10 Mathematics for pluralist economics

Steve Keen

Most traditional economics departments have one educational objective in mind: to produce believers in, and academic practitioners of, traditional economics. This "Darwinian" desire to reproduce blinds traditional academics to the reality that the vast majority of their students will never be academic economists. It also explains the relentless emphasis upon forever expanding teaching of technical quantitative subjects (and core subjects like microeconomics) that has contributed to the political economy backlash against mathematics in general.

If, as many of us have, you spend your time fighting against the encroachment of quantitative subjects on teaching slots once allocated to courses in economic history and the history of economic thought, you become reflexively opposed to mathematics whenever more of it is suggested. However, I hope that political economists can be less obsessed with self-reproduction and, perhaps ironically, more responsive to the job market for which most of our students need to be prepared. The vast majority of economics graduates are not going to become academic economists – or even private-sector or government economists – but will use some economic thinking while working in much more broadly defined roles in business, government, and the community sector.

For these students, *and also the minority who go on to actually practice as economists*, the fundamentals of a political economy education should be courses in economic history, the history of economic thought, and political economy – the study of the key literature in political economic thought. These subjects could be reintroduced to the many departments where they are no longer compulsory by eliminating the current who actually want to practice as economists (academic or otherwise), I argue that mathematics and computing courses are necessary. But this sets me against a significant perspective in political economy today, which is to eschew mathematics on the grounds that the use of mathematical methods is the major reason why traditional economics has failed. Tony Lawson, the founder of the influential "Critical Realism" movement within political economy, provides the most eloquent expression of this anti-mathematical view.[1]

Should political economists use mathematics?

Lawson argues against mathematics in economics, on the grounds that "the disarray of modern economics follows because methods of mathematical-deductive modelling are regularly applied in conditions for which they are not appropriate" (Lawson 2005: 428).

I concur with Lawson's critique of the deductive mathematical method as it has been practiced in economics. But Lawson applies this critique too widely. Though traditional economics in particular has used inappropriate mathematical methods, mathematics (and computing) can and should play a role in the development of political economy – albeit a role that is cognizant of the limitations of the mathematical method. It is precisely because traditional economics is undertaken largely in denial of – and sometimes in flagrant breach of – these limitations, that it is in disarray. The flaws in traditional economics arise partially from the use of mathematics, but predominantly from its abuse.

I have detailed the abuses extensively in *Debunking Economics* (Keen 2001). Here, I wish to identify the key ways in which I believe Lawson "doth protest too much" at mathematics, as well as to properly characterize the aspects of the traditional (ab)use of mathematics that Lawson rightly critiques.

Lawson notes that "the mainstream methods presuppose a closed atomistic reality, whereas heterodox conceptions can be shown to be based on a vision of social reality as open, structured, processual, highly internally related, among much else" (Lawson 2005: 435–436). Here we are in complete agreement about both the nature of reality and the inappropriateness of the mathematical methods the mainstream has chosen to use.

However, he later conflates the concept of *equilibrium* – and the traditional insistence on its relevance, versus the predominant opposition to it by political economists – with the mathematical method itself:

> It is equally possible to explain our remaining puzzle, the polarization of attitudes over the relevance of an equilibrium notion. I have already noted that attitudes have tended to divide along mainstream/heterodox lines, with the mainstream, unlike heterodoxy, insisting the equilibrium notion is essential, and with the heterodox opposition becoming increasingly marked over time. We now have before us the resources to understand why. Consider first the mainstream insistence that the notion be retained. The reason for this must now be clear. This mainstream project is defined by its insistence that mathematical methods be everywhere and always employed, despite the dearth of explanatory successes to date.
>
> (Lawson 2005: 435–436)

Lawson thus asserts that the dispute over "the relevance of an equilibrium notion" is the product of the mainstream insistence on mathematical methods, versus a heterodox opposition to them. This implies that mathematical methods and equilibrium are inseparable – but this is not the case.

Lawson's conflation of "equilibrium" with "mathematical methods" is thus, in general, mistaken. However, the initial conflation of "a closed world of isolated atoms" with "insisting the equilibrium notion is essential" is correct: if the former assumption is made, then the equilibrium concept *is* an essential aspect of mathematical reasoning.

But the second conflation of "insisting the equilibrium notion is essential" with the identification of the mainstream project "by its insistence that mathematical methods be everywhere and always employed" is false, if "mathematical methods" may be taken to mean "mathematics in general," rather than "mathematics as traditional economists employ it."

The former indeed appears to be what Lawson implies, since, when conceding that the traditional paradigm is still evolving, he notes as an instance of this evolution the use of complexity theory. He then states that "complexity theory is providing a new way to conceptualise equilibrium states" (Lawson 2006).

This may describe how some misguided traditional economists have attempted to employ complexity theory, but it is easily shown that complexity theory itself is effectively antithetical to the "equilibrium notion" as it is applied in traditional economics. Truly complex systems exhibit far from equilibrium behavior, and therefore cannot be analyzed in terms of their behavior in equilibrium – *because they will never be in equilibrium*.

