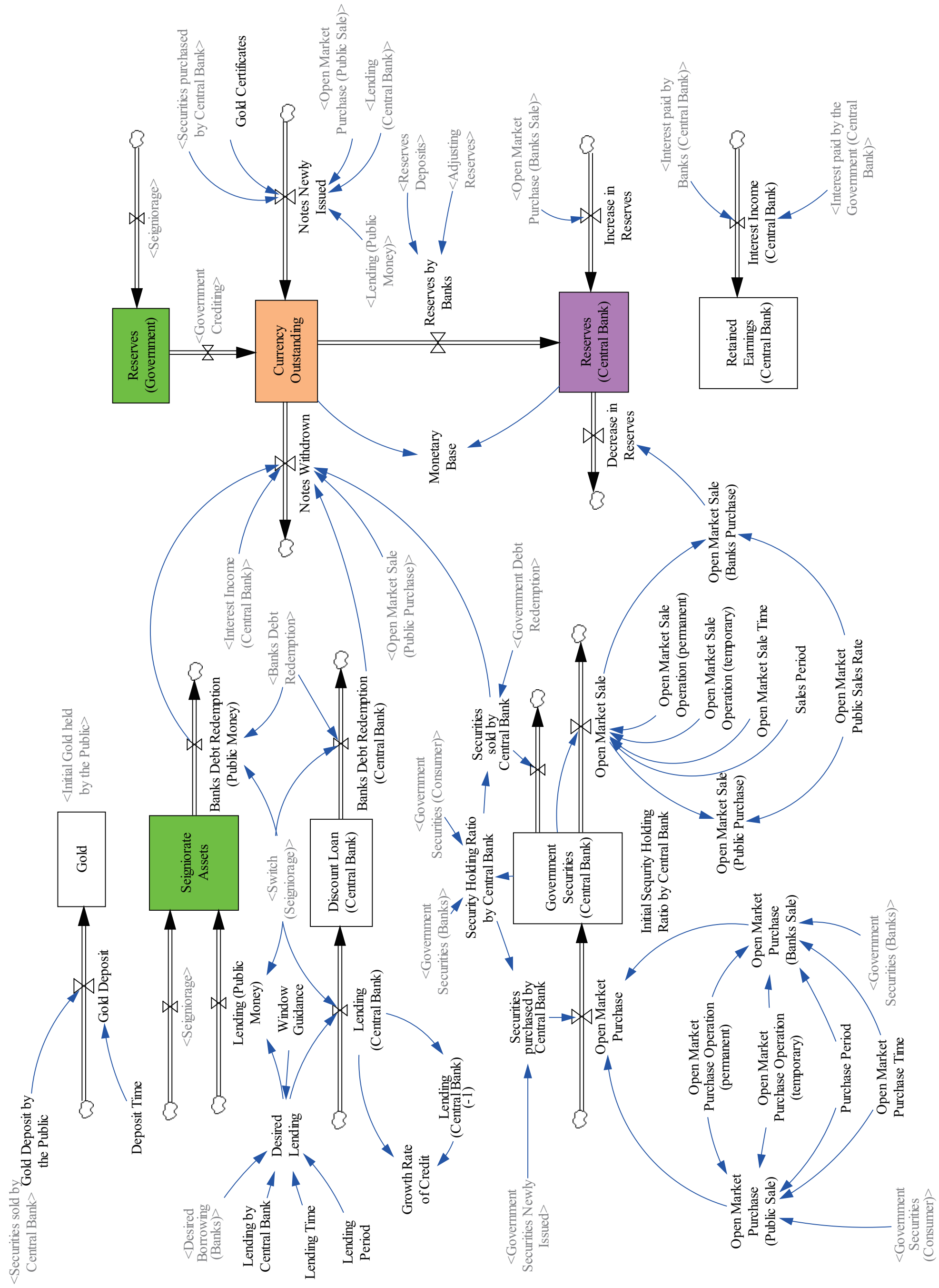

Figure 12.10: Transactions of the Public Money Administration

is applied, while line 4 shows a reduction of the government debt when 100% fraction (full reserves) is applied at the year 10. Both lines coincide, meaning that the abolishment methods of a fractional level do not affect the liquidation of the government debt, because banks are allowed to fill in the enough amount of cash shortage by borrowing from the PMA in the model.