The first complex systems model, the Lorenz model of convection currents in the atmosphere, provides a simple illustration of this.

The model itself is incredibly simple, with only three variables and three parameters,[2] but can generate very complex behavior – because it has multiple equilibria, all of which are unstable for some parameter values. Additionally, it is possible to alter some quantitative aspect of the model and drastically alter the system's tie path, without changing the system's equilibria at all.

Figure 10.1 shows two plots of the convection intensity variable ????? in the model for two different values of its relevant parameter ?????. The value of ????? does not determine any of ????? equilibria, but changes in ????? drastically alter the time path of the weather. An "equilibrium analysis" of the Lorenz model of the weather would then assert that convection intensity has no impact – which is manifestly false.

Figure 10.1 also illustrates one of the key reasons why standard statistical methods fail with complex systems – because of, amongst many other factors, the phenomenon of "sensitivity to initial conditions." Two initial conditions that differ infinitesimally result in time paths that, after a finite time, diverge completely. The possibilities for quantitative prediction are thus extremely limited for complex systems – though there are nonlinear techniques that provide a limited capacity to forecast – and modelers tend to focus on reproducing the qualitative aspects of empirical data rather than the tight fit to the empirical record that is the unattainable Holy Grail of conventional econometrics.

Finally, though the equilibria of the model can be helpful in analytically characterizing the model's *qualitative* behavior,[3] the system itself will never be in any of its equilibria – and neither the historic time average, nor any of the
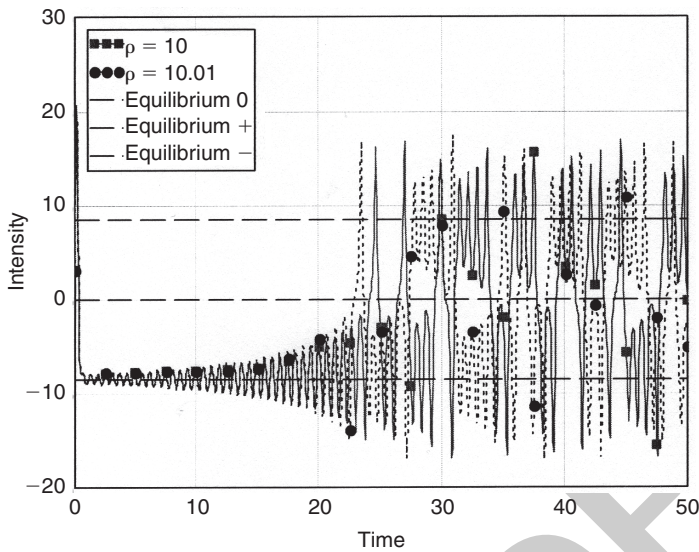
Figure 10.1 The irrelevance of equilibrium for complex systems.

equilibria, will have any capacity to predict the system's future path. These characteristics of this specific example are found in many mathematical models of complex systems.

It can also be shown that another of Lawson's conflations, of mathematical reasoning with atomism,[4] is not a characterization of mathematical methods in general, but of the traditional economics usage of inappropriate mathematics. Lawson notes that,

> The social realm is also highly interconnected and organic. Fundamental here is the prevalence of internal social relations. Relations are said to be internal when the relata are what they are and/or can do what they do, just in virtue of the relation to each other in which they stand. Obvious examples are relations holding between employer and employee … you cannot have the one without the other; each is constituted through its relation to the other. In fact, in the social realm it is found that it is social positions that are significantly internally related.
>
> (Lawson 2006: 495–496)

This is true, but it does not preclude appropriate nonlinear dynamic mathematical modeling – and that modeling can provide insights into such relations that cannot be gained by verbal logic alone. A classic instance of an organic social relation between employers and employees in economic literature is Marx's description of a cycle in employment and income distribution in Section 1 of Chapter 25 of *Capital*:

accumulation slackens in consequence of the rise in the price of labor, because the stimulus of gain is blunted. The rate of accumulation lessens; but with its lessening, the primary cause of that lessening vanishes, i.e., the disproportion between capital and exploitable labor-power. The mechanism of the process of capitalist production removes the very obstacles that it temporarily creates. The price of labor falls again to a level corresponding with the needs of the self-expansion of capital, whether the level be below, the same as, or above the one which was normal before the rise of wages took place.... To put it mathematically: the rate of accumulation is the independent, not the dependent, variable; the rate of wages, the dependent, not the independent, variable.