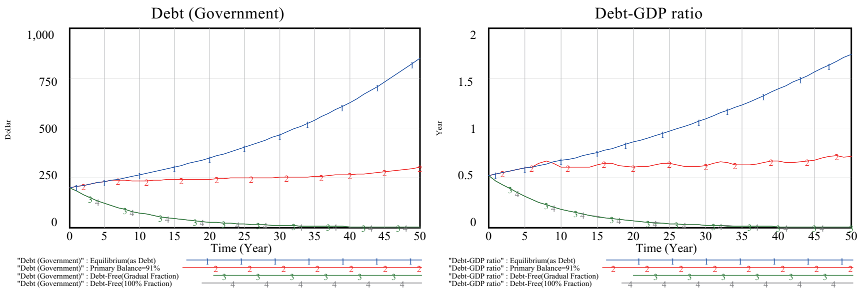

Figure 12.11: Liquidation of Government Debt and Debt-GDP Ratio

Under the system of debt-free money, a higher GDP is attained, to our surprise, than the one under the system of money as debt as illustrated by lines 3 & 4 in Figure 12.12. Moreover, it shows that the gradual abolishment of a fractional reserve banking system (line 3) attains a higher GDP than the one by a sudden abolishment of fractional reserves at the year 10 (line 4). In either case, system of debt-free money can be said to be a far better system because of the accomplishment of higher economic growth without inflation.

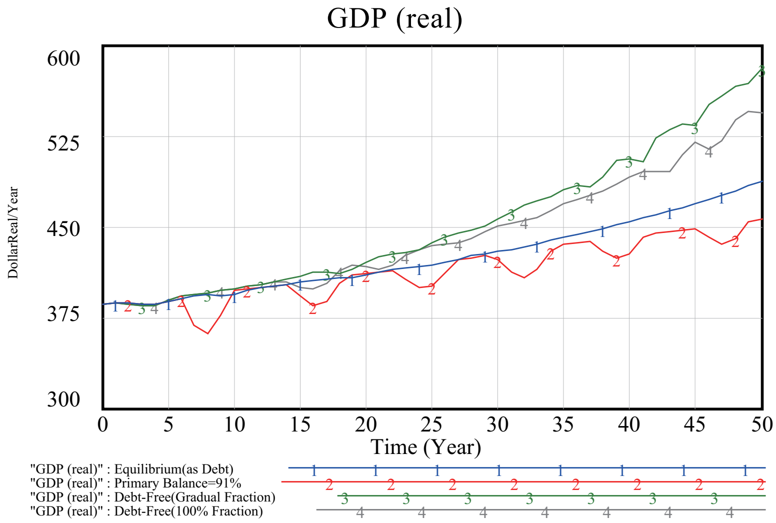

Figure 12.12: Comparison of GDP paths

In fact, left diagram in Figure 12.13 illustrates that price of line 3 increases at maximum to 1.6% at the year 17, and inflation rate of line 3 in the right diagram is constantly below 0.3% at maximum.

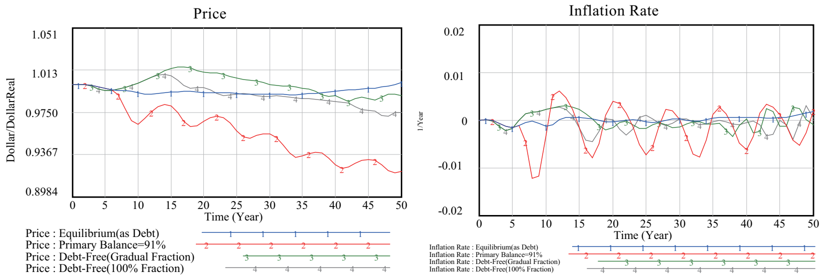

Figure 12.13: Price Level and Inflation Rate

Inflation and GDP Gap

Persistent objection to the system of debt-free money has been that government, once a free-hand power of issuing money is being endowed, tends to issue more money than necessary, which tends to bring about inflation eventually, though history shows the opposite [70].

Theoretically, under the existence of GDP gap, increase in the government expenditure by issuing new money would not cause inflation, but stimulate the economic growth instead. To examine this case, let us first create a GDP gap by changing the exponent of capital from 0.4 to 0.43 as illustrated by line 1 in Figure 12.14, in which the GDP gap is observed between the year 5 and 10. Faced with this recession, money as public utility is being newly issued by the amount of 23 for 10 years starting at the year 6. This corresponds to a continual inflow of money into circulation as proposed in the introduction. As a result equilibrium is attained again as illustrated by line 2 in the left diagram, yet inflation does not seem to appear as line 2 of the right diagram indicates.

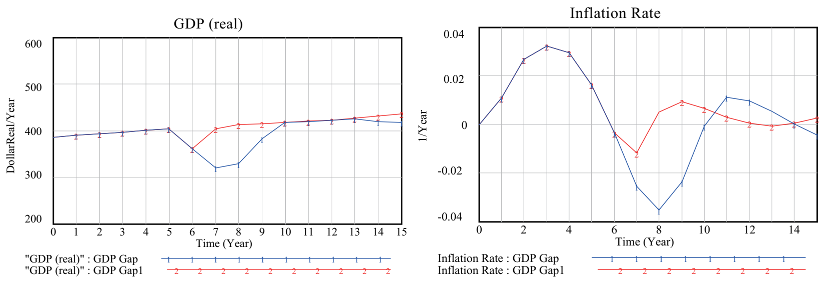

Figure 12.14: No Inflation under GDP Gap

Inflation could occur only when government mismanages the money supply. To examine this case, let us take a benchmark state attained by the gradual fraction equilibrium, then assume that the government increases its spending by mistakenly issuing new money by the amount of 10 for 3 years, starting at the year 10; that is, the government expenditure continues to increase to 74 from 64 for three years.

As being expected, the increase in the government expenditure under the equilibrium state surely causes inflation, 2% at the year 13, as illustrated by line 2 of the left diagram of Figure 12.15, followed by the deflation of -2% at the year 17. To be worse, this inflation triggers economic recession of -6.6% at the year 16 as illustrated by line 2 in the right diagram. Figure 12.16 shows

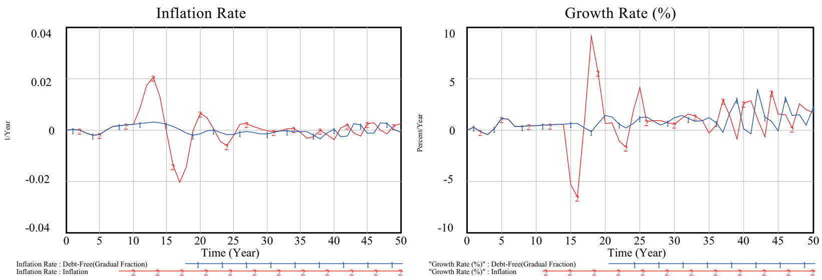

Figure 12.15: Inflation under No GDP Gap

business cycles caused by the mismanagement of the increase in money supply when no GDP gap exists.