(Marx 1867: 580)

Richard Goodwin realized that this argument was akin to the predator–prey population dynamic models developed by the biologist Alfred Lotka and mathematician Vito Volterra in the 1920s, and in 1967 he rendered it as a pair of coupled ordinary differential equations (ODEs) (Goodwin 1967). The model generates the same perpetual cycle Marx describes, because of precisely the kind of internal social relations emphasized by Lawson: workers' share of output depends on the investment decisions of capitalists, and the investment decisions of capitalists depend on the wage demands of workers.
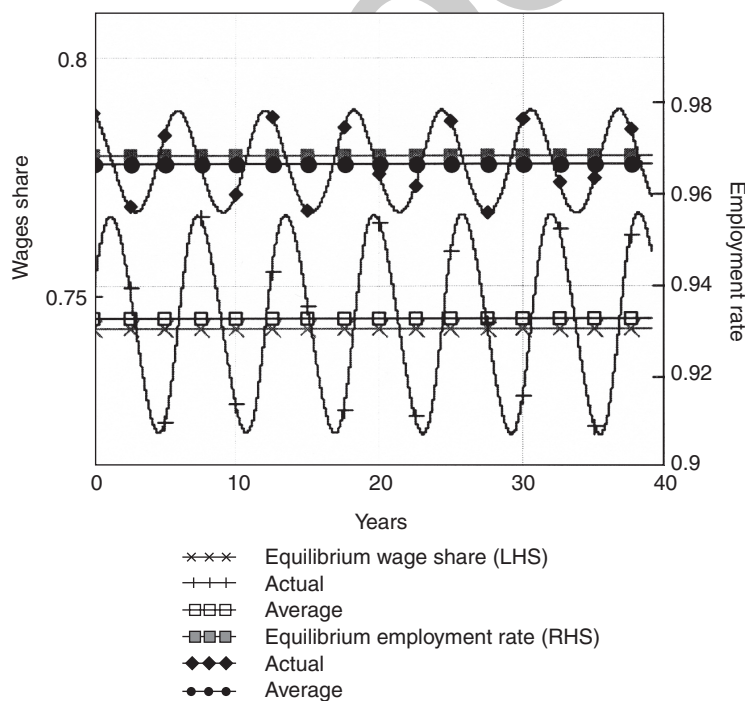

Figure 10.2 The time path, average and equilibrium of Goodwin's model.

With the nonlinear investment and wage demand functions used here, the model also illustrates another feature common to nonlinear dynamic systems: *the equilibrium and the average of the system do not coincide*. The equilibrium of the model could not therefore be used to predict even the system's average value over time, let alone its time path, which keeps it continually in *dis*equilibrium. With an extension to include finance, the model can, given suitable initial conditions, have neither an equilibrium, nor an average – but it remains a valid mathematical model (Keen 1995).[5]

The Lorenz and Goodwin models are just two examples of a whole class of mathematical models that are non-atomistic, non-ergodic,[6] non-equilibrium, structured, and internally related.

The one aspect of economic reality that mathematical methods cannot capture, according to Lawson, is openness. Mathematical models as such cannot capture the phenomenon of truly new entities appearing in a system, such as the development of new commodities or technologies in a productive system, or the development of new economic institutions over time. A model of production that explicitly includes all existing products, for example, can't suddenly sprout a new equation to cope with the development of a new industry.

Computer-based methods of modeling can cope with this to some extent – see, for example, Standish's Ecolab modeling environment, which models evolution as an open-dimensional process (so that new species and new phenotypic characteristics can appear over time) (Standish 2000), but this type of work is still tentative and likely to remain so for a substantial period.

There are, however, circumstances in which closure is appropriate. Modeling capitalism as if it always consists of only "n" commodities – a common technique in neo-Ricardian as well as traditional economics neoclassical practice – can be a denial of the open nature of capitalism (and be atomistic to boot, when linear equilibrium techniques are used to analyze the model). However, working at a level of aggregation at which there are a fixed number of sectors – such as worker consumption, investment, capitalist consumption – or a fixed classification of social classes – worker, capitalist, rentier – may be valid for some models of economic reality. Closure in this sense results from functional classification rather than a denial of the innate nature of social reality – and even verbal methods frequently force us to classify open processes into closed categories.

There is a final reason why heterodox economics needs mathematical methods: sometimes verbal logic alone fails us. The long-running controversy in Circuitist literature about whether capitalists in the aggregate can make profits is a classic instance of this.

The Circuit School and the need for mathematics

The Circuit School is a largely European group of heterodox economists, inspired by both Keynes and Marx, whose ambition was to provide a truly monetary model of a production economy. They developed a compelling explanation of why a monetary economy is fundamentally different to the barter model of

traditional economics, but then struggled to turn this explanation into a viable model of an economy.

Working from first principles, Graziani argued that a monetary economy must be using a token for money, and that the only way this could happen without seignorage was if "any monetary payment must therefore be a triangular transaction, involving at least three agents, the payer, the payee, and the bank" (Graziani 1989: 3). He and subsequent authors then tried to model the monetary circuit verbally – and occasionally mathematically, using the inappropriate tool of simultaneous equations – by tracing the process from the initial creation of credit (where Graziani assumed that initial credit requirements were equal to the wage bill (Graziani 1989: 4) to the alleged eventual destruction of money when the debt was repaid (Graziani 1989: 5).

Several conundrums arose. Circuitist economists concluded that firms were unable even to pay interest on debt, let alone make a profit – "Money will never be available for the payment of interest" (Graziani 1989: 17). Savings by workers implies that firms could not even repay the principal of loans in full, so that workers' savings forced firms to take on ever-higher debt – if wage-earners save, "the money stock and debt of firms becomes increasingly higher and higher" (Graziani 1989: 19).

Gallingly, though their ambition was to build a purely monetary model of the economy, it also appeared that barter could not be avoided, since it seemed that was how interest was paid – "In substance, what has taken place is a barter, firms having paid interest in kind" (Graziani 1989: 18).

All this arose from an attempt to, amongst other things, provide a monetary expression of Marx's circuits analysis of capitalism, when his logic was predicated on the concept that production generated a surplus. How could there be a physical surplus, but no monetary profits? Yet this seemed to be the conclusions of the Circuit approach. This implicit conflict between the Circuitist School's inspiration and its results led to the following lament,

> The existence of monetary profits at the macroeconomic level has always been a conundrum for theoreticians of the monetary circuit: not only are firms unable to create profits, they also cannot raise sufficient funds to cover the payment of interest. In other words, how can $_M$ become M^+.
>
> (Rochon 2005: 125)

In fact, all these conclusions from verbal argument are wrong: firms *can* pay interest and make a profit, savings by workers *do not* compromise firms' solvency, rising debt is *not* necessary to sustain constant economic activity, and interest *is* repaid, as it should be, in money rather than in kind. The Circuitist School is thus consistent with Marx, and does explain how a physical surplus is monetized – but reaching these correct results requires the application of the correct mathematical logic. As the conundrums of the Circuit make abundantly clear, verbal intuition frequently gets lost in the complexities of flows of a verbal model of a market process.

However, these are easily kept track of using the correct mathematical tool to analyze the dynamics of monetary flows: ordinary differential equations.[7] If we consider the simplest possible case of a stationary pure credit economy, financed by an initial loan from the banking sector to the firm sector of \$L, then the following operations apply:

1 The bank pays the firm interest at the rate r_D on the deposit account F that is created simultaneously with the loan and therefore starts with \$L in it.
2 The firm pays the bank the interest charged on its debt at the rate r_L, making a transfer from its deposit account F to the bank's account B.[8]
3 The firm then hires workers, making a transfer from its deposit account F to the workers' deposit account W at some rate (w) proportional to the current balance in the F account.
4 The banking sector pays the workers interest on their savings, making a transfer from its account B to the workers account W.
5 Bankers and workers then buy the products produced by the firm sector, making transfers from their accounts to the firm sector's deposit account at a rate proportional to the current balances in their accounts (respectively β and ω).

The equations of this system are easily derived by placing the above flows in a table akin to the double-entry book-keeping of accountancy.

The three equations of the system can then be written by simply adding up the entries in each column:

$$\frac{d}{dt}F = r_D \cdot F - r_L \cdot L - w \cdot F + \beta \cdot B + \omega \cdot W$$

$$\frac{d}{dt}B = -r_D \cdot F + r_L \cdot L - r_D \cdot W - \beta \cdot B \qquad (10.1)$$

$$\frac{d}{dt}W = +w \cdot F + r_D \cdot W - \omega \cdot W$$

With realistic values for its parameters, this model generates positive profits that are well in excess of interest payments, as well as wages and interest income for the two other classes (see the Appendix for the derivation of profits). Constant output and income levels can be sustained indefinitely without additional

Table 10.1 Account flow dynamics in the simplest Circuitist model

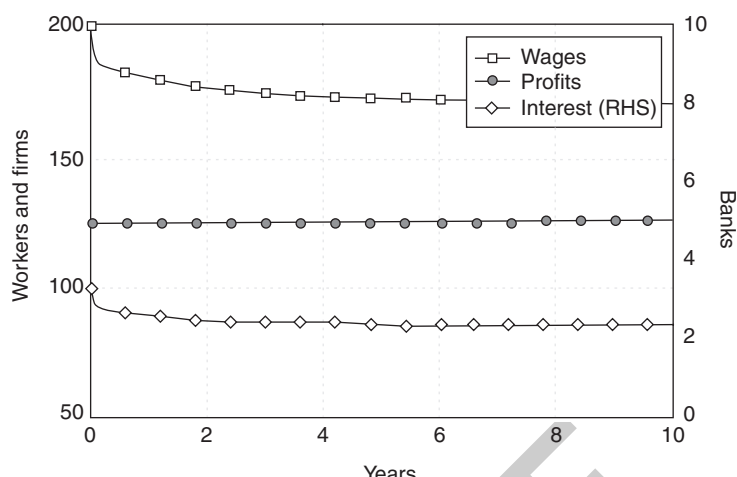| Accounts activity	Firm (F)	Bank (B)	Workers (W)
Interest payment on deposit	$+r_D \cdot F$	$-r_D \cdot F$	
Interest repayment on loan	$-r_L \cdot L$	$-r_L \cdot L$	
Wages	$-w \cdot F$		$+w \cdot F$
Interest payment on workers' account		$-r_D \cdot W$	$+r_D \cdot W$
Consumption by banks and workers	$+\beta \cdot B + \omega \cdot W$	$-\beta \cdot B$	$+ -\omega \cdot W$

Figure 10.3 Incomes in the simple circuit model.

injections, and positive bank balances for workers don't force losses on capitalists.