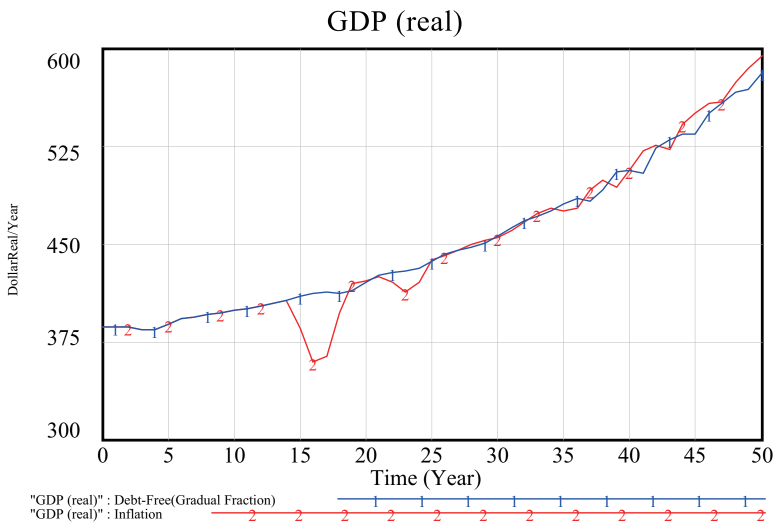

Figure 12.16: Business Cycles caused by Inflation under No GDP Gap

Maximum Tolerable Inflation

This could be a serious moral hazard lying under the system of debt-free money, because the incumbent government tries to cling to the power by unnecessarily stimulating the economy in the years of election as history demonstrates. Business cycle thus spawned is called *political business cycle*. "There is some evidence that such a political business cycle exits in the United States, and the Federal Reserve under the control of Congress or the president might make the cycle even more pronounced [41, p.353]." Proponents of the central bank take advantage of this cycle as an excuse for establishing the independence of the central bank from the intervention by the government. On the contrary, recent financial crises and runaway accumulation of government debt are caused, indeed, by the independence of the central bank under the system of money as debt.

How can we avoid the political business cycle, then, without resorting to the independence of the central bank? As a system dynamics researcher, I suggest an introduction, by law, of a feedback loop of stabilizing monetary value which forces a resignation of the government in case of higher inflation, or at least the head of the Public Money Administration to step down.

How high inflation, then, can we be tolerant of? The American Monetary Act stipulates the maximum interest rate of 8% per year, including all fees.

> Because before 1980/1981, forty nine States had "anti-usury" laws which limited normal interest rates to a maximum of between 6% and 10% p.a. (one state had 12%). The American Monetary Act takes the middle of this range to represent a restoration of the interest limits prevailing across the country prior to 1980/1981 [71, p.27].

From the following relation,

$$\text{Nominal interest rate} = \text{Real interest rate} + \text{Inflation rate} \tag{12.5}$$

we have, for non-negative real interest rate ≥ 0,

$$\text{Maximum Nominal interest rate } (= 8\%) \geq \text{Inflation rate} \tag{12.6}$$

That is to say, the maximum tolerable inflation rate becomes 8% under the system of debt-free money. The success of the system depends on the legalization of a forced step down of the government in case of an inflation rate higher than 8%.

Debt-free Monetary Policy

The role of the public money administration under the macroeconomic system of debt-free money is to maintain the monetary value, similar to the role assigned to the central banks under the system of money as debt. Interest rate is no longer controlled by the public money administration, and left to be determined in the market. History shows that an economic bubble and its burst have been caused by the purposive manipulation of the interest rates such as overnight call rate and federal fund rate by the privately-owned central bank for the benefits of financial elite. In this sense, we will be finally freed from the control of the central bank.

Accordingly, the only tool to stabilize the monetary value is through the public management of the amount of money in circulation. This could be carried out through the control of lending money to commercial banks and through the fiscal policy. Specifically, in case of an inflationary state, lending money to the banks may be curbed, or the money in circulation could be sucked back by raising taxes or cutting government spending. In case of deflation, demand for money by the banks would be weak, so that government has to take a strong leadership by spending more than tax revenues with newly issued money. In this way, complicated monetary policies such as the manipulation of required reserve ratio, discount ratio, and open market operations under the system of money as debt are no longer required.

Finally, it would be worth mentioning that system of debt-free money is ecologically friendly to the environment, because forced payment of interest will be replaced with interest-free money, and borrowers of money, mainly producers, need not be driven into forced economic growth at the cost of environmental destruction. System of debt-free money is indeed a foundation for sustainability.

12.6 Conclusion

This chapter investigates how to liquidate runaway government debt under the current financial crises. First, the current system is identified as a macroeconomic system of money as debt, under which the accumulation of government debt is built into the system by the Keynesian theory, and the reduction of debt-GDP ratio becomes, it is demonstrated, very costly, triggering economic recessions and business cycles.

Then, an alternative system is suggested as the system of debt-free money, in which only the government can issue money, and the government debt, it is shown, can to be gradually eliminated. Moreover, it turns out that higher economic growth is simultaneously attained.

In this sense, the alternative macroeconomic system, from a viewpoint of system design, seems to be worth being implemented if we wish to avoid accumulating government debt, unfair income distribution, repeated financial crises, war and environmental destruction.

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