All the conundrums of the Circuitist debate were therefore simply the result of verbal logic making it difficult to differentiate between stocks and flows. Perhaps also the mental difficulties of keeping verbal track of a dollar's circulation caused Circuit authors to forget Marx's fundamental insights about a turnover period between investing and receiving (and also the generation of a surplus).

There are thus good reasons why a heterodox education in economics should include mathematics in its curriculum – but as noted, this should be limited for the general cohort of students who do not plan to become professional economists. For these students, I would require only one mathematically oriented course in analytic methods – a subject that introduces modern approaches to dynamic modeling, and also, as an addendum, provides a warts-and-all critique of the equilibrium methodology that dominates neoclassical model building.

Analytic methods for heterodox economics

Far, far away from the obsession with equilibrium that dominates mathematical methods in economics, mathematicians, computer programmers, and engineers have developed a range of software programs that simplify the processes of developing and exploring models of complex physical, social, and biological processes. They fall into three main classes.

Direct entry and numerical simulation of dynamic models

Programs like *Mathcad* (www.ptc.com/products/mathcad/), *Mathematica* (www.wolfram.com/), and *Scientific Workplace* (www.mackichan.com/) give a user-friendly interface to these equations, which can be entered and run using standard mathematical notation (other mathematical programming environments like *Matlab*, *Scilab*, etc., have routines for the numerical simulation of systems of differential equations, but provide only awkward text-only interfaces for writing the equations).

Flowchart models of system dynamics

There is a plethora of commercial programs now, including *Simulink* (a component of *Matlab* – www.mathworks.com/), *Vensim* (www.vensim.com/), *Vissim* (www.vissol.com/), *Stella*, and *iThink* (www.iseesystems.com/), as well as free software implementations such as the system dynamics component of *NetLogo*, and *Scicos* (www.scicos.org/), a component of *Scilab* (www.scilab.org/), that lets you model dynamic processes using causal flowcharts. These models can often be built by tracing out a causal chain as one does in a standard flowchart diagram, and then filling in the mathematical links later.

Multi-agent simulations of complex interactions between heterogeneous populations

The most accessible of these – meaning the one that involves the smallest computer-programming learning curve – is *NetLogo* (http://ccl.northwestern.edu/netlogo/). It is also free, and contains a systems dynamics subset as well as a multi-agent environment.

This subject would illustrate dynamic modeling in economics using all these methodologies, with examples of their application to the economic literature and data encountered in the foundation subjects of heterodox economics. Here I will indicate what each of these look like using a Lotka-Volterra predator–prey model which, as noted above, is the class of model to which the Marx–Goodwin growth cycle model belongs.

As a pair of differential equations, the model is:

$$\frac{d}{dt} S = a \cdot S - b \cdot S \cdot W$$

$$\frac{d}{dt} W = -c \cdot W + d \cdot S \cdot W \tag{10.2}$$

where S is the number of prey (sheep) and W the number of predators (wolves), and a, b, c, d are positive constants. Implemented in *Mathcad* – one of the direct entry class of programs – the model is as shown in Figure 10.4. The equations are written exactly as they would be on paper, and simulated by the Odesolve function for different parameter values and initial conditions (S_0 and W_0 respectively).

Given $\dfrac{d}{dt} S(t) = a \cdot S(t) - b \cdot S(t) \cdot W(t)$ $S(0) = S_0$

$\dfrac{d}{dt} W(t) = -c \cdot W(t) + d \cdot S(t) \cdot W(t)$ $W(0) = W_0$

$\text{PredPrey}(a, b, c, d, S_0, W_0) := \text{Odesolve}\left[\begin{pmatrix} S \\ W \end{pmatrix}, t, 100\right]$

$\begin{pmatrix} S_e \\ W_e \end{pmatrix} := \text{PredPrey}\left(2, \dfrac{1}{10}, 1, \dfrac{1}{300}, 300, 20\right)$

$\begin{pmatrix} S \\ W \end{pmatrix} := \text{PredPrey}\left(2, \dfrac{1}{10}, 1, \dfrac{1}{300}, 290, 25\right)$

A flowchart representation of exactly the same system is shown in Figure 10.4, using the simulation program *Vissim*. The mathematical operators are now shown graphically, and in most instances of this class of software, the model is simulated dynamically on screen

Finally, a multi-agent Netlogo version of the model is shown in Figure 10.5. Here, as opposed to equations, the system is built by defining the behavior of the entities that comprise it – sheep, wolves, and, in this version, the grass that the
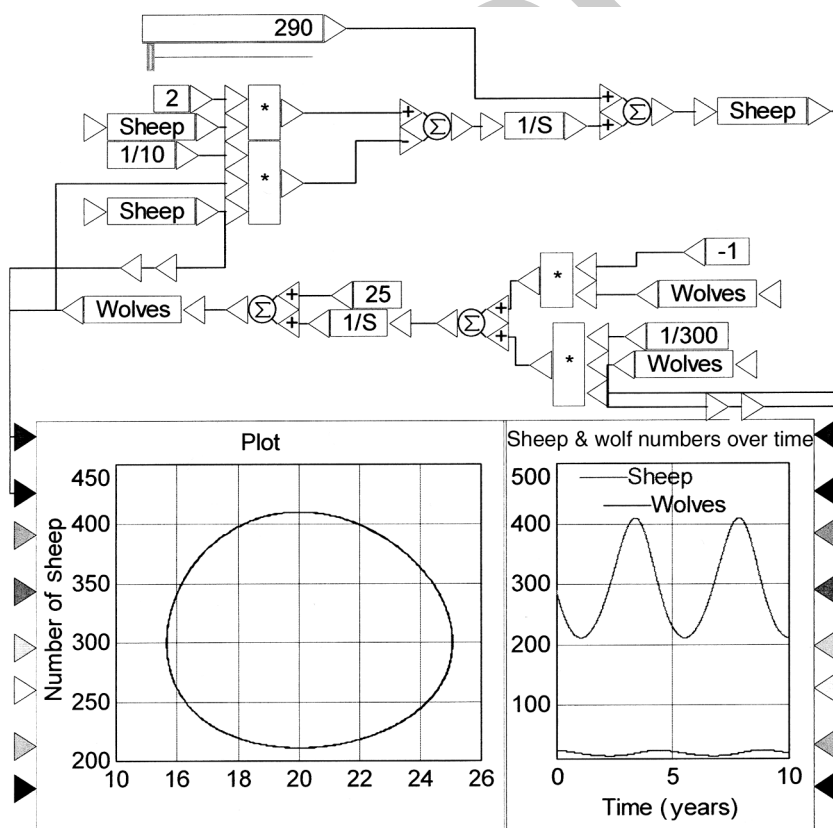

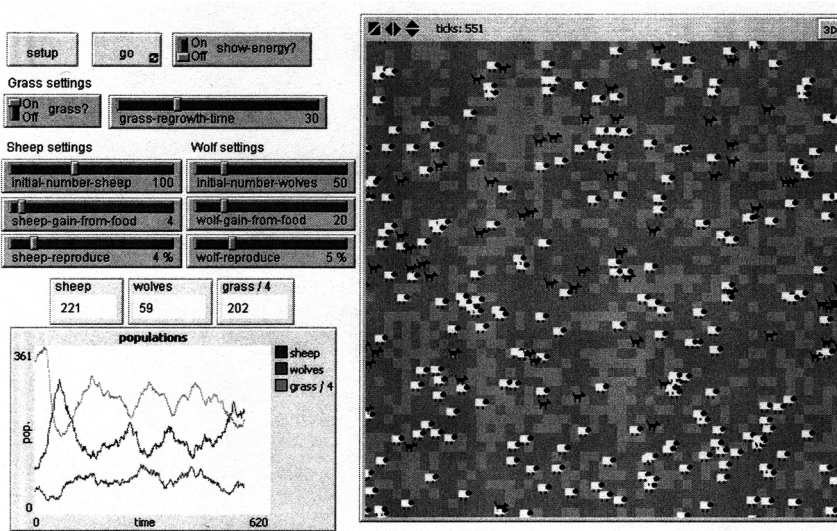*Figure 10.4* The predator–prey model implemented as a dynamic flowchart.

Figure 10.5 NetLogo multi-agent simulation of predator–prey interactions.

sheep themselves eat. Rather than modeling an entire population from the top down, the multi-agent approach works from the bottom up, and requires computer programming to instruct individual agents how to behave. A simulation is then run, in which large numbers of agents interact.

An overhead of learning how to program is unavoidable, but NetLogo drastically reduces this overhead when compared to any other programming environment. For example, the code that specifies wolf behavior in the above model is:

```
ask wolves [
move
set energy energy – 1; wolves lose energy as they move
catch-sheep
reproduce-wolves
death
]
```

The module catch-sheep is:

```
to catch-sheep;; wolf procedure
let prey one-of sheep-here;; grab a random sheep
if prey != nobody;; did we get one? if so,
[ ask prey [ die ];; kill it
set energy energy + wolf-gain-from-food ];; get energy from eating
end
```

This is extremely minimal – and intelligible – computer code, with operations like working out whether a wolf is on the same patch as a sheep (and can therefore catch one) simplified by the keywords "one of" and "here," that are built-in functions in NetLogo.

The question arises of who would teach such a course. Ideally, political economists themselves would already be equipped to do so but, with few exceptions, we are currently trapped between the bad training in inappropriate mathematical methods we received as students on the one hand, and a lack of exposure to modern mathematical and computer methods on the other. The initial solution would therefore be to recruit graduates from systems engineering schools – and there are many of these around the world, because modeling unstable dynamic processes is a bread-and-butter exercise in engineering these days. A list of institutions teaching systems engineering is maintained by the International Council on Systems Engineering.[9] This subject would also be undertaken by students intending to become professional economists, who therefore choose to do an economics degree with a larger mathematical component.

Mathematics for heterodox economics

The objective of the common modeling subject would be to enable those who do not intend to work as economists (whether in academia or industry) to understand dynamic economic models and how they are constructed, and to enable those who do plan to be professional economists to know how to design – and understand the limitations of – such models. This second professional cohort must have the deeper understanding of these methods that comes from doing specialized mathematics.

The essential topic here is differential equations – the study of processes of change which are described by equations of the form:

$$\frac{d}{dt} y - f(y) \tag{10.3}$$

This is not the simple calculus of optimization that forms the basis of the math courses traditional economists inflict on their students, which are based on equations of the form:

$$\frac{d}{dx} y - f(x) \tag{10.4}$$

For those who are not familiar with differential equations, the key differences between equations (10.3) and (10.4) are:

1   In (10.4), the rate of change of the dependent variable y is a function simply of the independent variable x; in (10.3), it is a function both of the independent variable t, *and its own value*.
2   The independent variable x in (10.4) is normally some other economic variable (L in a production function relating output Y as a function of labor input

L, for example); in (10.3), the ultimate independent variable is time itself, and therefore signified by *t*.

These two superficially minor differences make a world of difference to the practice of mathematics. First, whereas almost all equations like (10.4) can be solved, almost none like (10.3) can be. Second, and most important, these equations are designed to analyze processes when equilibrium does *not* apply. Conceptually, these equations are easily extended to include multiple variables (see the examples in the Appendices), and when systems have more than two variables and nonlinear relations, sustained far-from-equilibrium behavior becomes the norm. Prediction – especially by econometric techniques that assume linearity – then becomes impossible. The objective of modeling then switches from prediction to qualitative description of the model's behavior.[10]

In order to properly comprehend differential equations, foundation courses in calculus and linear algebra are necessary.

All three subjects should be taught by mathematics departments, and not by economists.

Economists, even mathematically savvy heterodox economists, lack the capacity to deliver these topics properly to students – in part simply because they are economists, not mathematicians, and are therefore isolated from the mainstream of developments in mathematics and modeling in the sciences and engineering. Many of the travesties in the use of mathematics in economics have resulted from precisely this isolation, and the self-referential way in which mathematical economics has evolved. We need to delegate the task of teaching fundamental mathematics to professional mathematicians, so that mathematics in economics can become what it is in other sciences and engineering – a good servant, rather than the poor master it has been in economics to date.

Practicing political economists may also need an introduction to differential equations – if only to realize that there are other ways to practice mathematical analysis than the econometrics they suffered as undergraduates. Here the most readable text I have encountered is Martin Braun's *Differential Equations and Their Applications: An Introduction to Applied Mathematics* (1992). A description given of it by a customer on Amazon.com captures its appeal very well:

> I have used this to teach DE's in a one to one tutoring context for a couple of years now, since I first picked it off a library shelf and felt literally like jumping for joy over how good it was. Not that I dislike Boyce/DiPrima, but suddenly I had a text that was really fun! Now, this is definitely a "what DE's are good for" kind of text, so if you feel that your students should suffer teething pains on a dry theoretical tome, this is NOT the book for you. Nevertheless, having had students chew up all their available time on this book, because they loved it, makes me recommend it highly. (Check www.amazon.com/Differential-Equations-Their-Applications-Introduction/dp/0387978941/ref=sr_1_1?ie=UTF8&qid=1226288501&sr=1–1 for more details.)

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Conclusion: of babies and bathwater

There is no doubt that mathematical economics as it has been practiced by neo-classical economics is a large part of why the neoclassical method has failed. But to eschew mathematical analysis in the development of heterodox economics as a result would be a classic case of throwing the baby out with the bathwater. Mathematics and computing will play important roles in the evolution of heterodox approaches to economics into a realism-based economics of the future.

Appendix I: Lorenz's model

The equations of Lorenz's model are:

$$\frac{d}{dt} x = \rho \cdot (y - x)$$
$$\frac{d}{dt} y = x \cdot (\tau - z) - y \qquad (10.5)$$
$$\frac{d}{dt} z = x \cdot y - \beta \cdot y$$

x, y, and z are variables (respectively the convection intensity, the temperature difference between rising and falling currents, and the degree to which the vertical temperature profile of the air cell differs from linearity) while a, b, and c are parameters. The equilibria of the model, all of which are unstable for the parameter values used in this chapter, are:

$$\begin{matrix} x_e \\ y_e \\ z_e \end{matrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sqrt{\beta \cdot (\tau - 1)} \\ \sqrt{\beta \cdot (\tau - 1)}, \\ (\tau - 1) \end{bmatrix}, \begin{bmatrix} -\sqrt{\beta \cdot (\tau - 1)} \\ -\sqrt{\beta \cdot (\tau - 1)} \\ (\tau - 1) \end{bmatrix} \qquad (10.6)$$

Notice that none of the equilibria depend on the value of the parameter ????? – and therefore they do not change when ????? changes. Yet changing ????? has a dramatic effect upon the time path of the system.
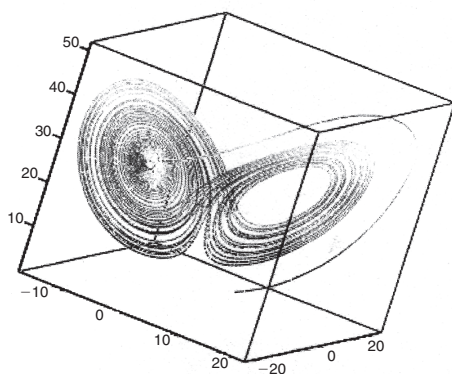

Figure 10A.1 3D map of the Lorenz position.

Goodwin's model

The equations of Goodwin's model are:

$$\frac{d}{dt}\,\omega = \omega \cdot (P[\lambda] - a)$$
$$\frac{d}{dt}\,\lambda = \lambda \cdot \left(\frac{\Pi\left[\frac{1-\omega}{v}\right]}{v} - y - \alpha - \beta \right) \tag{10.7}$$

where ????? is the workers' share of output, ????? is the employment rate, ????? is a relationship between the rate of change of wages and the employment rate, ????? is an investment function, v is the accelerator, ????? is the rate of productivity growth, ????? the rate of population growth, and ????? is depreciation.

The empirical fit of the Goodwin model was somewhat criticized in Harvie (2000). In fact there was a simple schoolboy error in the econometrics (Harvie's words in personal correspondence) of not converting percentages into decimal points. With this error corrected, the empirical fit of the model to OECD data was quite good – and it improves further with a nonlinear Phillips curve and investment function, as used in the simulation shown in Figure 10.2.

Circuitist model

The differential equations of the Circuitist model are the sum of the columns of Table 10.1. F, B, and W are the deposit accounts for firms, banks, and workers respectively, while L is the initial loan; r_D is the rate of interest on deposits, r_L the rate on loans; w is a parameter indicating the rate at which wages are paid to workers (this can be shown to be equivalent to the share of surplus from production that goes to workers as wages ?????, divided by the time delay ????? measured in years between the outlay of M to finance production, and the receipt of ????? – Marx's concept of a turnover period); ????? and ????? are consumption rate parameters for bankers and workers respectively.

With parameter values of ?????, ?????, ?????, ?????, ????? and ?????, the model generates the dynamics for wages, profits and gross interest income shown in Figure 10.3.

Notes

1 For those unfamiliar with Tony Lawson and Critical Realism, the latter philosophy argues that while there is an independent, objective reality, our sensory perceptions of it includes components that do not necessarily represent objective entities. Lawson and others apply this to economics, in particular to the constructs of econometrics and mathematical deductivism. The Critical Realist movement has become influential amongst political economists, though it is far from universally accepted.

2 The variables are the intensity of convection in a weather cell, the temperature difference between ascending and descending currents, and the degree of curvature of the temperature gradient across the cell.

3 The stability properties of the equilibria can be characterized using eigenvalue analysis, and the probable qualitative nature of orbits can be implied for different parameter combinations.

4 Reliance on methods of mathematical deductive modeling more or less necessitates a focus on conceptions of atomistic individuals and closure (Lawson 2004: 334).

5 This is an instance of the inverse tangent route to chaos (Schuster and Just 2006: 69–88). Such systems have no equilibrium for some initial conditions, but equilibria for others. In general, they have to be analyzed using non-equilibrium methods.

6 There has been considerable confusion over the term ergodic in the political economy literature. An intelligible, accurate definition of ergodicity is given in Wikipedia: all accessible microstates are equally probable over a long period of time (WikiErgodicHypothesis). On this definition, the Lorenz system and comparable systems are *non-ergodic* – since there is probability zero that the system will enter the area around the equilibria (and other regions in the feasible phase space). See Figure 10.A1 on p. 000.

7 This is why the attempts by several Circuitist writers, including Graziani, to apply mathematical logic to the monetary circuit failed: they used the inappropriate method of solving simultaneous equations – the comparative static method that infests economics – rather than the more appropriate method of differential equations illustrated in this chapter. Comparative static methods failed because they abstract from flows, when income is all about flows: GDP is a flow of output over time, for instance, as is the demand that is used to purchase output – even when it is financed by borrowed money.

8 I have omitted the loan account here, since I am not considering debt repayment.

9 See www.incose.org/educationcareers/academicprogramdirectory.aspx. One omission from that list is the Norwegian Institute of Technology (www.itk.ntnu.no).

10 There are techniques, such as the Levenberg-Marquardt algorithm, that can estimate parameters that enable a nonlinear model to empirically fit a dataset, but even here the emphasis is upon description rather than prediction.

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Wikipedia "Ergodic Hypothesis," http://en.wikipedia.org/wiki/Ergodic_hypothesis.